1. Show that there is a language $B \in \text{EXP}$ such that $\text{NP}^B \neq P^B$.

2. The class $\text{ZPP}$ (zero-error probabilistic polynomial time) is another variant on $\text{BPP}$:

   **Definition.** $L \in \text{ZPP}$ if there exists a probabilistic TM (PTM) $M$ such that if
   
   \begin{align*}
   x \in L &\iff \Pr[(M(x) = 1)] = 1 \tag{1} \\
   x \notin L &\iff \Pr[(M(x) = 1)] = 0 \tag{2}
   \end{align*}

   and for all $x$, $M(x)$ terminates in polynomial time *on average*.

   The idea with $\text{ZPP}$ is that it always outputs the right answer, and usually it takes
   polynomial time, but it can sometimes take much longer. However, the likelihood of it taking
   a long time is small.

   Another way of defining $\text{ZPP}$, which we’ll call $\text{ZPP}_2$ is as follows:

   **Definition.** $L \in \text{ZPP}$ if there exists a probabilistic TM (PTM) $M$ that can output the
   symbols $\{0, 1, ?\}$, where if
   
   \begin{align*}
   x \in L &\rightarrow M(x) \in \{1, ?\} \tag{3} \\
   x \notin L &\rightarrow M(x) \in \{0, ?\} \tag{4}
   \end{align*}

   and for all $x$, the probability that $M(x)$ outputs ‘?’ is less than $1/2$, and $M$ runs in polynomial
   time.

   (a) Prove that $\text{ZPP} = \text{ZPP}_2$
   (b) Explain the significance of part (a).
   (c) Prove $\text{ZPP} \in \text{RP} \cap \text{coRP}$. Note that $\text{RP}$ always terminates in polynomial time.