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Intro to Algorithm Complexity

Important function in C.S.: worst case time complexity of an algorithm

$T: \mathbb{N} \rightarrow \mathbb{N}$

↑ input size

↑ # of operations performed by algorithm in worst case.

unless parallel computing, this tells you the time the computer will take to run the algorithm. Just multiply by time to do 1 operation

Linear Search

- Input: $(a_1, a_2, \ldots a_n)$, x ⟸ Input size is n
- Output: j if $a_j = x$, 0 otherwise

1) $i = 1$
2) while ($i \leq n$ and $x \neq a_i$)
3) $i = i+1$
4) if $i \leq n$: return i
5) else: return 0

Q: What is $T(n)$ for linear search? (Hint: n is not correct)

Report by

By group:

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Issues:

- too fine-grained / detailed
  - different computers might do operations differently
  - when n gets large, don't care about 100000 vs 100001
- too difficult to count every operation

Big-O to Rescue!

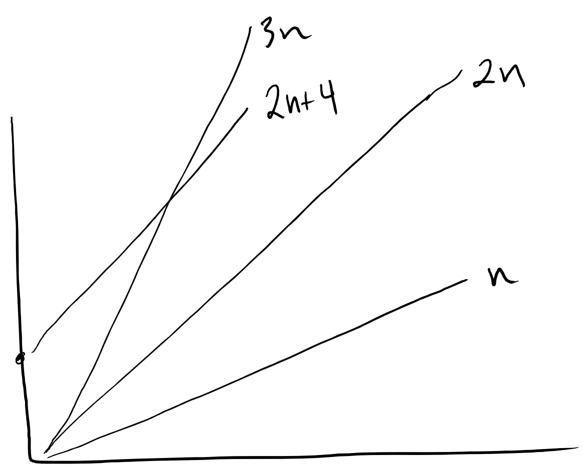
↑ special notation to describe how functions grow

<u>def</u>: Let $f, g: \mathbb{Z} \to \mathbb{R}$. Then $f(x)$ is $O(g(x))$ if there exist constants $k \in \mathbb{Z}$ and $C \in \mathbb{R}$ such that when $x \geq k$, then

$$|f(x)| \leq C|g(x)|$$

← We will always consider positive functions, so can remove abs. value.

ex: $2n+4$ is $O(n)$



Let's choose $C=3$.

When cross? $3n = 2n+4$
$n = 4$

Proof:
When $n \geq 4$, we have

$2n+4 \leq 2n+n = 3n$, so

$2n+4 = O(n)$ with $k=4$, $C=3$

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Q: Prove other functions for linear search # of operations is $O(n)$

Starting to see why big-O is good for algorithm time complexity:

- Small differences in how you calculate don't matter
- Not too fine grained

However big-O is only upper bound:

ex: $7x+1$ is $O(x^2)$

Pf: $7x+1 \le 7x+x$ for all $x \ge 1$

$7x+x = 8x \le x^2$ for all $x \ge 8$

Thus with $k=8$, $C=1$, $7x+1 = O(x^2)$

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Q: Prove $10x^2$ is not $O(x)$. This means There do not exist constants k, C, such that $10x^2 \leq Cx$ $\forall x \geq k$.

Pf: For contradiction, assume k, C exist. Then $\forall x \geq k$, we have

$$10x^2 \leq Cx$$

When $x > 0$, we have $x \leq \frac{C}{10}$. Thus, this inequality holds only when $0 < x \leq \frac{C}{10}$, which contradicts that it should hold for all $x \geq k$.