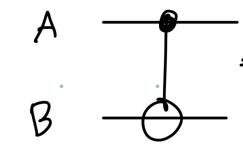
Learning Goals

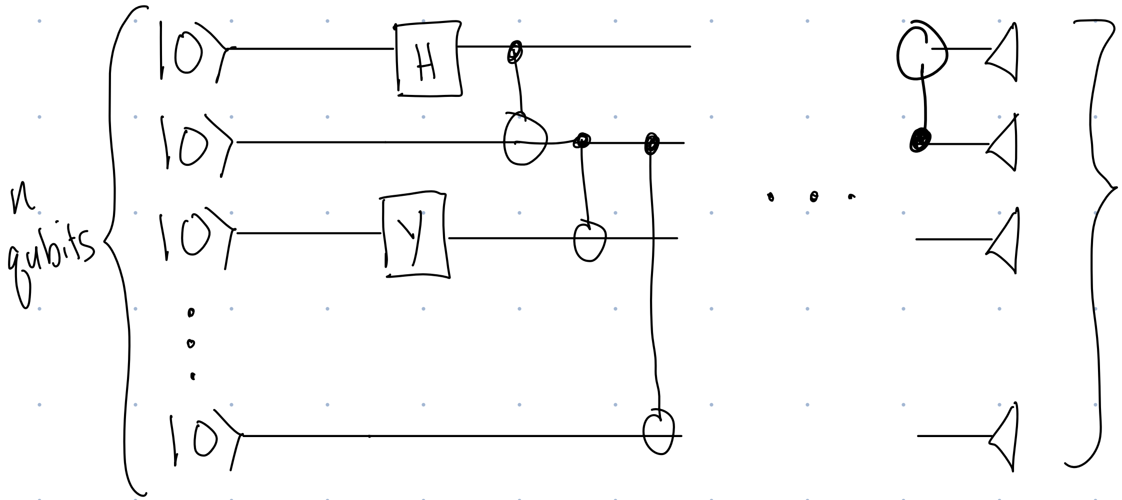
- Describe multiqubit algorithms using circuits + kets

Multiqubit Circuit



= CNOT

If $A = |1\rangle$, $\rightarrow XB$



Measure each qubit in standard basis

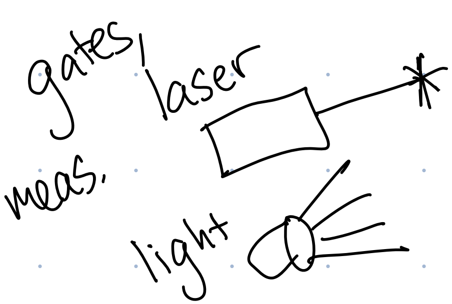
$M = \{|0\rangle, |1\rangle\}$

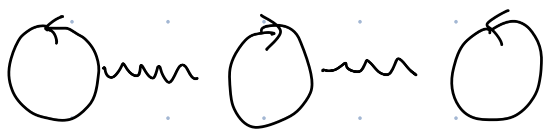
$|0\rangle$ prep, $\triangleleft$ meas

$\Downarrow$

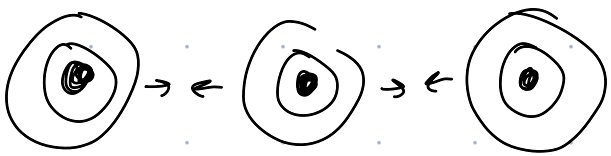
for ease of implementation

Physically:

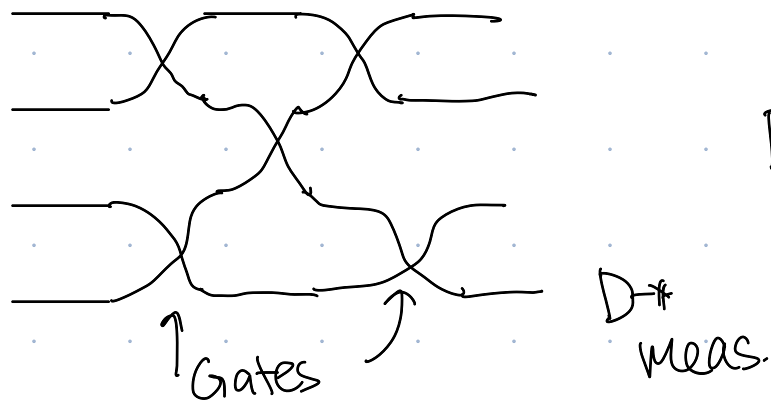




Superconducting circuits

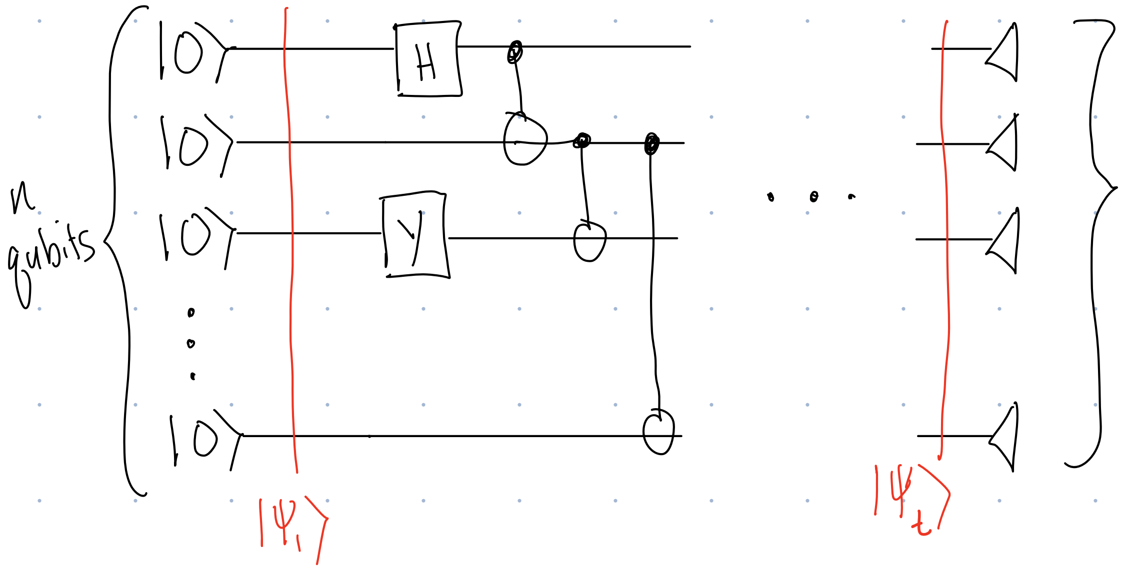


trapped ions/atoms



Photon paths

Multiqubit Circuit



Measure each qubit in standard basis

$$\underbrace{M = \{|0\rangle, |1\rangle\}}_{\text{effective}}$$

$$M = \{|000\ldots0\rangle, |000\ldots1\rangle, |000\ldots10\rangle$$

$$|\psi_1\rangle = |0\rangle|0\rangle|0\rangle\cdots|0\rangle = |\underbrace{000\ldots0}_{n \text{ 0's}}\rangle$$

At the end of the algorithm, state can be written as e.g.

$$|\psi_t\rangle = \frac{1}{\sqrt{10}}|011000\rangle + \frac{i}{\sqrt{10}}|110101\rangle + \sqrt{\frac{3}{10}}\, e^{2\pi i/3}|100010\rangle + \frac{1}{\sqrt{2}}|111101\rangle$$

6 qubit state

standard basis states

amplitudes

Representing + Measuring n-qubit States

amplitude

$$|\psi\rangle = \sum_{s \in \{0,1\}^n} a_s |s\rangle \quad \text{s.t.} \quad a_s \in \mathbb{C}, \quad \sum_{s \in \{0,1\}} |a_s|^2 = 1$$

standard basis state

- When measure $|\psi\rangle$ in standard basis, get outcome $|s\rangle$ with probability $|a_s|^2$

ex:

$$|\psi_t\rangle = \frac{1}{\sqrt{10}}|011000\rangle + \frac{i}{\sqrt{10}}|110101\rangle + \sqrt{\frac{3}{10}}\, e^{2\pi i/3}|100010\rangle + \frac{1}{\sqrt{2}}|111101\rangle$$

What is the probability of measuring $|100010\rangle$

A) $\sqrt{\frac{3}{10}}\, e^{2\pi i/3}$ B) $\sqrt{\frac{3}{10}}$ C) $\frac{3}{10}$ D) 0

$$\left|\sqrt{\frac{3}{10}}\, e^{2\pi i/3}\right|^2 = \sqrt{\frac{3}{10}}\, e^{2\pi i/3} \cdot \sqrt{\frac{3}{10}}\, e^{2\pi(-i)/3} = \frac{3}{10}\, e^{\cancelto{0}{2\pi i/3 - 2\pi i/3}}$$

$$= \frac{3}{10}$$

$a^0 = 1$ $a^x \cdot a^y = a^{x+y}$

Partial measurement

- When measure first k qubits in standard basis,
  → Rewrite $|\psi\rangle$ by pulling out standard basis states of measured qubits

$$|\psi\rangle = |\underbrace{000..0}_{k}\rangle \left(\sum_{i \in \{0,1\}^{n-k}} a_{i,0000} |i\rangle \right) + |\underbrace{000..1}_{k}\rangle \left(\sum_{i \in \{0,1\}^{n-k}} a_{i,000..1} |i\rangle \right) + \dots$$

- Probability of outcome $|000\dots1\rangle$ is $\sum_{i \in \{0,1\}^{n-k}} |a_{i,0001}|^2 = p_{000..1}$

- Remaining qubits collapse to $\frac{1}{\sqrt{p_{000..1}}} \sum_{i \in \{0,1\}^{n-k}} a_{i,000..1} |i\rangle$

ex:

$$|\psi_t\rangle = \frac{1}{\sqrt{10}}|011000\rangle + \frac{i}{\sqrt{10}}|110101\rangle + \sqrt{\frac{3}{10}}e^{2\pi i/3}|100010\rangle + \frac{1}{\sqrt{2}}|111101\rangle$$

Measure 1^{st} 2 qubits:

$$|\psi_t\rangle = |01\rangle\left(\frac{1}{\sqrt{10}}|1000\rangle\right) + |11\rangle\boxed{\left(\frac{i}{\sqrt{10}}|0101\rangle + \frac{1}{\sqrt{2}}|1101\rangle\right)} + |10\rangle\left(\sqrt{\frac{3}{10}}e^{2\pi i/3}|0010\rangle\right)$$

Prob of outcome $|11\rangle$

$$\left|\frac{i}{\sqrt{10}}\right|^2 + \left|\frac{1}{\sqrt{2}}\right|^2$$

$$\frac{1}{10} + \frac{1}{2} = \frac{6}{10} = \frac{3}{5}$$

Collapses to

$$\frac{\sqrt{10}}{\sqrt{6}}\left(\boxed{\frac{i}{\sqrt{10}}|0101\rangle + \frac{1}{\sqrt{2}}|1101\rangle}\right)$$

$$\frac{1}{\sqrt{\frac{6}{10}}} = \sqrt{\frac{10}{6}}$$

$$= \frac{i}{\sqrt{6}}|0101\rangle + \sqrt{\frac{5}{6}}|1101\rangle$$

Universal Gate Set

- Classically, NOT + AND are a universal gate set

$\Downarrow$

any Boolean function can be created using AND + NOT (not efficiently)

- Quantumly $\underbrace{\text{CNOT, H, T}}$ universal gate set

$\downarrow$

Problem? Countably infinitely possibility

$\not\,$ Uncountably infinitely many 1-qubit rotations.

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$\boxed{\text{def: Give } \varepsilon > 0\text{, a universal gate set can create any gate to precision } \varepsilon. \text{ (Not efficiently)}}$

BQP

T

$|0\rangle \to |0\rangle$

$|1\rangle \to e^{i\pi/4}|1\rangle$

e.g. $X =$

Circular:

$|0\rangle \to |\circlearrowright\rangle$

$|1\rangle \to |\circlearrowleft\rangle$

Circ $=$