Learning Goals

- Describe quantum gates using ket action
- Describe sufficient + necessary properties of gates
- Apply ket formalism to analyze novel gate applications

[QC4]

Announcements

Exit Tickets

Previous examples of gates (also called operations/unitaries)

Beamsplitter

$$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$$

$$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$$

$$|0\rangle_P|V\rangle_D \rightarrow |0\rangle_P|V\rangle_D$$

$$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$$

θ Wave plate

$$|0\rangle \rightarrow \cos\theta|0\rangle + \sin\theta|1\rangle$$

$$|1\rangle \rightarrow -\sin\theta|0\rangle + \cos\theta|1\rangle$$

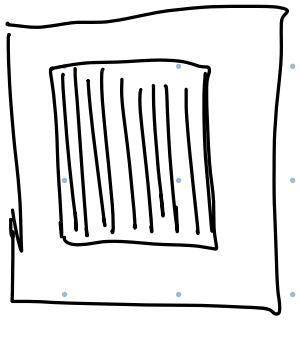
Properties of Gates (Necessary + Sufficient)

- Take quantum states to quantum states
- Reversible

Are these gates? Why/ Why not?

$|0\rangle \rightarrow |0\rangle + |1\rangle$

$|1\rangle \rightarrow |0\rangle - |1\rangle$



Gate is a unitary operation

- a unitary is a multidimensional generalization of reflections and rotations
- Must take an orthonormal set to an orthonormal set

$|0\rangle \rightarrow |\psi_1\rangle$

$|1\rangle \rightarrow |\psi_2\rangle$

} Valid if $\{|\psi_1\rangle, |\psi_2\rangle\}$ orthonormal

$|00\rangle \rightarrow |\psi_1\rangle$

$|01\rangle \rightarrow |\psi_2\rangle$

$|10\rangle \rightarrow |\psi_3\rangle$

$|11\rangle \rightarrow |\psi_4\rangle$

} Valid if $\langle\psi_i|\psi_j\rangle = \delta_{ij}$ (orthonormal)

Famous Gates:

Paulis

I
$$|0\rangle \to |0\rangle$$
$$|1\rangle \to |1\rangle$$

X
$$|0\rangle \to |1\rangle$$
$$|1\rangle \to |0\rangle$$

Y
$$|0\rangle \to i|1\rangle$$
$$|1\rangle \to -i|0\rangle$$

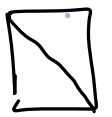
Z
$$|0\rangle \to |0\rangle$$
$$|1\rangle \to -|1\rangle$$

Hadamard

H
$$|0\rangle \to |+\rangle$$
$$|1\rangle \to |-\rangle$$

CNOT

$$|00\rangle \to |00\rangle$$
$$|01\rangle \to |01\rangle$$
$$|10\rangle \to |11\rangle$$
$$|11\rangle \to |10\rangle$$

$\approx$ 

controlled (based) on 1st qubit, apply NOT to 2nd

T

$$|0\rangle \to |0\rangle$$
$$|1\rangle \to e^{i\frac{\pi}{4}}|1\rangle$$

Act to right

$$HY|0\rangle = H(Y|0\rangle) = H(i|1\rangle) = iH|1\rangle = i|-\rangle$$

Group Work: QC1

What is $TH|-\rangle$?

Hadamard

H

$|0\rangle \to |+\rangle$

$|1\rangle \to |-\rangle$

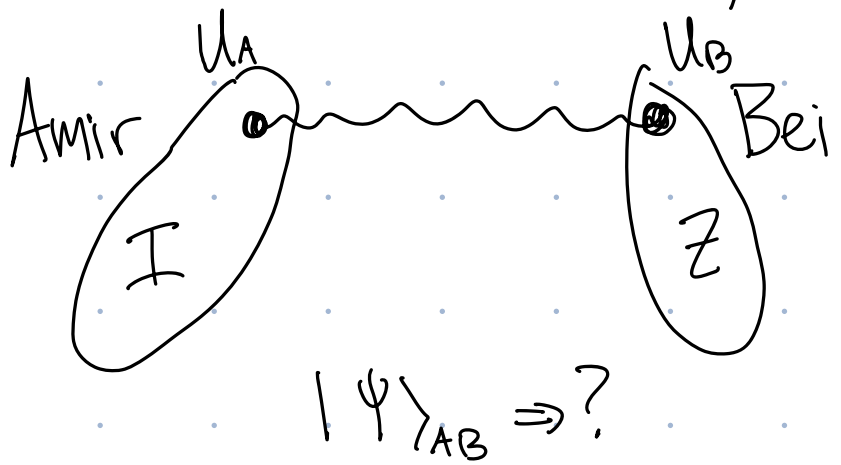
T

$|0\rangle \to |0\rangle$

$|1\rangle \to e^{i\frac{\pi}{4}}|1\rangle$

Single Qubit Gates on 2-qubit States

$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$



$|\psi\rangle_{AB} \Rightarrow ?$