Goals

- Describe qubits + quantum measurement using kets
- Connect ket notation to physical intuition
- Analyze novel situations using kets

Announcements

- WIDTS on Friday @ 12:30 75 SHS 102
- First In-class Exam 1 week from today 9/25
    - No note sheet (cheat sheat)
    - 15 min
    - QI1, QI2
- More OH: Tues 12:30-1:30 (most weeks)

Exit Tickets

Qubit = quantum bit ↙ e^- energy, e^- magnetic spin.

Single photon polarization can encode one qubit

Some familiar qubit states

Photon	Ket Notation	English
↕	$\lvert 0\rangle$	"ket 0" "0 state"
↔	$\lvert 1\rangle$	"ket 1" "1 state"
⤢	$\lvert +\rangle = \frac{1}{\sqrt{2}}\lvert 0\rangle + \frac{1}{\sqrt{2}}\lvert 1\rangle$	"plus state"
⤡	$\lvert -\rangle = \frac{1}{\sqrt{2}}\lvert 0\rangle - \frac{1}{\sqrt{2}}\lvert 1\rangle$	"minus state

$\boxed{\text{Why "Qubit"?}}$

Measurements → 2 outcomes
(1 bit extracted)

$\{\lvert 0\rangle, \lvert 1\rangle\}$ – standard basis

$\{\lvert +\rangle, \lvert -\rangle\}$ – Hadamard basis

Qubit State:

"amplitudes"

$a_0, a_1 \in \mathbb{C}$

psi

"multiplication"

"ket psi"

"state psi"

$\psi \sim \chi$

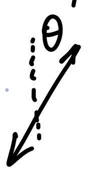
$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle$$

normalization: $|a_0|^2 + |a_1|^2 = 1$

$$|b|^2 = b \cdot b^*$$

ex: $|0\rangle : 1|0\rangle + \cancel{0|1\rangle}$

$|-\rangle : \frac{1}{\sqrt{2}}|0\rangle + \left(\frac{-1}{\sqrt{2}}\right)|1\rangle$



$: \cos\theta|0\rangle + \sin\theta|1\rangle$

See online notes for vector notation

"Superposition" - is a combination of $|0\rangle$ and $|1\rangle$

$|+\rangle$ is a superposition of $|0\rangle$ and $|1\rangle$

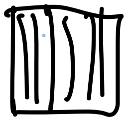
Qubit Measurement

Represented mathematically by orthonormal pair of kets

measurement　　measurement outcome　　orthogonal

$M = \{ |\phi_1\rangle, |\phi_2\rangle \}$ "phi"　　$\langle \phi_1 | \phi_2 \rangle = 0$

$\langle \phi_1 | \phi_1 \rangle = \langle \phi_2 | \phi_2 \rangle = 1$

ex:  $\Rightarrow \{ |0\rangle, |1\rangle \}$

If measure state $|\psi\rangle$ with $M = \{ |\phi_1\rangle, |\phi_2\rangle \}$:　　normalized

- With probability $|\langle \phi_1 | \psi \rangle|^2$ get outcome $|\phi_1\rangle$, $|\psi\rangle$ collapses to $|\phi_1\rangle$
- With probability $|\langle \phi_2 | \psi \rangle|^2$ get outcome $|\phi_2\rangle$, $|\psi\rangle$ collapses to $|\phi_2\rangle$

Can't control outcome of measurement —

Bras

complex conjugates

If $|\psi\rangle = a_0|0\rangle + a_1|1\rangle \overset{\text{Bra}}{\Longrightarrow} \langle\psi| = a_0^*\langle 0| + a_1^*\langle 1|$

"bra psi"

Brakets/Inner Products

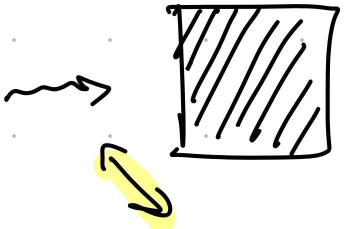
Basic Rules:

-
-
-
-
-
-

Example

$\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$ ↻

$M = \{|+\rangle, |-\rangle\}$



Probability of no photon exiting $= \frac{1}{2}$

Collapse: $|-\rangle$

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \overset{\text{bra}}{\Longrightarrow} \left(\frac{1}{\sqrt{2}}\right)^* \langle 0| + \left(\frac{-1}{\sqrt{2}}\right)^* \langle 1| = \frac{1}{\sqrt{2}}\langle 0| - \frac{1}{\sqrt{2}}\langle 1|$$

$|\langle - | \circlearrowright \rangle|^2$

"inner product"

mult. mult.

$$\langle - | \circlearrowright \rangle = \langle - | \cdot | \circlearrowright \rangle = \left(\frac{1}{\sqrt{2}}\langle 0| - \frac{1}{\sqrt{2}}\langle 1|\right)\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\cancelto{1}{\langle 0|0\rangle} + \frac{i}{2}\cancelto{0}{\langle 0|1\rangle} - \frac{1}{2}\cancelto{0}{\langle 1|0\rangle} - \frac{i}{2}\cancelto{1}{\langle 1|1\rangle}$$

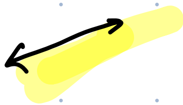
Rule: $\langle 0|0\rangle = \langle 1|1\rangle = 1 \quad \langle 0|1\rangle = \langle 1|0\rangle = 0$

$$= \frac{1}{2}\cdot 1 + \frac{-i}{2}$$

$$|\langle - | \circlearrowright \rangle|^2 = \left(\frac{1}{2} - \frac{i}{2}\right)\cdot\left(\frac{1}{2} + \frac{i}{2}\right) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

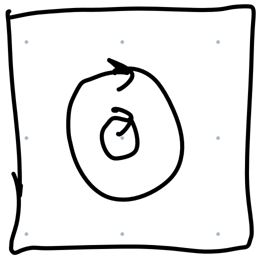
$\boxed{\text{QIZ}}$

1:



Clockwise Counter-clockwise

$\frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}\,|1\rangle \rightarrow$

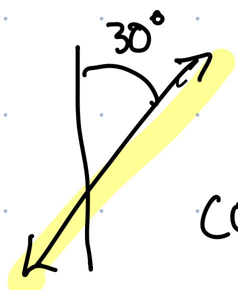


$M = \left\{ \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle,\ \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle \right\}$

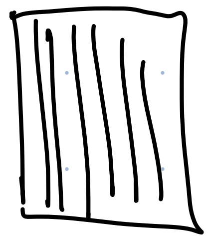
clockwise polarized filter.

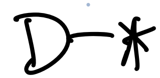
What is probability a photon emerges + what polarization?

2.



$\cos 30^\circ|0\rangle + \sin 30^\circ|1\rangle \rightsquigarrow$

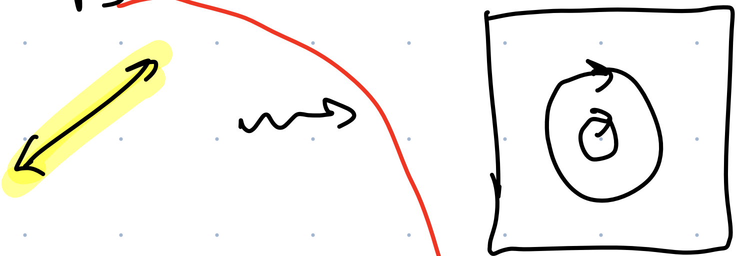




What is prob of detection / no detection?

3. If state $|\psi\rangle = a_0|0\rangle + a_1|1\rangle$, what is $\langle\psi|\psi\rangle$?

1: $\frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}\,|1\rangle \to$



Right circ. , Left circ.

$$M = \left\{ \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle,\ \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle \right\}$$

$$\downarrow$$

$$\frac{1}{\sqrt{2}}\langle 0| + \frac{-i}{\sqrt{2}}\langle 1|$$

Right circularly polarized filter.

What is probability a photon emerges + what polarization?

↓

clockwise

$$\left(\frac{1}{\sqrt{2}}\langle 0| - \frac{i}{\sqrt{2}}\langle 1|\right)\left(\frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}\,|1\rangle\right)$$

$$\frac{1}{\sqrt{6}}\langle 0|0\rangle^{1} + 0 + 0 \quad \frac{-i}{\sqrt{3}}\langle 1|1\rangle^{1}$$

$$\downarrow$$

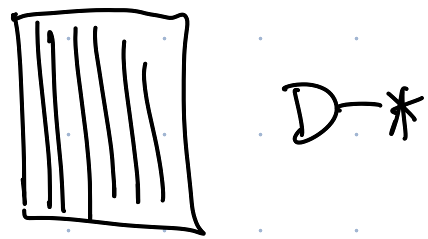
$$\left|\frac{1}{\sqrt{6}} - \frac{i}{\sqrt{3}}\right|^2 = \left(\frac{1}{\sqrt{6}} - \frac{i}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{6}} + \frac{i}{\sqrt{3}}\right) = \frac{1}{6} + \frac{i\cdot(-i)}{3}$$

$$i|0\rangle = |0\rangle$$

$$= \frac{1}{6} + \frac{1}{\ } = \frac{1}{2}$$

2.

$i\cos 30°|0\rangle + \sin 30°|1\rangle \rightsquigarrow$



What is prob of detection / no detection?

Detection:

$|\langle 0|(i\cos 30°|0\rangle + \sin 30°|1\rangle)|^2$

$= |i\cos 30°|^2$

$= 0.87$

No detection

$|\langle 1|(i\cos 30°|0\rangle + \sin 30°|1\rangle)|^2$

$= |i\cos 30°\langle 1|0\rangle + \sin 30°\langle 1|1\rangle|^2$

$= |\sin 30°|^2$

$= .13$

Standard basis → take abs val sq. of each amplitude

3. If state $|\psi\rangle = a_0|0\rangle + a_1|1\rangle$, what is $\langle\psi|\psi\rangle$?

$$\langle\psi|\psi\rangle = \left(a_0^*\langle 0| + a_1^*\langle 1|\right)\left(a_0|0\rangle + a_1|1\rangle\right)$$

$$= a_0 a_0^* \langle 0|0\rangle + a_0^* a_1 \langle 0|1\rangle + a_1^* a_0 \langle 1|0\rangle + a_1^* a_1 \langle 1|1\rangle$$

$$= a_0 a_0^* + a_1 a_1^*$$

$$= |a_0|^2 + |a_1|^2$$

$$= 1$$

☆☆ $|a_0|^2 = a_0 a_0^*$

By normalization condition

A properly normalized state has inner product 1 with itself.