Goals

• Mathematical definition of “Problem”
• Compare Decision and function problems
• Understand connection between Decision Problems and Languages

Language:

Set of all languages: $\mathcal{L}$

Example Problem: Addition

- Input: $\langle x, y \rangle$
- Output: $x + y$

Instead: “Decide” Problem: $\langle x, y \rangle$ is Problem

Addition (Decision)

- Input: $\langle x, y \rangle$
- Output: 1 if $x + y$ is odd, 0 otherwise

Sets are smaller (mathematically) than functions.

Addition (Language): $L = \{ \langle x,y \rangle : x+y \text{ is odd} \}$

- Input: $\langle x, y \rangle$
- Output: $1$ if $x + y$ is odd, $0$ otherwise

Alternate: $L = \{ \langle x,y \rangle : x+y \text{ is odd} \}$

Group Work:

Function Problem: Which vertex in $G - \{v\}$ has the most edges?

- Decision Problem?
- Language?
- Write language only using math:

Input: $\langle G, v \rangle$

Output: $1$ if $v$ has most edges in $G$, $0$ otherwise

$L = \{ \langle G, v \rangle : v \text{ has most edges in } G \}$

Venn Diagram

Important notation/terminology

- TM, $M$, decides a language $L$
- Input Tape
- Output Tape
- Input size := $|x| = n$: # of bits on input tape

Addition (Language): $L = \{ \langle x,y \rangle : x+y = 2 \}$

What is $|\langle x \rangle|$?

A) $O(1)$  B) $O(\log(x+y))$  C) $O(\log x \times \log y)$  D) $O(\log(x+y))$

$x = 15 \rightarrow 11101$  
$\log_{10} 15 \rightarrow 1.176$  
$\log_{10} x \times \log_{10} y \rightarrow \log_{10}(15^2)$