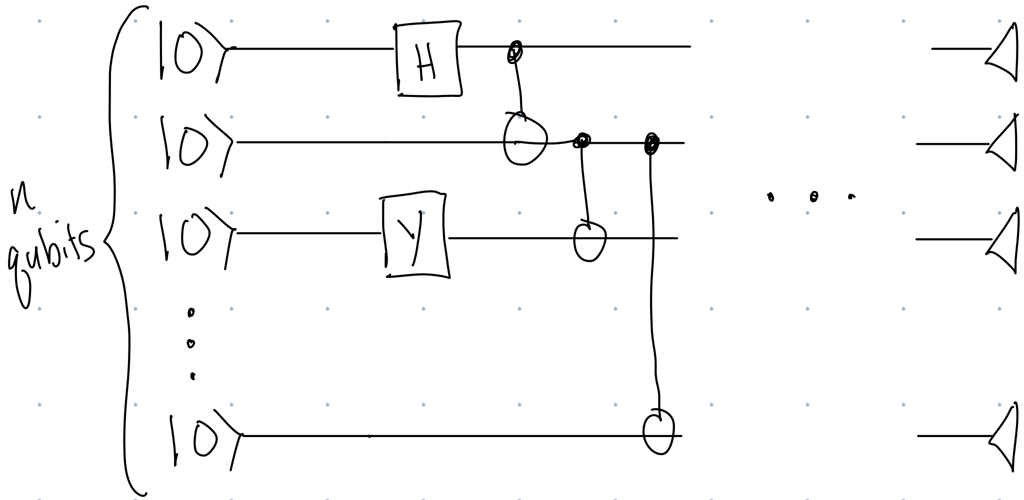
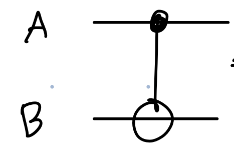
Many-Qubit Circuits

Learning Goals

- Describe multiqubit algorithms using circuits + kets

Multiqubit Circuit



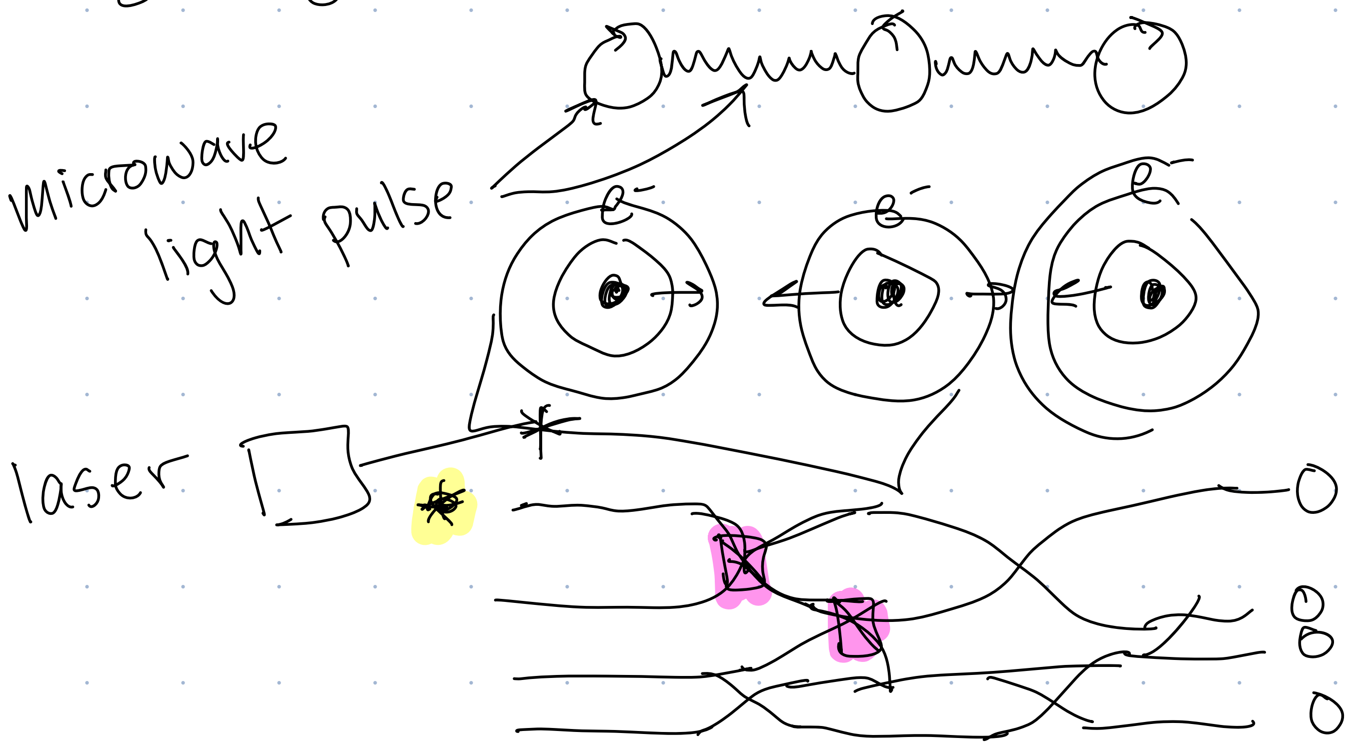


= CNOT

If $A = |1\rangle$,
$\rightarrow XB$

Physically:

Architectures

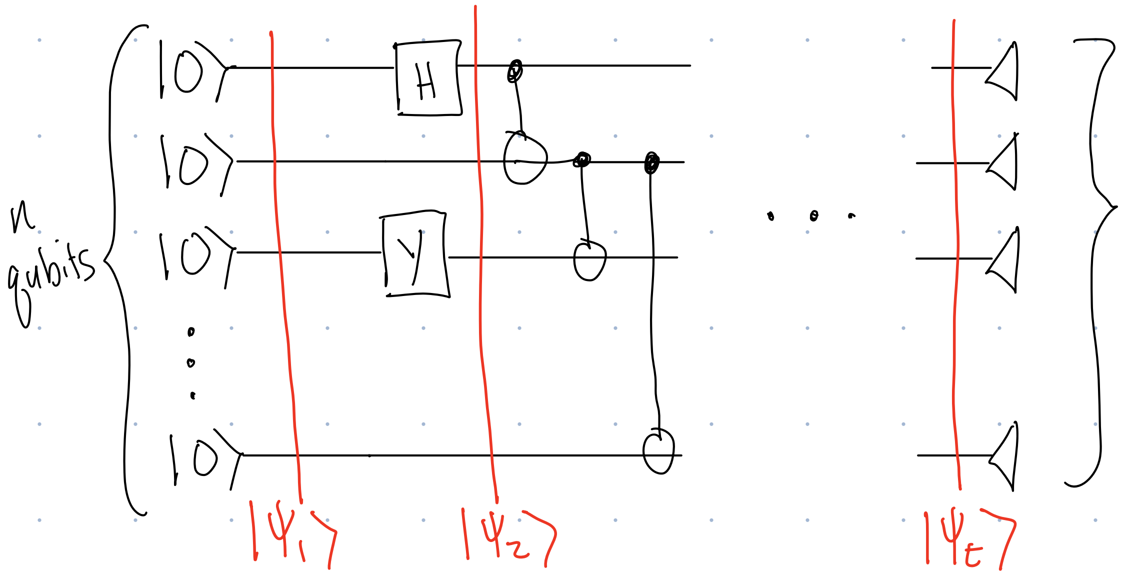


Superconducting Circuit

Trapped Ion/atoms

Photon paths

Multiqubit Circuit



Measure each qubit in standard basis

$M = \{|0\rangle, |1\rangle\}$

$M = \{ |0000\ldots0\rangle$
$|000\ldots1\rangle$
$|000\ldots10\rangle$
$\vdots$
$\}$

$$|\psi_1\rangle = \underbrace{|0\rangle\otimes|0\rangle\otimes|0\rangle\cdots\otimes|0\rangle}_{n} = |0\rangle^{\otimes n} = |0\rangle|0\rangle|0\rangle\cdots|0\rangle = |\underbrace{00\cdots0}_{n}\rangle$$

At the end of the algorithm, state can be written as

$$|\psi_t\rangle = \frac{1}{\sqrt{100}}|\underbrace{011001}_{n\text{-bit}}\rangle + i\sqrt{\frac{3}{50}}|\underbrace{101100}_{n\text{-bit}}\rangle + \cdots$$

$$|+\rangle|-\rangle = \frac{1}{2}|00\rangle + \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle - \frac{1}{2}|01\rangle$$

For n qubits, 2^n standard basis states. Can't write all

Representing + Measuring n-qubit States

$$|\psi\rangle = \sum_{s\in\{0,1\}^n} a_s|s\rangle \quad \text{s.t.} \quad a_s \in \mathbb{C}, \quad \sum_{s\in\{0,1\}^n} |a_s|^2 = 1$$

- When measure $|\psi\rangle$ in standard basis, get outcome $|s\rangle$ with prob. $|a_s|^2$
- When measure first k qubits in standard basis, get outcome $|j\rangle$ (for $j \in \{0,1\}^k$) w/ probability

$$\rightarrow \boxed{\sum_{s:\, s=(j,\ldots)} |a_s|^2}$$

(sum of abs. val. squared amplitudes of s.b states whose first k bits are j)

$$\xrightarrow{\text{collapse}} \frac{1}{\sqrt{\boxed{P(j)}}} \sum_{s:(j,\ldots)} a_s|s\rangle$$

Prob of outcome $|j\rangle$

$|00100\rangle$, $|00110\rangle$
$|00101\rangle$ $|00111\rangle$

Universal Gate Set

- Classically, NOT + AND are a universal gate set

$\Downarrow$

any Boolean function can be created with AND + NOT (not efficiently)

- Quantumly - CNOT, H, T are a universal gate set.

mismatch → Countably many possible

mismatch → Uncountably many 1-qubit rotations

$\boxed{\text{def: Given } \varepsilon > 0\text{, a quantum universal gate set can create any gate to precision } \varepsilon\text{. (Not efficiently)}}$

$$\boxed{\begin{array}{l} \quad T \\ |0\rangle \to |0\rangle \\ |1\rangle \to e^{i\pi/4}|1\rangle \end{array}}$$

e.g. $X = HTTTTH$

$$\left.\begin{array}{l} |0\rangle \to \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle \\ |1\rangle \to \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle \end{array}\right\} TTH$$