Learning Goals

- Describe quantum gates using ket action
- Describe sufficient + necessary properties of gates
- Apply ket formalism to analyze novel gate applications

Announcements

- No OH today

Exit Tickets

- Always have a prob of exploding bomb? Point of Bomb?
- Gates vs. Measurement?
- Normalization?

Previous examples of gates (also called operations/unitaries)

⧅ Beamsplitter

$$|0\rangle_P |H\rangle_D \rightarrow |0\rangle_P |H\rangle_D$$
$$|1\rangle_P |H\rangle_D \rightarrow |1\rangle_P |V\rangle_D$$
$$|0\rangle_P |V\rangle_D \rightarrow |0\rangle_P |V\rangle_D$$
$$|1\rangle_P |V\rangle_D \rightarrow |1\rangle_P |H\rangle_D$$

$\textcircled{\theta}$ Wave plate

$$|0\rangle \rightarrow \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$$
$$|1\rangle \rightarrow -\sin\theta\,|0\rangle + \cos\theta\,|1\rangle$$

Take q states to q states:

$$⧅\left(\frac{1}{\sqrt{5}}|0\rangle|H\rangle + \sqrt{\frac{2}{5}}\,i\,|1\rangle|H\rangle + \sqrt{\frac{3}{5}}\,|1\rangle|V\rangle\right)$$

$$\textcircled{\theta}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)$$
$$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta\,|0\rangle + \frac{1}{\sqrt{2}}\sin\theta\,|1\rangle$$

Reversible $|\psi\rangle$ ⧅ ⧅ $|\psi\rangle$

$\textcircled{\theta}$ $\textcircled{-\theta}$

Properties of Gates (Necessary + Sufficient) Iff Gate

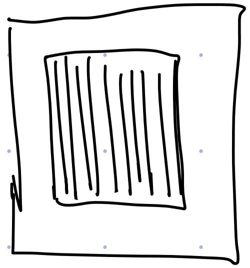
- Take quantum states to quantum states
- Reversible

iff

Are these gates? Why/Why not?

$$|0\rangle \to |0\rangle + |1\rangle$$
$$|1\rangle \to |0\rangle - |1\rangle$$

X $q \to q$

$|\psi\rangle$  $\updownarrow \to \overset{?}{O} \to |\psi\rangle$

X reversible

Gate is a unitary operation

- a unitary is a multidimensional genaralization of reflections and rotations
- Must take an orthonormal set to an orthonormal set

$$\left.\begin{aligned} |0\rangle &\to |\psi_1\rangle \\ |1\rangle &\to |\psi_2\rangle \end{aligned}\right\} \text{Valid if } \{|\psi_1\rangle, |\psi_2\rangle\} \text{ orthonormal}$$

$$\left.\begin{aligned} |00\rangle &\to |\psi_1\rangle \\ |01\rangle &\to |\psi_2\rangle \\ |10\rangle &\to |\psi_3\rangle \\ |11\rangle &\to |\psi_4\rangle \end{aligned}\right\} \text{Valid if } \langle\psi_i|\psi_j\rangle = \delta_{ij} \text{ (orthonormal)}$$

Famous Gates:

Paulis

NOT
=
X: $|0\rangle \to |1\rangle$, $|1\rangle \to |0\rangle$

I: $|0\rangle \to |0\rangle$, $|1\rangle \to |1\rangle$

Y: $|0\rangle \to i|1\rangle$, $|1\rangle \to -i|0\rangle$

Z: $|0\rangle \to |0\rangle$, $|1\rangle \to -|1\rangle$

Hadamard

H: $|0\rangle \to |+\rangle$, $|1\rangle \to |-\rangle$

CNOT - control not

$|00\rangle \to |00\rangle$
$|01\rangle \to |01\rangle$
$|10\rangle \to |11\rangle$
$|11\rangle \to |10\rangle$

controlled (based) on 1^{st} qubit, apply NOT to 2^{nd}

T

$|0\rangle \to |0\rangle$
$|1\rangle \to e^{i\pi/4}|1\rangle$

Apply left to right / right to left

$$HY|0\rangle = H\left(Y|0\rangle\right) = Hi|1\rangle = iH|1\rangle = i|-\rangle$$

Inside out / Like a function

Hadamard

H $\boxed{\begin{matrix}|0\rangle \to |+\rangle \\ |1\rangle \to |-\rangle\end{matrix}}$

What is $H|+\rangle$?

$$H\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}H|0\rangle + \frac{1}{\sqrt{2}}H|1\rangle$$

$$= \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) + \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)$$

$$= \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle + \frac{1}{2}|0\rangle - \frac{1}{2}|1\rangle$$

$$= |0\rangle$$

Single Qubit Gates on 2-qubit States

$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

tensor (product)

$U_A \otimes U_B$

U_A U_B

Amir  Bei

I Z

$U_{AB} = I_A \otimes Z_B$

$|\psi\rangle_{AB}$

$$U_{AB}|\psi\rangle_{AB} = (I_A \times Z_B)\left(\frac{1}{\sqrt{2}}|00\rangle_{AB} + \frac{1}{\sqrt{2}}|11\rangle_{AB}\right)$$

$$= \frac{1}{\sqrt{2}} I_A \otimes Z_B \underbrace{|00\rangle_{AB}}_{|0\rangle_A \otimes |0\rangle_B} + \frac{1}{\sqrt{2}} I_A \otimes Z_B \underbrace{|11\rangle_{AB}}_{|1\rangle_A \otimes |1\rangle_B}$$

$$= \frac{1}{\sqrt{2}}(I_A|0\rangle_A) \otimes (Z_B|0\rangle_B) + \frac{1}{\sqrt{2}}(I_A|1\rangle_A) \otimes (Z_B|1\rangle_B)$$

$$= \frac{1}{\sqrt{2}}|0\rangle_A \otimes |0\rangle_B + \frac{1}{\sqrt{2}}^{*}|1\rangle_A \overset{*}{\otimes} (-1)^{*}|1\rangle_B$$

$$= \frac{1}{\sqrt{2}}|00\rangle_{AB} - \frac{1}{\sqrt{2}}|11\rangle_{AB} \neq |-\rangle|-\rangle \Rightarrow \frac{1}{2}|01\rangle$$

$|-\rangle \to |0\rangle$, $|-\rangle \to |1\rangle$

Group Problem

Suppose Amir + Bei share the 2-qubit state $|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.
What Paulis should they each apply to create the state $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$? $Z \otimes X$, $X \otimes Z$, $I \otimes (XZ)$, $Y \otimes I$

Write this effective 2 qubit gate as a transformation of standard basis states:

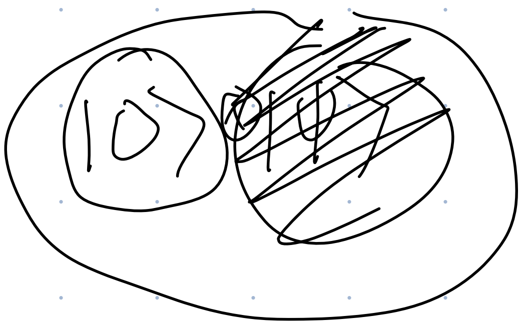
$Y \otimes I$

$|00\rangle \to i|10\rangle$

$|01\rangle \to i|11\rangle$

$|10\rangle \to -i|00\rangle$

$|11\rangle \to -i|01\rangle$



$$Y \otimes I \left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle\right)$$

$$= \frac{1}{\sqrt{2}} Y \otimes I |00\rangle + \frac{1}{\sqrt{2}} Y \otimes I |11\rangle$$

$$= \frac{1}{\sqrt{2}}(i)|1\rangle|0\rangle + \frac{1}{\sqrt{2}}(-i)|0\rangle|1\rangle$$

$$= \left(\frac{i}{\sqrt{2}}|10\rangle - \frac{i}{\sqrt{2}}|01\rangle\right) =$$

i

$$\frac{-1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|01\rangle$$