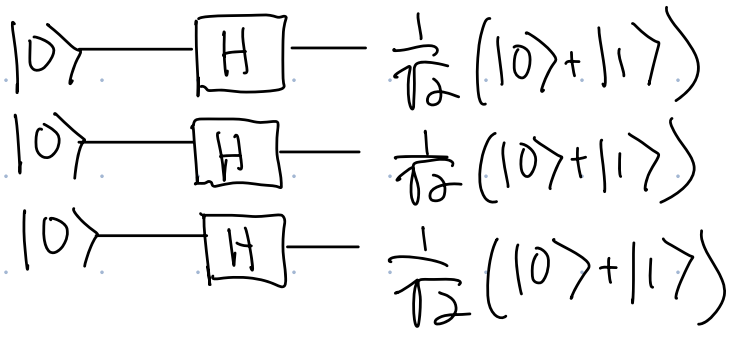
Learning Goals

- Describe how quantum algorithms gain an advantage over probabilistic algorithms.
- Analyze circuits using the path integral formalism

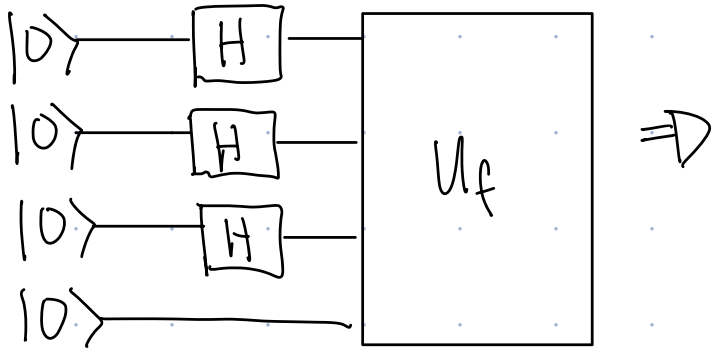
Quantum Secret Sauce?

Superposition?



$$\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\otimes\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\otimes\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$$

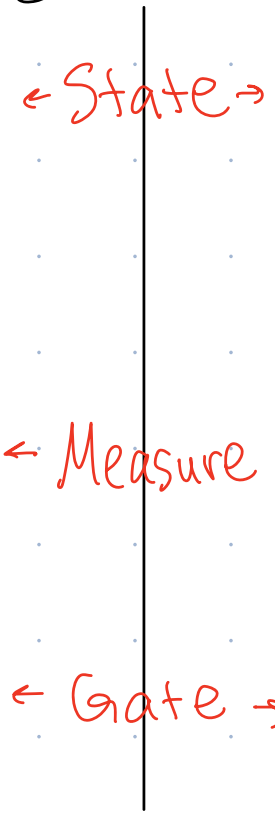
$$=\frac{1}{2\sqrt{2}}(|000\rangle+|001\rangle+|010\rangle+|010\rangle+|100\rangle+|101\rangle+|110\rangle+|111\rangle)$$



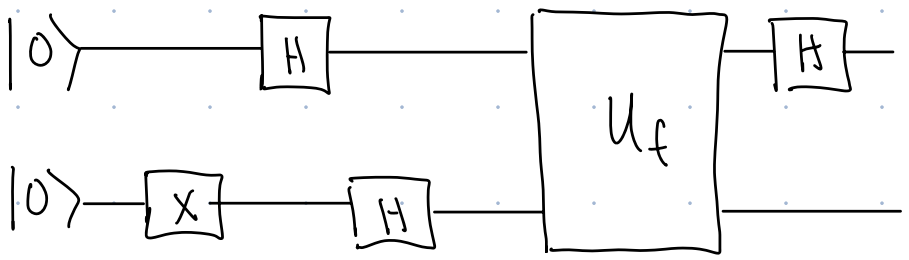
But actually, this exponential scaling is not special

ex:

Quantum Computing vs Probabilistic Computing



Deutsch's Alg (Probabilistic Version)



X:

$|0\rangle \to |1\rangle$

$|1\rangle \to |0\rangle$

Unitary

Left Stoch.

U_f:

$|0\rangle|0\rangle \to |0\rangle|f(0)\rangle$

$|0\rangle|1\rangle \to |0\rangle|\overline{f(0)}\rangle$

$|1\rangle|0\rangle \to |1\rangle|f(1)\rangle$

$|1\rangle|1\rangle \to |1\rangle|\overline{f(1)}\rangle$

Unitary

Left Stoch.

H:

$|0\rangle \to \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

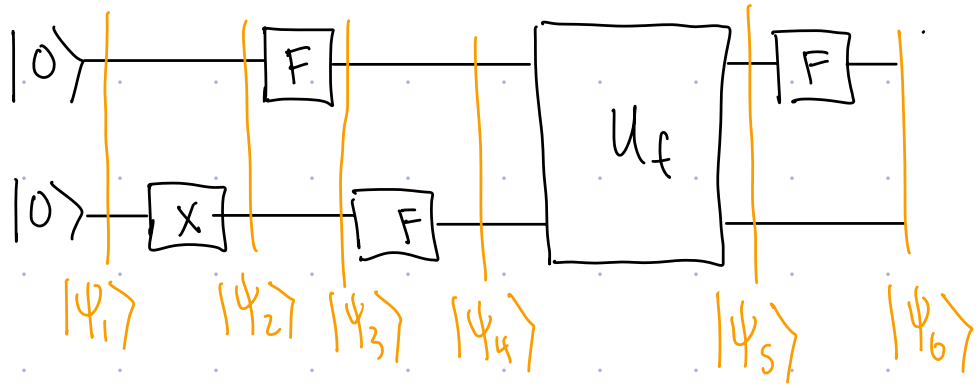
$|1\rangle \to \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

Unitary

Left Stoch.

Instead: F :

Path Integral Analysis (Probabilistic)



For exercise, set

$$f(0) = f(1) = 1$$

What is probability of 1st bit being 1?

$|00\rangle$ _____

$|01\rangle$ _____

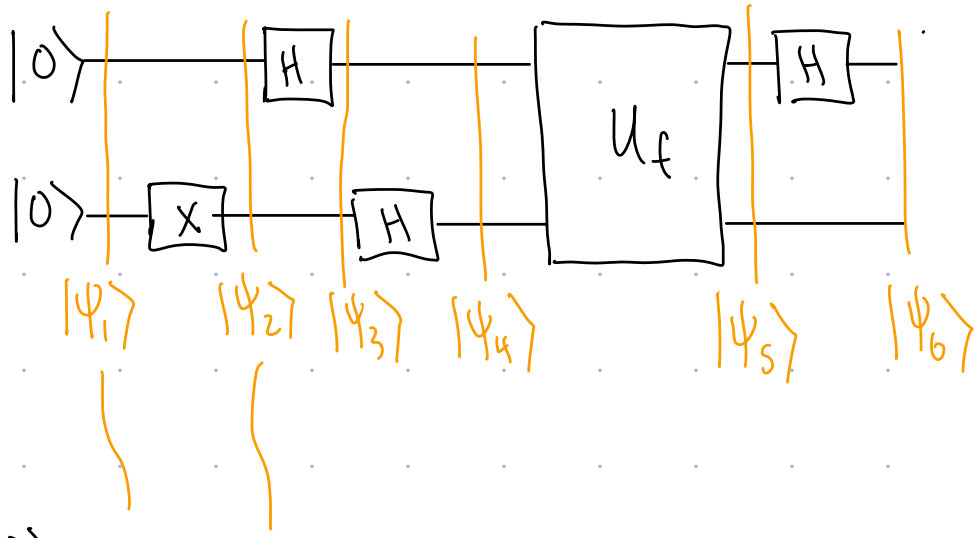
$|10\rangle$ _____

$|11\rangle$ _____

To analyze

- Multiply probabilities on a path to get prob of path
- Add probabilities of all paths terminating at state to get probability of that outcome

Path Integral Analysis (Quantum)



For exercise, set

$f(0) = f(1) = 1$

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$|00\rangle$ ———————————————

$|01\rangle$ ———————————————

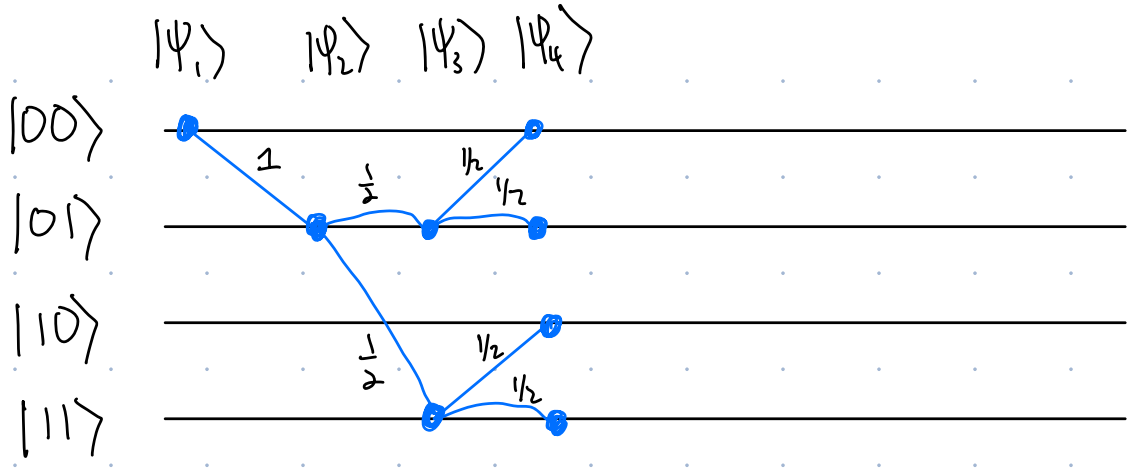
$|10\rangle$ ———————————————

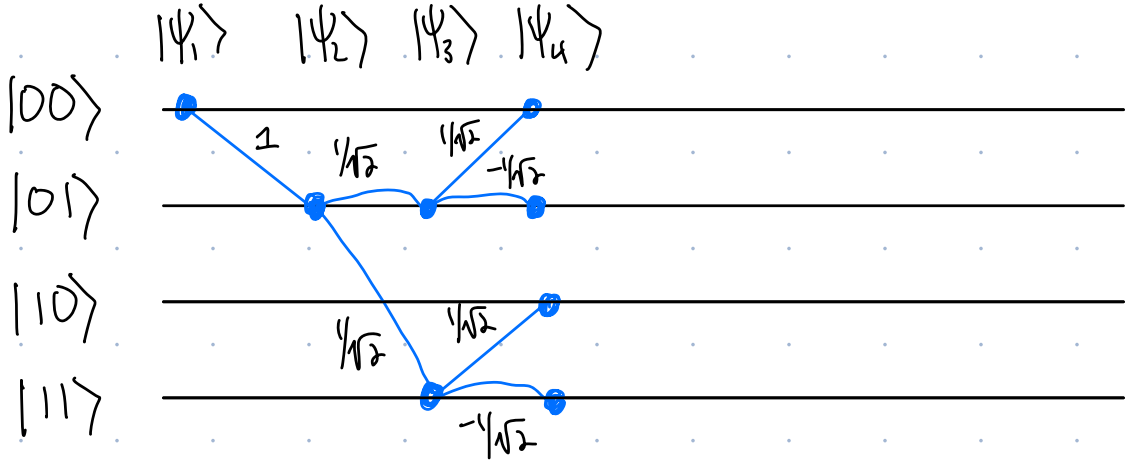
$|11\rangle$ ———————————————

To analyze

- Multiply probabilities on a path to get prob of path
- Add probabilities of all paths terminating at state to get probability of that outcome

Quantum Secret Sauce (Algorithms)


$|\psi_1\rangle$ $|\psi_2\rangle$ $|\psi_3\rangle$ $|\psi_4\rangle$
$|00\rangle$
$|01\rangle$
$|10\rangle$
$|11\rangle$
1
$\frac{1}{2}$
$\frac{1}{2}$
$\frac{1}{2}$
$\frac{1}{2}$
$\frac{1}{2}$


$|\psi_1\rangle$ $|\psi_2\rangle$ $|\psi_3\rangle$ $|\psi_4\rangle$
$|00\rangle$
$|01\rangle$
$|10\rangle$
$|11\rangle$
1
$1/\sqrt{2}$
$1/\sqrt{2}$
$-1/\sqrt{2}$
$1/\sqrt{2}$
$1/\sqrt{2}$
$-1/\sqrt{2}$