

Goals

- Describe qubits + quantum measurement using kets
 - Connect ket notation to physical intuition
 - Analyze novel situations using kets
- QI 2

Announcements

Next Thursday 10/1 in class

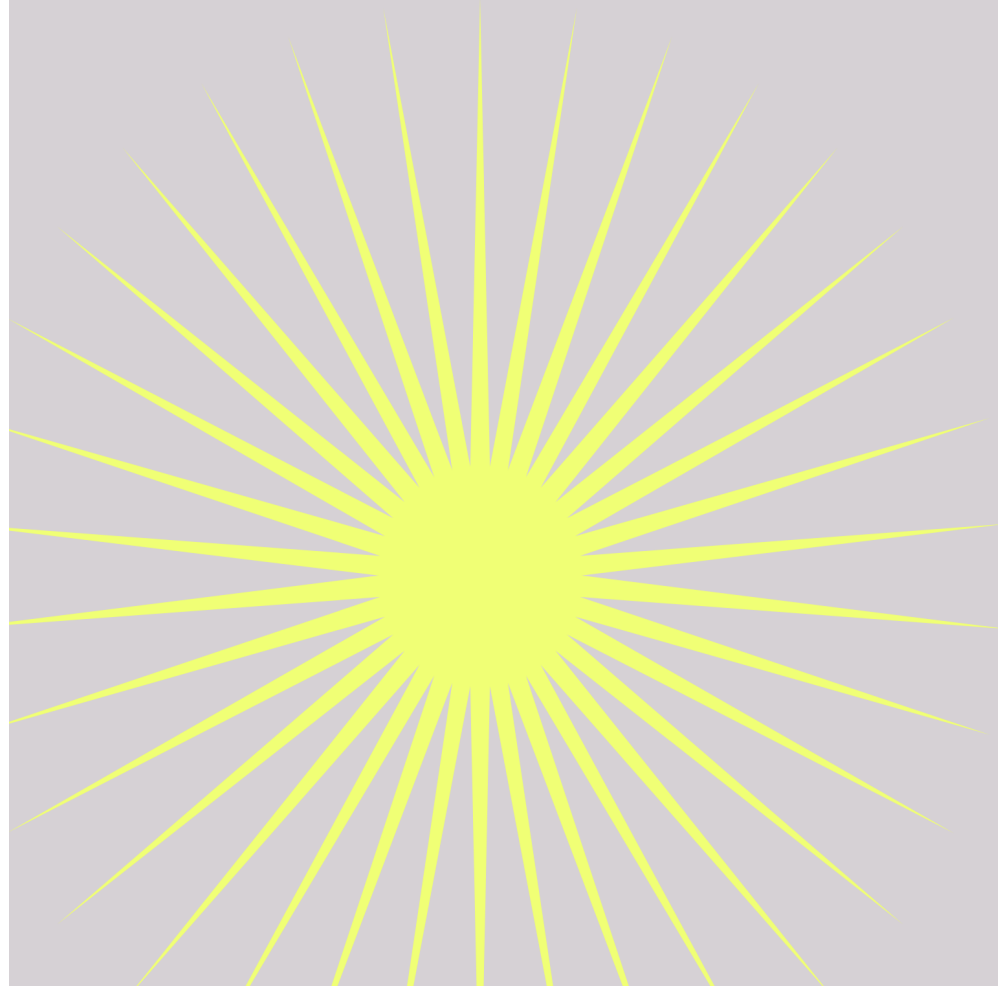
- Exam proctored - 15 min, QI 1, QI 2, no cheat sheet
- CS Help! go/cshelp + good vibes for all!
+ bribe

Exit Tickets

- Public vs Private in BB84?
- Optimal strategy for Eve?

**The Computer
Science
Department
Presents**

The 2026-27 LUNCHTIME SEMINAR SERIES



Join faculty and
students to catch up
over lunch and hear
stories of what folks
got up to this summer.
All are welcome!

**FREE PIZZA
FOR ALL!**

SESSIONS 1&2:
***What I Did
This Summer***

**9/18 & 9/25
12:20-1:20PM
75 SHANNON ST
102**

Qubit = quantum bit

Single photon polarization can encode one qubit

Some familiar qubit states	<u>Photon</u>	<u>Ket Notation</u>	<u>English</u>
		$ 0\rangle$	"ket 0" "0 state"
		$ 1\rangle$	"ket 1" "1 state"
		$ +\rangle$	"ket plus" "+ state"
		$ -\rangle$	"ket minus" "- state"

$\{|0\rangle, |1\rangle\} \Rightarrow$ standard basis

$\{|+\rangle, |-\rangle\} \Rightarrow$ Hadamard basis

Why "qubit"?

- Can represent any qubit state in terms of $|0\rangle, |1\rangle$
- Meas $\rightarrow$ 2 outcome

Qubit State:

$$x =$$

$$x \cdot y = xy$$

$$y =$$

"ket" ψ = "sign"

"state" ψ

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle \quad a_0, a_1 \in \mathbb{C}$$

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

amplitudes

$$\text{normalization: } |a_0|^2 + |a_1|^2 = 1$$

θ

$$|\theta\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$$

Not superposition: $|0\rangle, |1\rangle$

See online notes for
vector notation

"Superposition" - state is in a combination of
standard basis states.

Qubit Measurement

Represented mathematically by orthonormal pair of kets

measurement
↓

outcomes
↓

$$M = \{ |\phi_1\rangle, |\phi_2\rangle \}$$

$$\langle \phi_1 | \phi_1 \rangle = 1$$

$$\langle \phi_1 | \phi_2 \rangle = 0$$

$$\langle \phi_2 | \phi_2 \rangle = 1$$

$$\langle \phi_2 | \phi_1 \rangle = 0$$

ex: $\text{IIII} = \{ |0\rangle, |1\rangle \}$
"phi"

If measure state $|\psi\rangle$ with $M = \{ |\phi_1\rangle, |\phi_2\rangle \}$:

$$|\langle \psi | \phi_i \rangle|^2$$

- With probability $|\langle \phi_1 | \psi \rangle|^2$ get outcome $|\phi_1\rangle$,
and $|\psi\rangle$ collapses to $|\phi_1\rangle$.
- With probability $|\langle \phi_2 | \psi \rangle|^2$ get outcome $|\phi_2\rangle$
and $|\psi\rangle$ collapses to $|\phi_2\rangle$.

Can't control outcome of measurement — Probabilistic

Bra ket

If $|\psi\rangle = a_0|0\rangle + a_1|1\rangle \Rightarrow \langle\psi| = a_0^* \langle 0| + a_1^* \langle 1|$

bracket

"bra psi"

amplitudes

x multip.

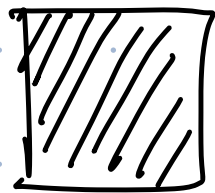
Brackets/Inner Products

- amplitudes commute
- bra/kets don't commute
- $\langle 0|0\rangle = \langle 1|1\rangle = 1$ $\langle 0|1\rangle = \langle 1|0\rangle = 0$

Example

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$$

$$M = \{ |+\rangle, |-\rangle \}$$



QI2

Probability of no photon exiting = $\frac{1}{2}$ Collapse: $|-\rangle$

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \xrightarrow{\text{bra}} \frac{1}{\sqrt{2}}\langle 0| - \frac{1}{\sqrt{2}}\langle 1|$$

$$|\langle -|0\rangle|^2$$

inner product

$$\left(\frac{1}{\sqrt{2}}\langle 0| - \frac{1}{\sqrt{2}}\langle 1| \right) \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle \right)$$

$$= \frac{1}{\sqrt{2}}\langle 0| \frac{1}{\sqrt{2}}|0\rangle$$

$$= \frac{1}{2}\langle 0|0\rangle - \frac{1}{2}\langle 1|0\rangle + \frac{i}{2}\langle 0|1\rangle - \frac{i}{2}\langle 1|1\rangle$$

$$= \frac{1}{2} - \frac{i}{2}$$

$$\left| \frac{1}{2} - \frac{i}{2} \right|^2 = \left(\frac{1}{2} - \frac{i}{2} \right) \left(\frac{1}{2} + \frac{i}{2} \right) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

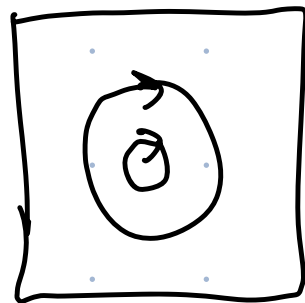
1:

Q12

Clockwise

Counter-clockwise

$$\frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle \rightarrow$$



$$M = \left\{ \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle, \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle \right\}$$

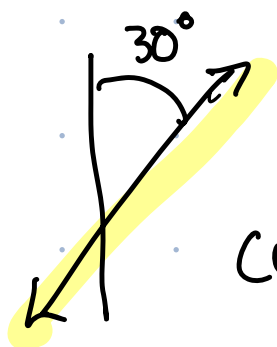
clockwise polarized filter.

What is probability a photon emerges + what polarization?

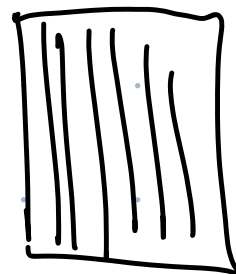
2.

$$\frac{1}{2}$$

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$$



$$\cos 30^\circ |0\rangle + \sin 30^\circ |1\rangle$$

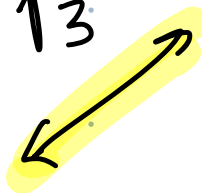


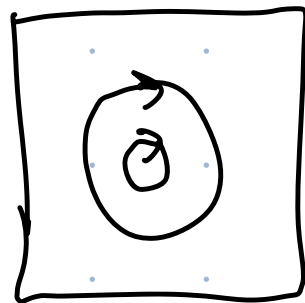
D*

What is prob of detection/no detection?

3. If state $|\psi\rangle = a_0|0\rangle + a_1|1\rangle$, what is $\langle\psi|\psi\rangle$?

1: $\frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle$





clockwise

counterclockwise

$$M = \left\{ \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle, \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle \right\}$$

Right circularly polarized filter.

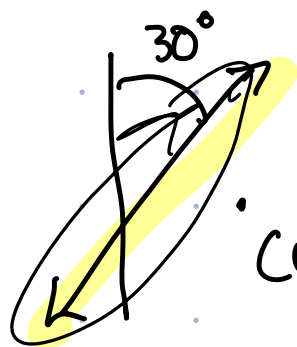
What is probability a photon emerges + what polarization?

↓
 $\frac{1}{2}$

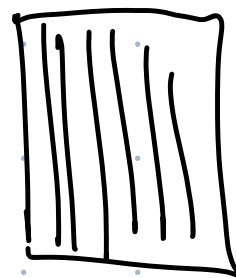
↓
 $\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$

2.

$$M = \{|0\rangle, |1\rangle\}$$



$$\cos 30^\circ |0\rangle + \sin 30^\circ |1\rangle$$



D*

What is prob of detection / no detection?

Detection:

$$\left| (\cos 30^\circ \langle 0| + \sin 30^\circ \langle 1|) |0\rangle \right|^2$$

$$|\cos 30^\circ \langle 0|0\rangle|^2$$

$$|\cos 30^\circ|^2 = \frac{3}{4}$$

No detection

Outcome $|1\rangle$:



$$|\sin 30^\circ|^2 = \frac{1}{4}$$

3. If state $|\psi\rangle = a_0|0\rangle + a_1|1\rangle$, what is $\langle\psi|\psi\rangle$?

$$(a_0|0\rangle + a_1|1\rangle)(a_0^*\langle 0| + a_1^*\langle 1|) \leftarrow \text{WRONG ORDER}$$

$$a_0|0\rangle a_0^*\langle 0| = |a_0|^2 |0\rangle\langle 0| \quad \times$$

Need bra first, then ket

Correct order:

$$(a_0^*\langle 0| + a_1^*\langle 1|)(a_0|0\rangle + a_1|1\rangle)$$

$$= |a_0|^2 \langle 0|0\rangle + |a_1|^2 \langle 1|1\rangle$$

$$= |a_0|^2 + |a_1|^2 = 1 \quad (\text{by normalization condition})$$

So $\langle\psi|\psi\rangle$ always is 1 for a qubit state.