Grover's Search Algorithm

Learning Goals

- Practice analyzing a new (2nd most famous) q. algorithm w/ geometric technique
- Learn an important q. algorithmic subroutine

Search Problem

Input:

Output:

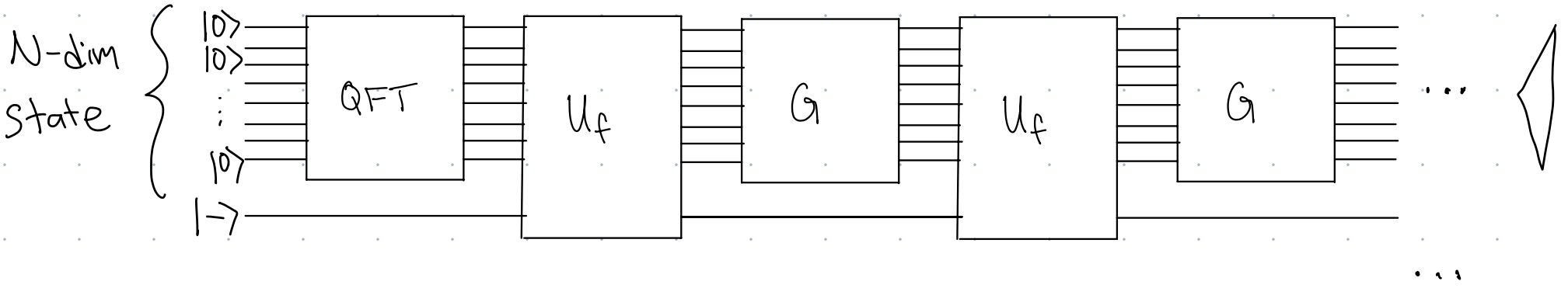
What is the classical query complexity (deterministic)?

A) $O(1)$ B) $O(\log N)$ C) $O(N)$ D) $O(2^N)$

What is the classical query complexity (probabilistic)?

A) $O(1)$ B) $O(\log N)$ C) $O(N)$ D) $O(2^N)$

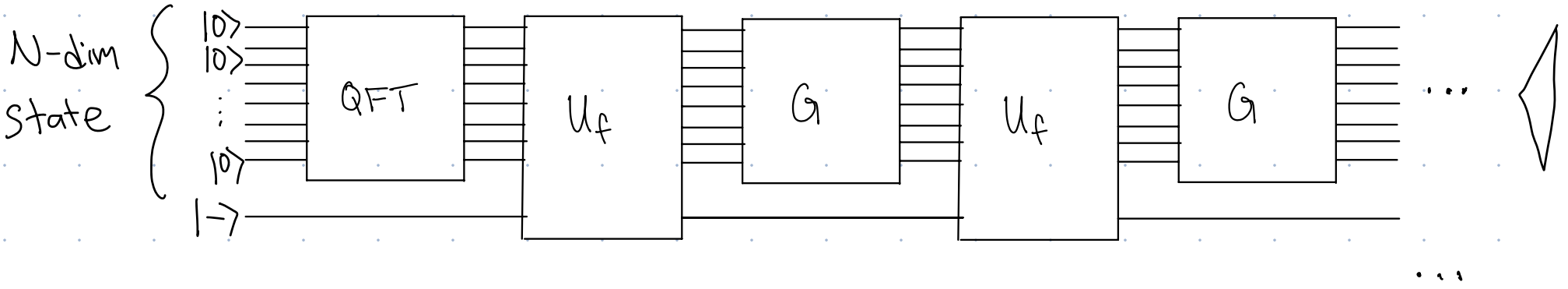
Grover's Algorithm (Quantum Search Alg)



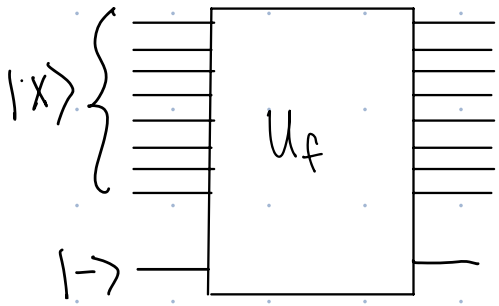
How many qubits are needed to search N possible inputs to f?

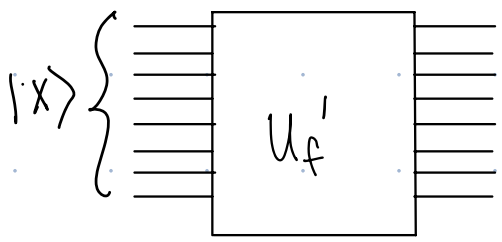
A) $\log_2 N$ B) $\log_2 N + 1$ C) N D) $N + 1$

Grover's Algorithm (Quantum Search Alg)

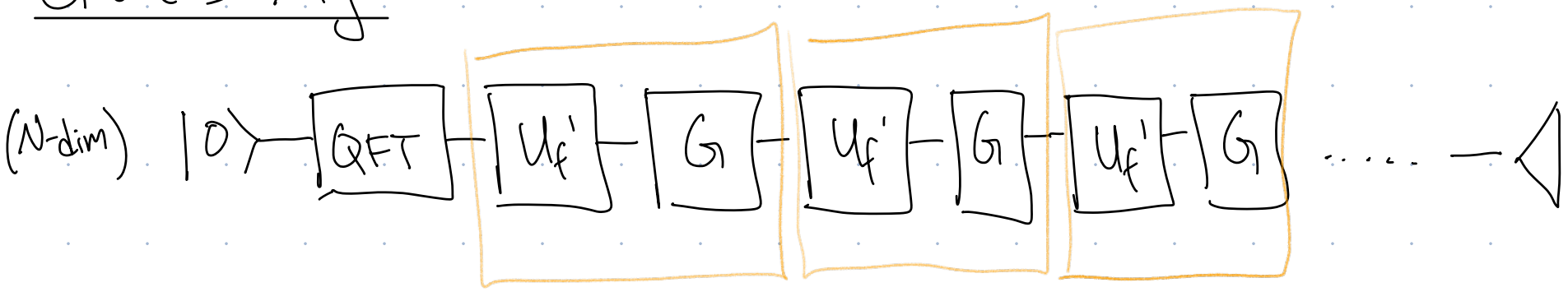


U_f:



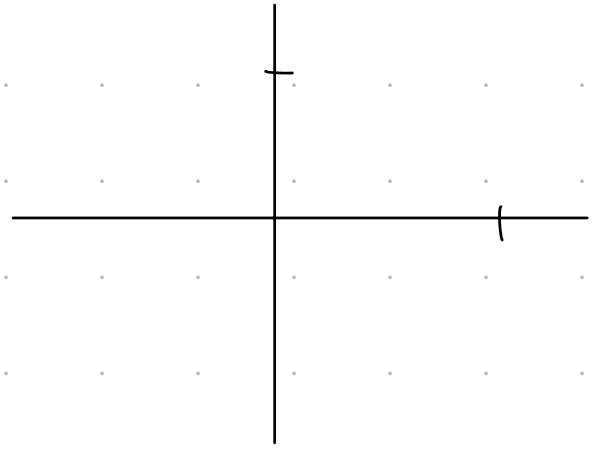


Grovers Alg:

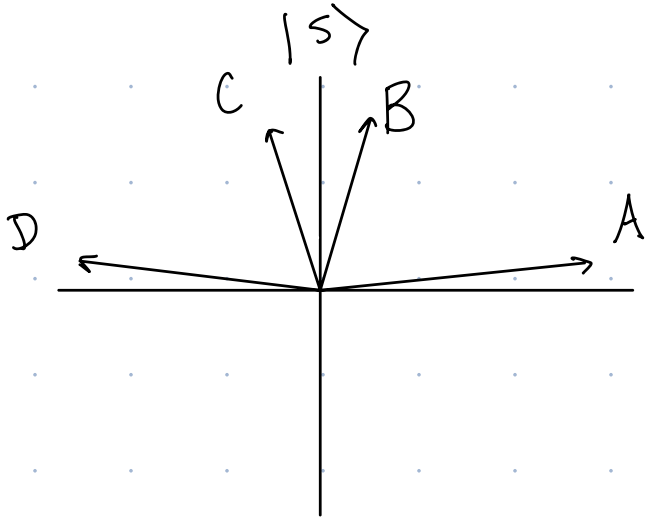


In particular:

We can express $|\psi_i\rangle = a_i|\beta\rangle + b_i|s\rangle$ as a vector in 2D:

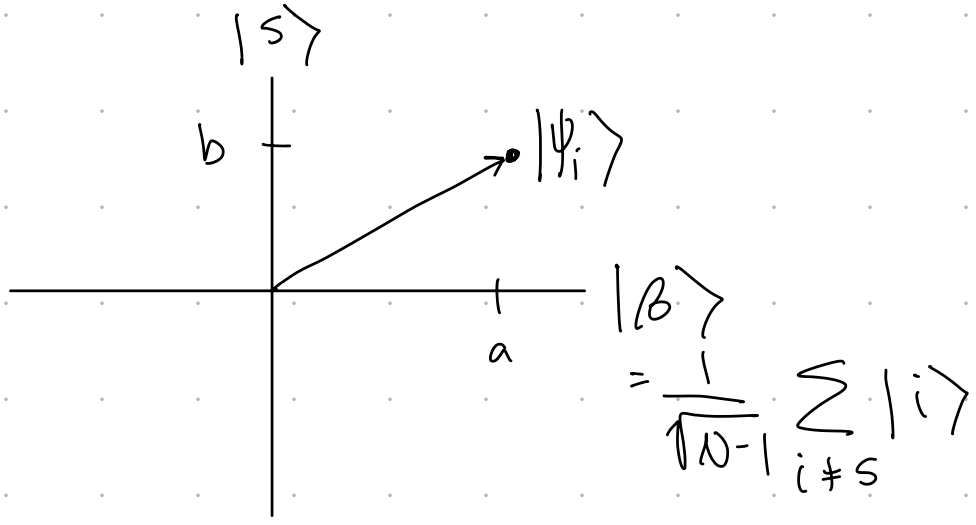


Where is $|\alpha\rangle = \frac{1}{\sqrt{N}} \sum_i |i\rangle$? (assume N is big)



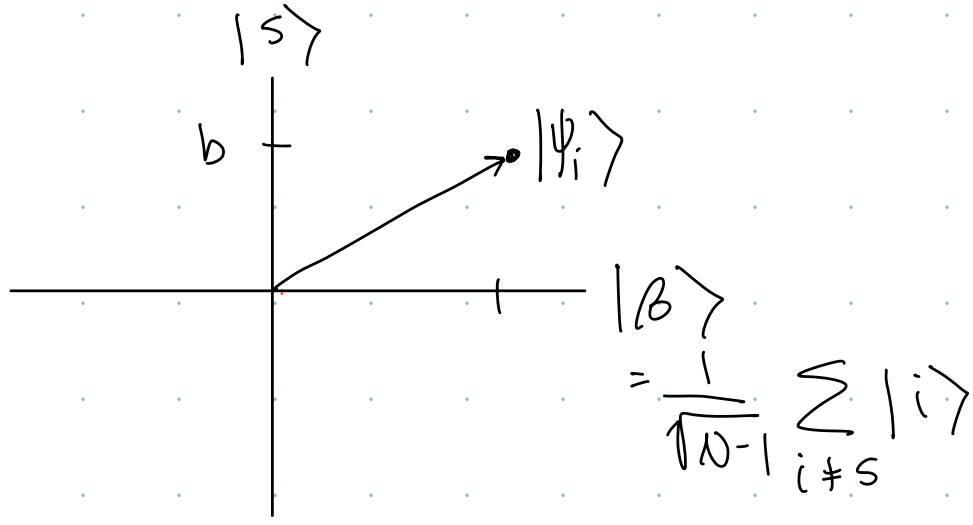
$$|\beta\rangle = \frac{1}{\sqrt{N-1}} \sum_{i \neq s} |i\rangle$$

Effect of $U_f' = I - 2|s\rangle\langle s|$



$$|\beta\rangle = \frac{1}{\sqrt{N-1}} \sum_{i \neq s} |i\rangle$$

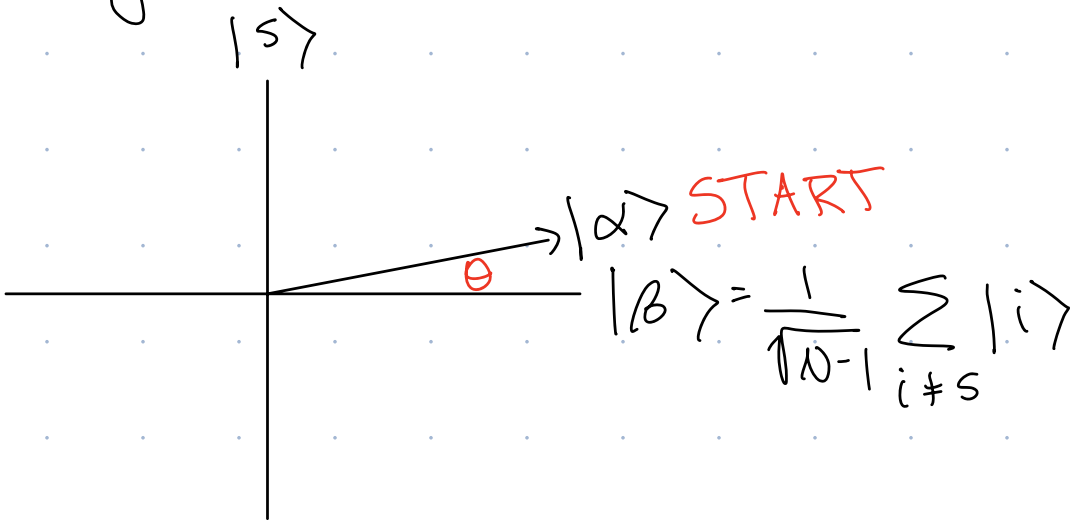
Effect of $G = -I + 2|\alpha\rangle\langle\alpha|$



$$|\beta\rangle = \frac{1}{\sqrt{N-1}} \sum_{i \neq s} |i\rangle$$

How many iterations before state becomes $|s\rangle$?

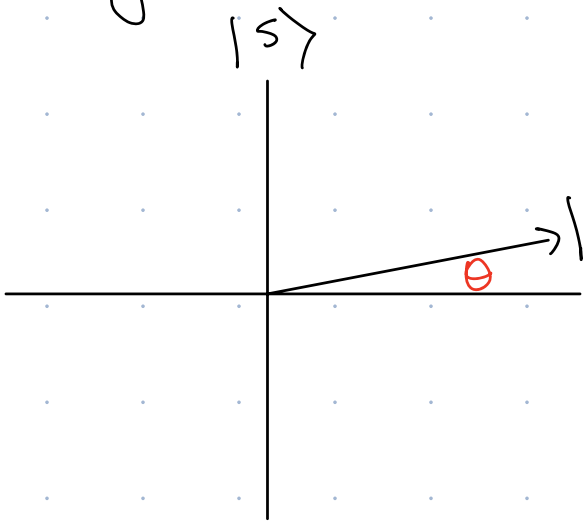
$(U_f', G, U_f', G, \ldots)$



$$|\beta\rangle = \frac{1}{\sqrt{N-1}} \sum_{i \neq s} |i\rangle$$

$\tan^{-1}(x) \approx x$ for $x \ll 1$

How many iterations before state becomes $|s\rangle$?



START

$$|\beta\rangle = \frac{1}{\sqrt{N-1}} \sum_{i \neq s} |i\rangle$$

Note

Application

ex: