1. What class ($\Sigma_2$ or $\Pi_2$) is the following language in:

$$L = \{ \phi : \text{there is exactly one satisfying assignment to the Boolean formula } \phi \}. \quad (1)$$

2. The class $\text{DP}$ is the set of languages $L$ for which there exist two languages $L_1 \in \text{NP}$ and $L_2 \in \text{coNP}$ such that $L = L_1 \cap L_2$. Let

$$\text{EXACT INDSET} = \{ \langle G, k \rangle : \text{the largest set of vertices where no vertex in the set has an edge to any other vertex in the set has size } k \}. \quad (2)$$

Prove

(a) $\text{EXACT INDSET} \in \Pi_2^p$
(b) $\text{EXACT INDSET} \in \text{DP}$
(c) Prove $\text{DP} \subseteq \Pi_2^p$.

3. In class, to prove that $\text{BPP} \in \Sigma_2 \cap \Pi_2$, we only prove that $\text{BPP} \in \Sigma_2$. We said that this implies the main result because $\text{BPP} = \text{coBPP}$, which you prove in Quiz 9. Use the facts that $\text{BPP} = \text{coBPP}$ and $\text{BPP} \subseteq \Sigma_2$ to prove $\text{BPP} \subseteq \Sigma_2 \cap \Pi_2$.

4. Prove that if $3\text{SAT} \leq_p 3\overline{\text{SAT}}$, then $\text{PH} = \text{NP}$. (The $\text{PH}$ collapses to $\text{NP}$).