

Previous examples of gates (also called operations/unitaries)

◻ Beamsplitter

$$\begin{aligned} |0\rangle_P |H\rangle_D &\rightarrow |0\rangle_P |H\rangle_D \\ |1\rangle_P |H\rangle_D &\rightarrow |1\rangle_P |V\rangle_D \\ |0\rangle_P |V\rangle_D &\rightarrow |0\rangle_P |V\rangle_D \\ |1\rangle_P |V\rangle_D &\rightarrow |1\rangle_P |H\rangle_D \end{aligned}$$

⊙ Wave plate

$$|0\rangle \rightarrow \cos\theta |0\rangle + \sin\theta |1\rangle$$

$$|1\rangle \rightarrow -\sin\theta |0\rangle + \cos\theta |1\rangle$$



$$|1\rangle|V\rangle \xrightarrow{\text{BS}} |1\rangle|H\rangle \xrightarrow{\text{BS}} |1\rangle|V\rangle$$

# Properties of Gates (Necessary + Sufficient)

- (1) Takes a quantum state to another quantum state
- (2) Reversible (without knowing initial state)

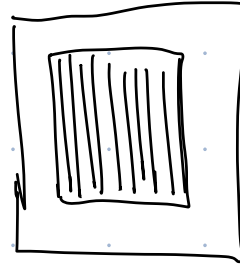
iff

Are these gates? Why/Why not?

X

$$\begin{array}{l} |0\rangle \rightarrow |0\rangle + |1\rangle \\ |1\rangle \rightarrow |0\rangle - |1\rangle \end{array}$$

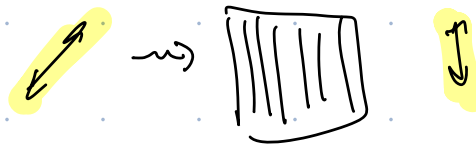
Violates (1)



Violates (2)

Unitary Matrices

$$U^\dagger U = I$$



$$|0\rangle \rightarrow$$

$$|1\rangle \rightarrow$$

$$|00\rangle \rightarrow$$

$$|01\rangle \rightarrow$$

$$|10\rangle \rightarrow$$

$$|11\rangle \rightarrow$$