

Learning Goals

- Describe quantum gates using ket action.
- Describe sufficient + necessary properties of gates
- Apply ket formalism to analyze novel gate applications

Announcements

Exit Tickets

Previous examples of gates (also called operations/unitaries)

◻ Beamsplitter

$$|0\rangle_P |H\rangle_D \rightarrow |0\rangle_P |H\rangle_D$$

$$|1\rangle_P |H\rangle_D \rightarrow |1\rangle_P |V\rangle_D$$

$$|0\rangle_P |V\rangle_D \rightarrow |0\rangle_P |V\rangle_D$$

$$|1\rangle_P |V\rangle_D \rightarrow |1\rangle_P |H\rangle_D$$

⊙ Wave plate

$$|0\rangle \rightarrow \cos\theta |0\rangle + \sin\theta |1\rangle$$

$$|1\rangle \rightarrow -\sin\theta |0\rangle + \cos\theta |1\rangle$$

Take q states to q states:

$$\square \left(\frac{1}{\sqrt{5}} |0\rangle |H\rangle + \sqrt{\frac{2}{5}} i |1\rangle |H\rangle + \sqrt{\frac{3}{5}} |1\rangle |V\rangle \right)$$

$$\odot \left(\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \right)$$

$$\Rightarrow \frac{1}{\sqrt{2}} \cos\theta |0\rangle + \frac{1}{\sqrt{2}} \sin\theta |1\rangle$$

Reversible

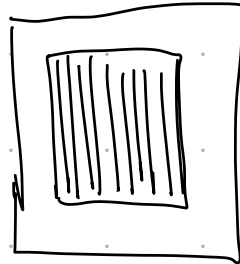
Properties of Gates (Necessary + Sufficient)

- Take quantum states to quantum states
- Reversible

iff $\updownarrow$

Are these gates? Why/Why not?

$$\begin{array}{l} |0\rangle \rightarrow |0\rangle + |1\rangle \\ |1\rangle \rightarrow |0\rangle - |1\rangle \end{array}$$



Gate is a unitary operation

- a unitary is a multidimensional generalization of reflections and rotations
- Must take an orthonormal set to an orthonormal set

$$\left. \begin{array}{l} |0\rangle \rightarrow |\psi_1\rangle \\ |1\rangle \rightarrow |\psi_2\rangle \end{array} \right\} \begin{array}{l} \text{Valid if} \\ \{|\psi_1\rangle, |\psi_2\rangle\} \\ \text{orthonormal} \end{array}$$

$$\left. \begin{array}{l} |00\rangle \rightarrow |\psi_1\rangle \\ |01\rangle \rightarrow |\psi_2\rangle \\ |10\rangle \rightarrow |\psi_3\rangle \\ |11\rangle \rightarrow |\psi_4\rangle \end{array} \right\} \begin{array}{l} \text{Valid if} \\ \langle \psi_i | \psi_j \rangle = \delta_{ij} \\ \text{(orthonormal)} \end{array}$$

Famous Gates:

Paulis

$$I \quad \begin{array}{|l} |0\rangle \rightarrow \\ |1\rangle \rightarrow \end{array}$$

$$X \quad \begin{array}{|l} |0\rangle \rightarrow \\ |1\rangle \rightarrow \end{array}$$

$$Y \quad \begin{array}{|l} |0\rangle \rightarrow \\ |1\rangle \rightarrow \end{array}$$

$$Z \quad \begin{array}{|l} |0\rangle \rightarrow \\ |1\rangle \rightarrow \end{array}$$

Hadamard

$$H \quad \begin{array}{|l} |0\rangle \rightarrow \\ |1\rangle \rightarrow \end{array}$$

CNOT

$$\begin{array}{|l} |00\rangle \rightarrow \\ |01\rangle \rightarrow \\ |10\rangle \rightarrow \\ |11\rangle \rightarrow \end{array}$$

controlled (based)
on 1st qubit,
apply NOT to 2nd

Apply left to right

$$HY|0\rangle =$$

Hadamard

H

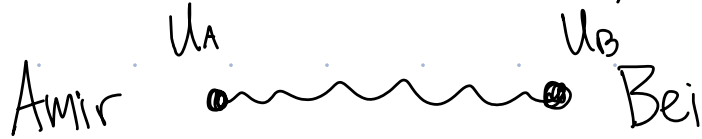
$$|0\rangle \rightarrow |+\rangle$$

$$|1\rangle \rightarrow |-\rangle$$

What is $H|+\rangle$?

Single Qubit Gates on 2-qubit States

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$



$$U_{AB}|\psi\rangle_{AB} =$$

Group Problem

Suppose Amir + Bei share the 2-qubit state $|\Psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

What Paulis should they each apply to create the state $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$?

Write this effective 2 qubit gate as a transformation of standard basis states:

$$|00\rangle \rightarrow$$

$$|01\rangle \rightarrow$$

$$|10\rangle \rightarrow$$

$$|11\rangle \rightarrow$$