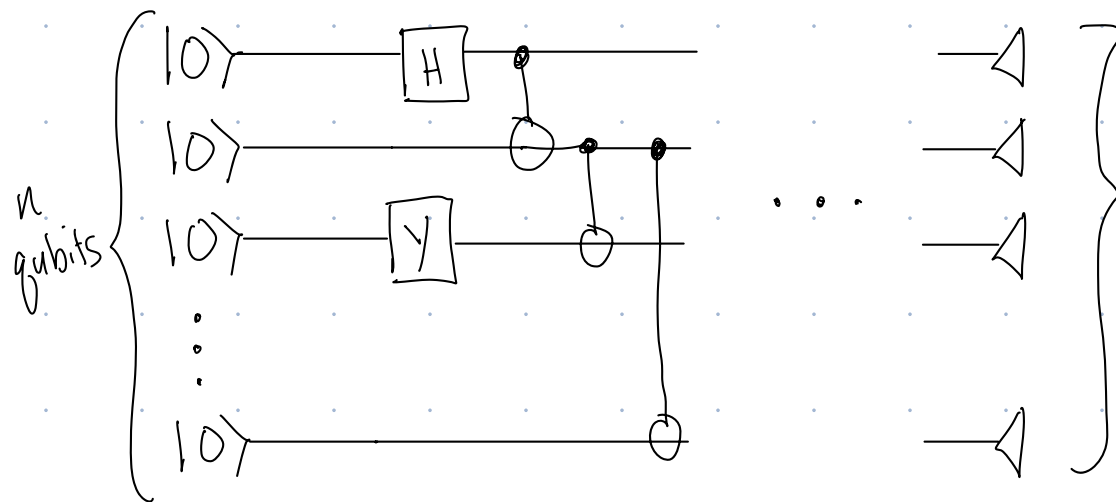


Learning Goals

- Describe multiqubit algorithms using circuits + kets

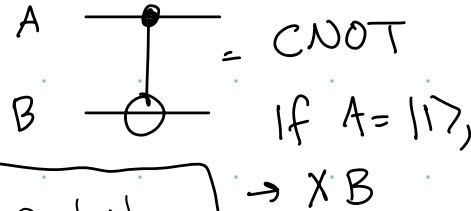
Multigubit Circuit



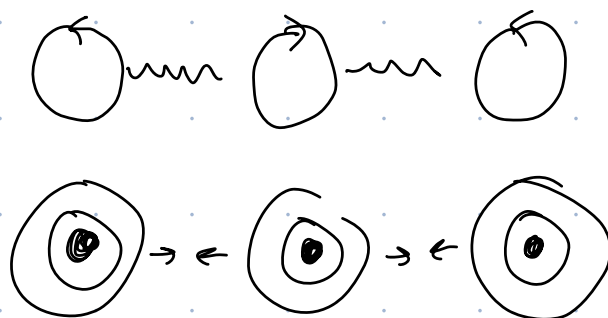
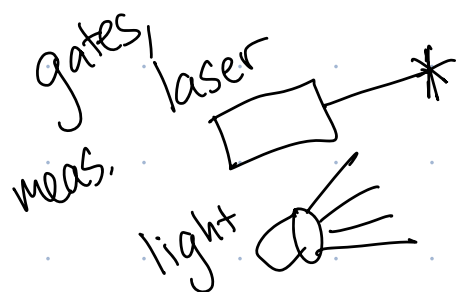
Measure each qubit
in standard basis

$$M = \{|0\rangle, |1\rangle\}$$

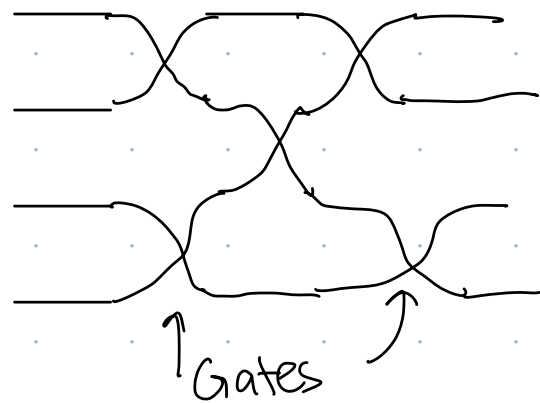
$|0\rangle$ prep, Δ meas
 $\Downarrow$
for ease of implementation



Physically:



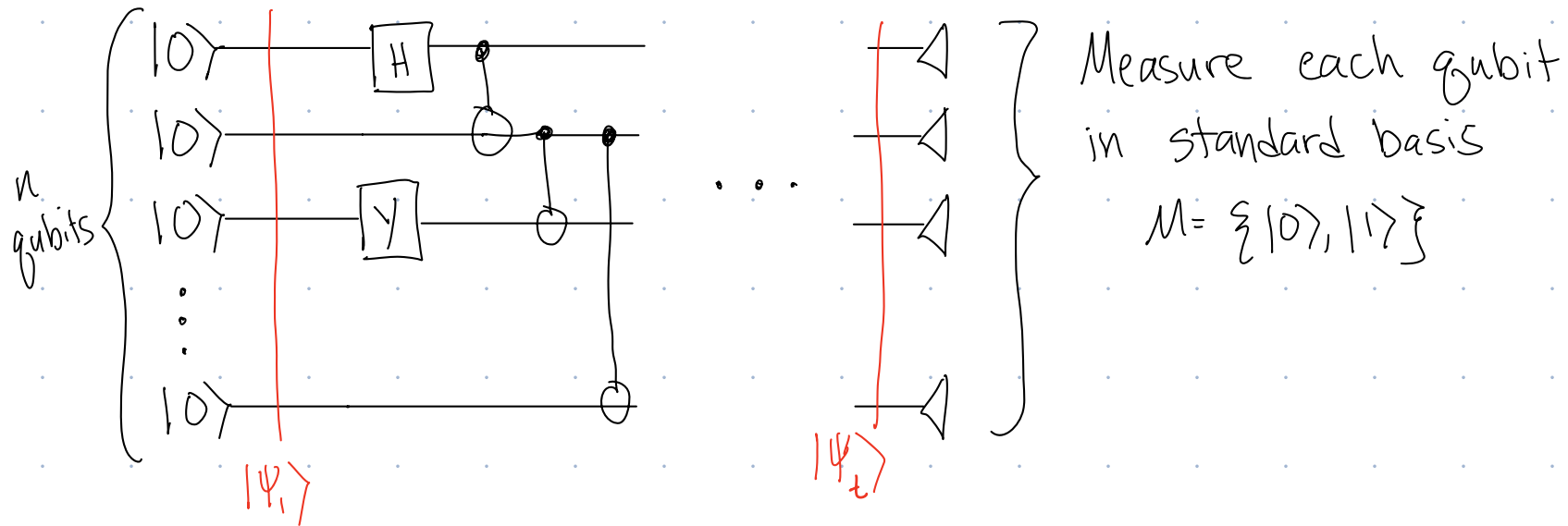
Superconducting circuits
trapped ions/atoms



Photon paths

D^*
meas.

Multigubit Circuit



At the end of the algorithm, state can be written as e.g.

$$|\psi_t\rangle = \frac{1}{\sqrt{10}} |011000\rangle + \frac{i}{\sqrt{10}} |110101\rangle + \sqrt{\frac{3}{10}} e^{2\pi i/3} |100010\rangle + \frac{1}{\sqrt{2}} |111101\rangle$$

Representing + Measuring n -qubit States

$$|\psi\rangle = \sum_{s \in \{0,1\}^n} a_s |s\rangle \quad \text{s.t.} \quad a_s \in \mathbb{C}, \quad \sum_{s \in \{0,1\}^n} |a_s|^2 = 1$$

- When measure $|\psi\rangle$ in standard basis, get outcome $|s\rangle$ with probability $|a_s|^2$

ex:

$$|\psi_+\rangle = \frac{1}{\sqrt{10}} |011000\rangle + \frac{i}{\sqrt{10}} |110101\rangle + \sqrt{\frac{3}{10}} e^{2\pi i/3} |100010\rangle + \frac{1}{\sqrt{2}} |111101\rangle$$

What is the probability of measuring $|100010\rangle$

A) $\sqrt{\frac{3}{10}} e^{2\pi i/3}$

B) $\sqrt{\frac{3}{10}}$

C) $\frac{3}{10}$

D) 0

- When measure first k qubits in standard basis,
 → Rewrite $|\psi\rangle$ by pulling out standard basis states of measured qubits

Partial measurement

$$|\psi\rangle = \underbrace{|000\dots 0\rangle}_k \left(\sum_{i \in \{0,1\}^{n-k}} a_{i,000\dots 0} |i\rangle \right) + \underbrace{|000\dots 1\rangle}_k \left(\sum_{i \in \{0,1\}^{n-k}} a_{i,000\dots 1} |i\rangle \right) + \dots$$

□ Probability of outcome $|000\dots 1\rangle$ is $\sum_{i \in \{0,1\}^{n-k}} |a_{i,000\dots 1}|^2 = p_{000\dots 1}$

□ Remaining qubits collapse to $\frac{1}{\sqrt{p_{000\dots 1}}} \sum_{i \in \{0,1\}^{n-k}} a_{i,000\dots 1} |i\rangle$

ex:

$$|\psi_+\rangle = \frac{1}{\sqrt{10}} |011000\rangle + \frac{i}{\sqrt{10}} |110101\rangle + \sqrt{\frac{3}{10}} e^{2\pi i/3} |100010\rangle + \frac{1}{\sqrt{2}} |111101\rangle$$

Measure 1st 2 qubits:

$$|\psi_+\rangle =$$

Prob of outcome $|11\rangle$

Collapses to

Universal Gate Set

- Classically, NOT + AND are a universal gate set



any Boolean function can be created using AND + NOT (not efficiently)

- Quantumly

Problem?

def :

$$T$$

$$|0\rangle \rightarrow |0\rangle$$

$$|1\rangle \rightarrow e^{i\pi/4}|1\rangle$$

e.g. $X =$

Circular:

$$|0\rangle \rightarrow |\odot\rangle$$

$$|1\rangle \rightarrow |\ominus\rangle$$

Circ =