Learning Goals

- Describe quantum gates using ket action
- Describe sufficient + necessary properties of gates
- Apply ket formalism to analyze novel gate applications

[QC1]

Announcements

- Exam 2: 10/22, 75 min in class
  QI2-5, QC1
  1 side of 3"x5" notecard

Previous examples of gates (also called operations/unitaries)

Beamsplitter

$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$

$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$

$|0\rangle_P|V\rangle_D \rightarrow |0\rangle_P|V\rangle_D$

$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$

θ Wave plate

$|0\rangle \rightarrow \cos\theta|0\rangle + \sin\theta|1\rangle$

$|1\rangle \rightarrow -\sin\theta|0\rangle + \cos\theta|1\rangle$

① Take qb state to quantum state

$|\psi\rangle_{AB} \rightarrow \square \rightarrow |\psi'\rangle_{AB}$

② Reversible

$|\psi\rangle_{AB} \rightarrow \square \xrightarrow{|\psi'\rangle_{AB}} \square \rightarrow |\psi\rangle_{AB}$

$|\psi\rangle \rightarrow \theta \rightarrow -\theta \rightarrow |\psi\rangle$

Properties of Gates (Necessary + Sufficient)

- Take quantum states to quantum states
- Reversible

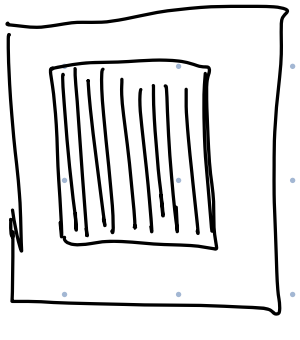
Reversible ✓

Reversible ✗

Are these gates? Why/Why not?

$|0\rangle \to |0\rangle + |1\rangle$

$|1\rangle \to |0\rangle - |1\rangle$



Take state → state ✓

Reversible

Gate is a unitary operation

- a unitary is a multidimensional generalization of reflections and rotations
- Must take an orthonormal set to an orthonormal set

$|0\rangle \to |\psi_1\rangle$

$|1\rangle \to |\psi_2\rangle$

} Valid if $\{|\psi_1\rangle, |\psi_2\rangle\}$ orthonormal

$|00\rangle \to |\psi_1\rangle$

$|01\rangle \to |\psi_2\rangle$

$|10\rangle \to |\psi_3\rangle$

$|11\rangle \to |\psi_4\rangle$

} Valid if $\langle\psi_i|\psi_j\rangle = \delta_{ij}$ (orthonormal)

Famous Gates:

$Z|+\rangle = Z\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) = \frac{1}{\sqrt{2}}Z|0\rangle + \frac{1}{\sqrt{2}}Z|1\rangle =$
$= \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle = |-\rangle$

Paulis

NOT = X

I
$|0\rangle \to |0\rangle$
$|1\rangle \to |1\rangle$

X
$|0\rangle \to |1\rangle$
$|1\rangle \to |0\rangle$

Y
$|0\rangle \to i|1\rangle$
$|1\rangle \to -i|0\rangle$

Z
$|0\rangle \to |0\rangle$
$|1\rangle \to -|1\rangle$

Hadamard

H
$|0\rangle \to |+\rangle$
$|1\rangle \to |-\rangle$

CNOT = control-NOT

$T\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)$

$T|1\rangle$

$|00\rangle \to |00\rangle$
$|01\rangle \to |01\rangle$
$|10\rangle \to |11\rangle$
$|11\rangle \to |10\rangle$

$\approx$ 

controlled (based) on 1st qubit, apply NOT to 2nd

$|0\rangle \to |0\rangle$
$|1\rangle \to e^{i\pi/4}|1\rangle$

Act to right

$$HY|0\rangle = H(Y|0\rangle) = H(i|1\rangle) = iH|1\rangle = i|-\rangle \Rightarrow |-\rangle$$

Group Work: QC1

What is $TH|-\rangle$?

$HXZH|1\rangle$

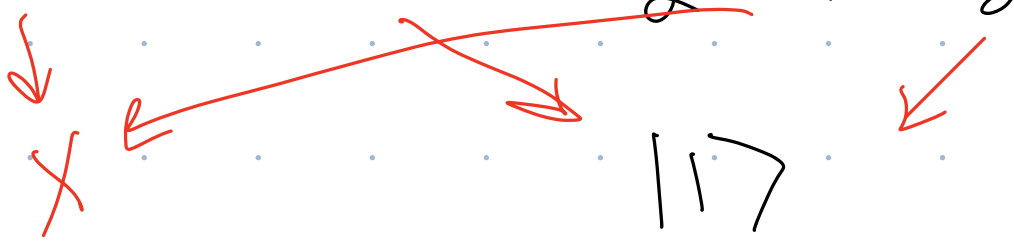
$T H |-\rangle$

$T\left(H\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)\right)$

$T\left(\frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle\right)$

$T\left(\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)\right)$ T

$T\left(\frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle - \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle\right)$



$|1\rangle$

$T|1\rangle = e^{i\pi/4}|1\rangle \Rightarrow |1\rangle$

Hadamard

H

$\|0\rangle \to \|+\rangle$
$\|1\rangle \to \|-\rangle$

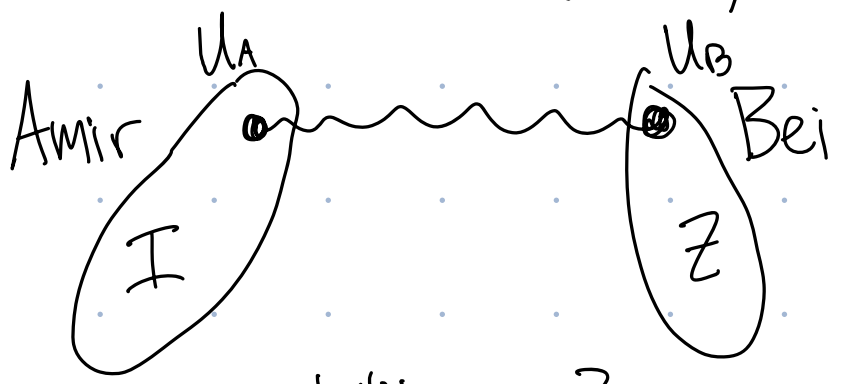
T

$\|0\rangle \to \|0\rangle$
$\|1\rangle \to e^{i\frac{\pi}{4}}\|1\rangle$

Single Qubit Gates on 2-qubit States

QC2

$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$



$|\psi\rangle_{AB} \Rightarrow ?$

$\leftarrow U_{AB} = U_A \otimes U_B$

$= I_A \otimes Z_B$

$U_{AB}|\psi\rangle_{AB} = (I_A \otimes Z_B)\left(\frac{1}{\sqrt{2}}|00\rangle_{AB} + \frac{1}{\sqrt{2}}|11\rangle_{AB}\right)$ Distribute

$= \frac{1}{\sqrt{2}} I_A \otimes Z_B |0\rangle_A \otimes |0\rangle_B + \frac{1}{\sqrt{2}} I_A \otimes Z_B |1\rangle_A \otimes |1\rangle_B$

$= \frac{1}{\sqrt{2}} I_A|0\rangle_A \otimes Z_B|0\rangle_B + \frac{1}{\sqrt{2}} I_A|1\rangle_A \otimes Z_B|1\rangle_B$

$= \frac{1}{\sqrt{2}}|0\rangle_A \otimes |0\rangle_B + \frac{1}{\sqrt{2}}|1\rangle_A \otimes (-1)|1\rangle_B$

$= \frac{1}{\sqrt{2}}|00\rangle_{AB} - \frac{1}{\sqrt{2}}|11\rangle_{AB}$