

**Syllabary**

- Separations
- Thursday, April 7, 2022
- P ≠ EXP

**Tool:** Universal TM $U \mapsto \bar{U}$ simulates any TM differently.

**Proof:**
- Let $x \in \{0, 1\}^*$, $a$ represents a TM $M_a$.
- $U$ is a TM, hence an infinitely many $a$.
- $M_a$ inputs $x$.
- $U$ takes as input $\langle x, a \rangle$ and $U(x, a) = U(a)(x)$.
- $M_a$ is a constant $c$.
- $M_a(x)$ takes time $c$.
- Thus $U(x,a)$ takes time $c$.

**Algorithm:** $\text{TIME}(c) \subseteq \text{TIME}(n^c)$

**Idea:** Create $L$ s.t. $L \in \text{TIME}(n^c)$ but not $L \notin \text{TIME}(c)$.

**Step 1:** Create $L$.

$f(x) = \begin{cases} 1 & \text{if } U(a)(x) \text{ does not terminate in } |x|^c \text{ steps, or reject in } |x|^c \text{ steps} \\ 0 & \text{if } U(a)(x) \text{ accepts in } |x|^c \text{ steps} \end{cases}$

$L = \{ x : f(x) = 0 \}$ or $L = \{ x : f(x) = 1 \}$.

**Step 2:** $L \notin \text{TIME}(c)$.

- Run $U(x,a)$ for $|x|^c$ steps.
- Accept if $U(x,a)$ has not terminated, or reject if $U(x,a)$ accepts.

**Step 3:** $L \notin \text{TIME}(c)$.

- For contradiction, assume $L \in \text{TIME}(c)$.
- Then $\exists$ a TM $M$ s.t. $M(a)$ accepts $\langle x, a \rangle$ and $M(a)$ always terminates in $|x|^c$ steps for some constant $c$.

- This means $\forall x \in \{0, 1\}^*$, $U(x,a)$ terminates in $c \cdot |x|^c$ steps.

**Case 1:** $a \in L$.

- By def of $L$, $f(x, a^x)^x = 0$.
- By def of $f$, $U(x, a^x)$ doesn't terminate in $|x|^c$ steps, or rejects.
- By $a \in L$, $U(x, a^x)$ terminates in less than $|x|^c$ steps, so $U(x, a^x)$ must reject in at most $|x|^c$ steps.
- By def, $U(x, a^x)$ is $M_{\bar{c}}(x)$.

**Case 2:** $a \notin L$.

- By def of $L$, $f(x, a^x)^x = 1$.
- By def of $f$, $U(x, a^x)$ accepts in $|x|^c$ steps.
- By def, $U(x, a^x)$ is $M_{\bar{c}}(x)$.

Thus, $U(x, a^x)$ is $M_{\bar{c}}(x)$.

**Theorem:** $P = \text{NP}$.

**Proof:**
- Since $U(x, a^x)$ is $M_{\bar{c}}(x)$, $M_{\bar{c}}(x)$ is $M_{\bar{c}}(x)$
- Thus, $P = \text{NP}$.