Error Correction

Learning Goals

- Describe errors in quantum/classical systems
- Describe repetition codes + why they work
- Analyze how a repetition code corrects errors

If Q. Computers are so great, why haven't we built one?

- Classical Computers
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  -

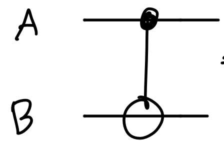
Solution: Repetition Code

- Sources of Quantum Errors
  - control lasers not perfect
    (shape, frequency, focus, intensity)
  - imperfect vacuum
  - Stray magnetic, electric fields
  - Heat (anomalous heating)

Why won't repetition code work for quantum?

A) MAJ not reversible
B) Doesn't correct phase errors (Z)
C) No fanout because can't copy quant
D) There are uncountably infinitely many errors to deal with

Shor's Repetition Code



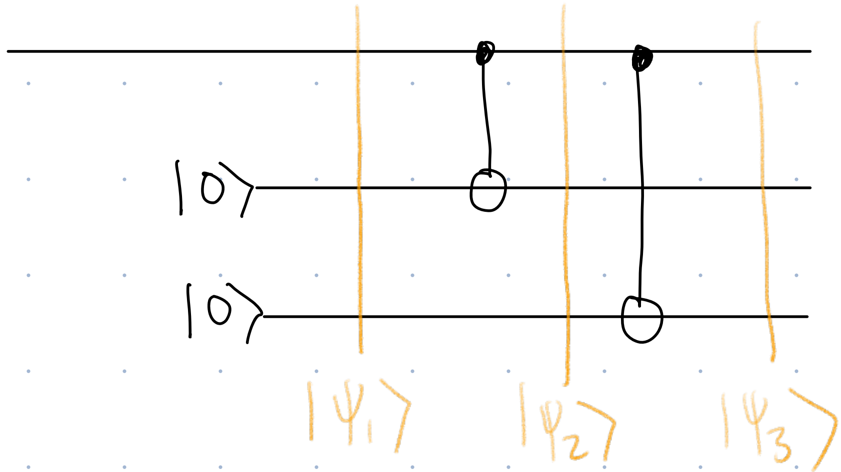
= CNOT

If $A = |1\rangle$,

$\rightarrow X B$

Encoding:

$|\psi\rangle = a|0\rangle + b|1\rangle$



original state to protect from errors. Don't know a, b.

$$|\psi_1\rangle = (a|0\rangle + b|1\rangle)\,|0\rangle\,|0\rangle = a|000\rangle + b|100\rangle$$

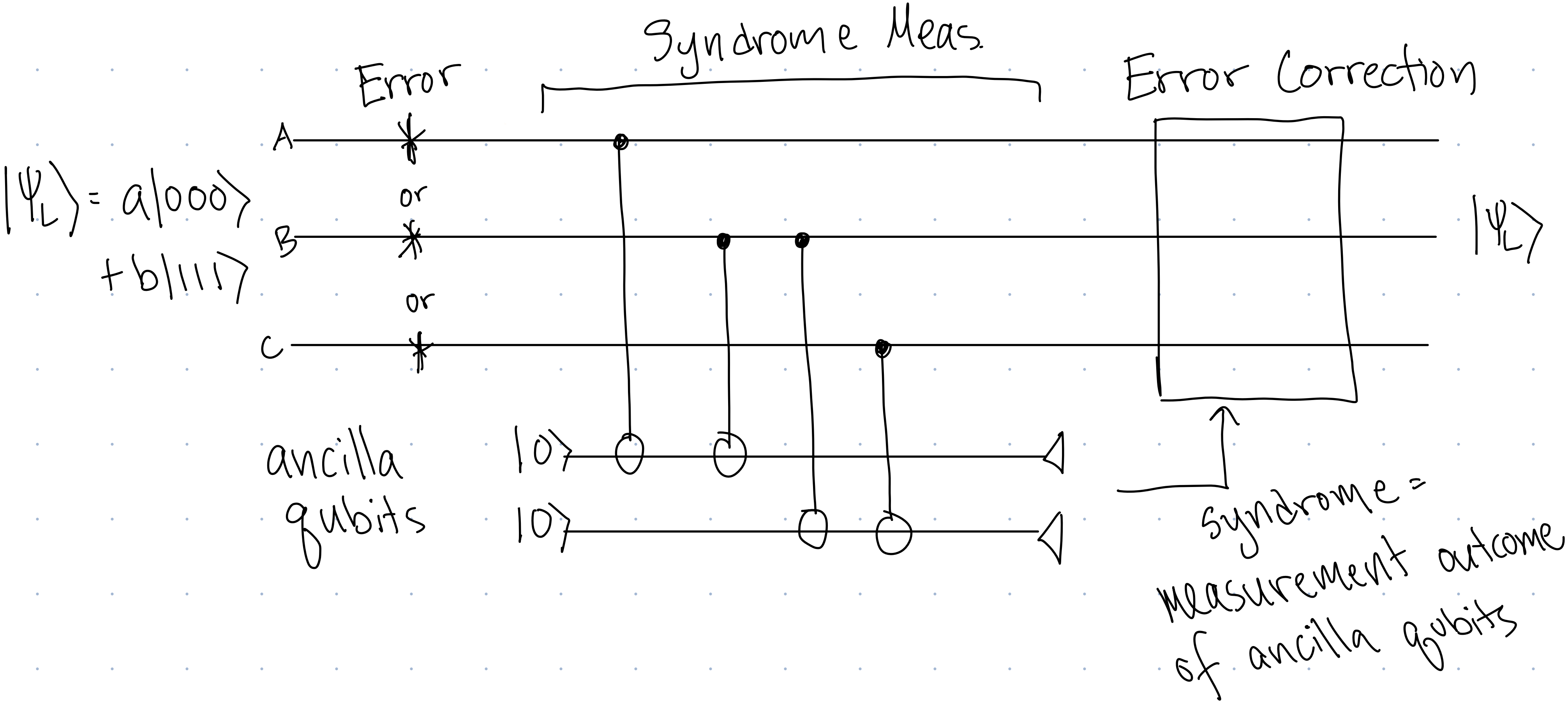
$|\psi_2\rangle =$

$=$

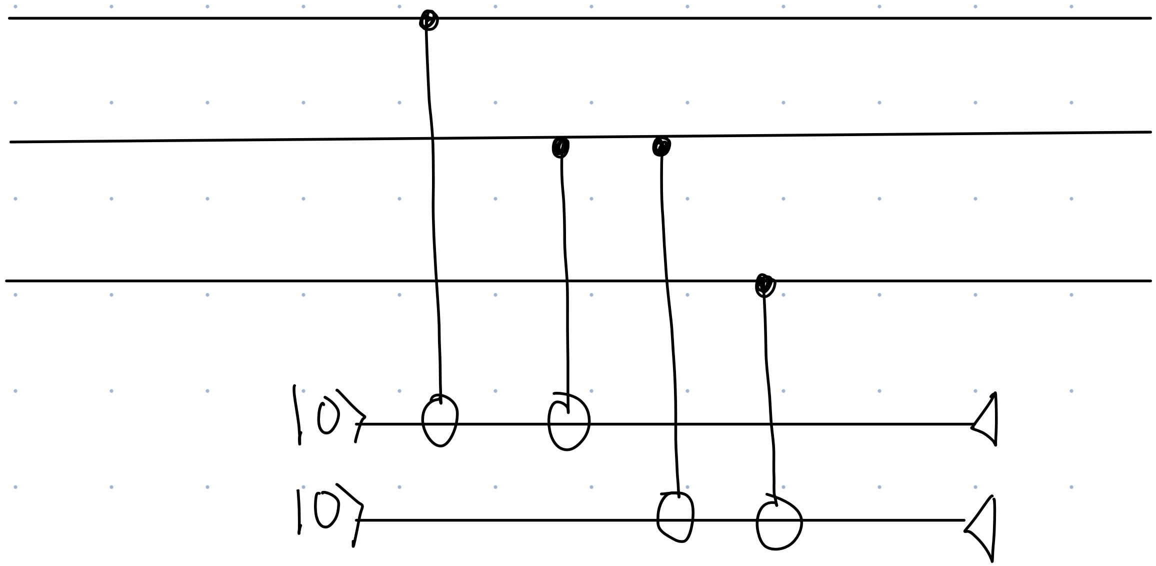
$|\psi_3\rangle =$

$=$

Error Correction Circuit



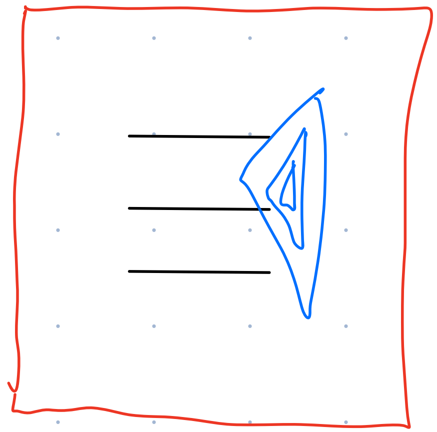
Syndrome Meas.



$\approx$

Projective Measurement

1.



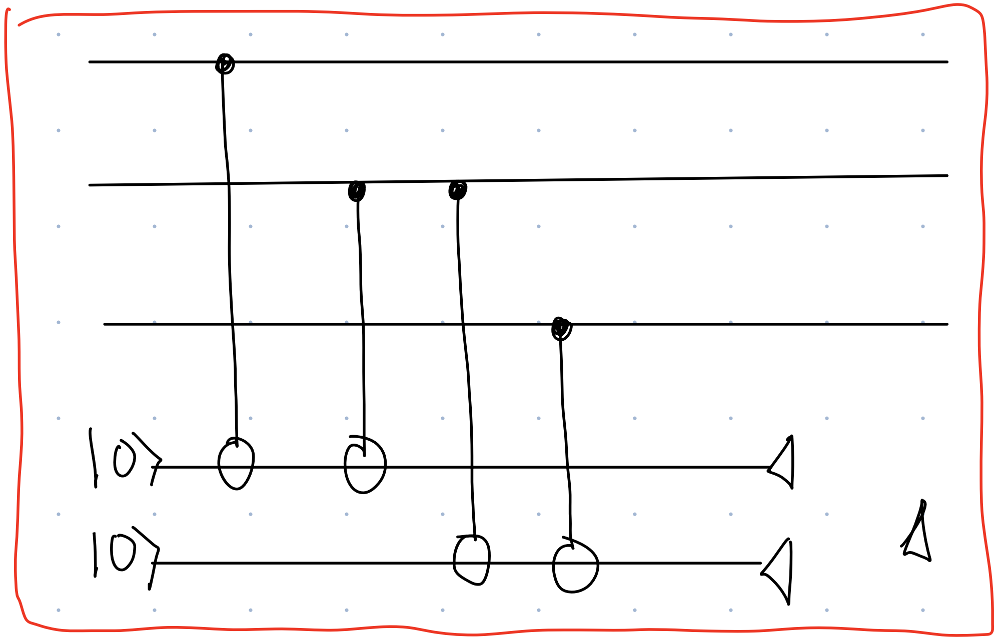
2.

3.

Projective Measurement:

4. Probability of outcome P_i

After outcome i, state collapses to



$\approx$

If measure $|\psi\rangle = \frac{1}{\sqrt{6}}|000\rangle + \sqrt{\frac{2}{6}}|001\rangle + \sqrt{\frac{3}{6}}|110\rangle$ with $\mathcal{M}$, which outcomes are possible?

A) P_0

B) P_1

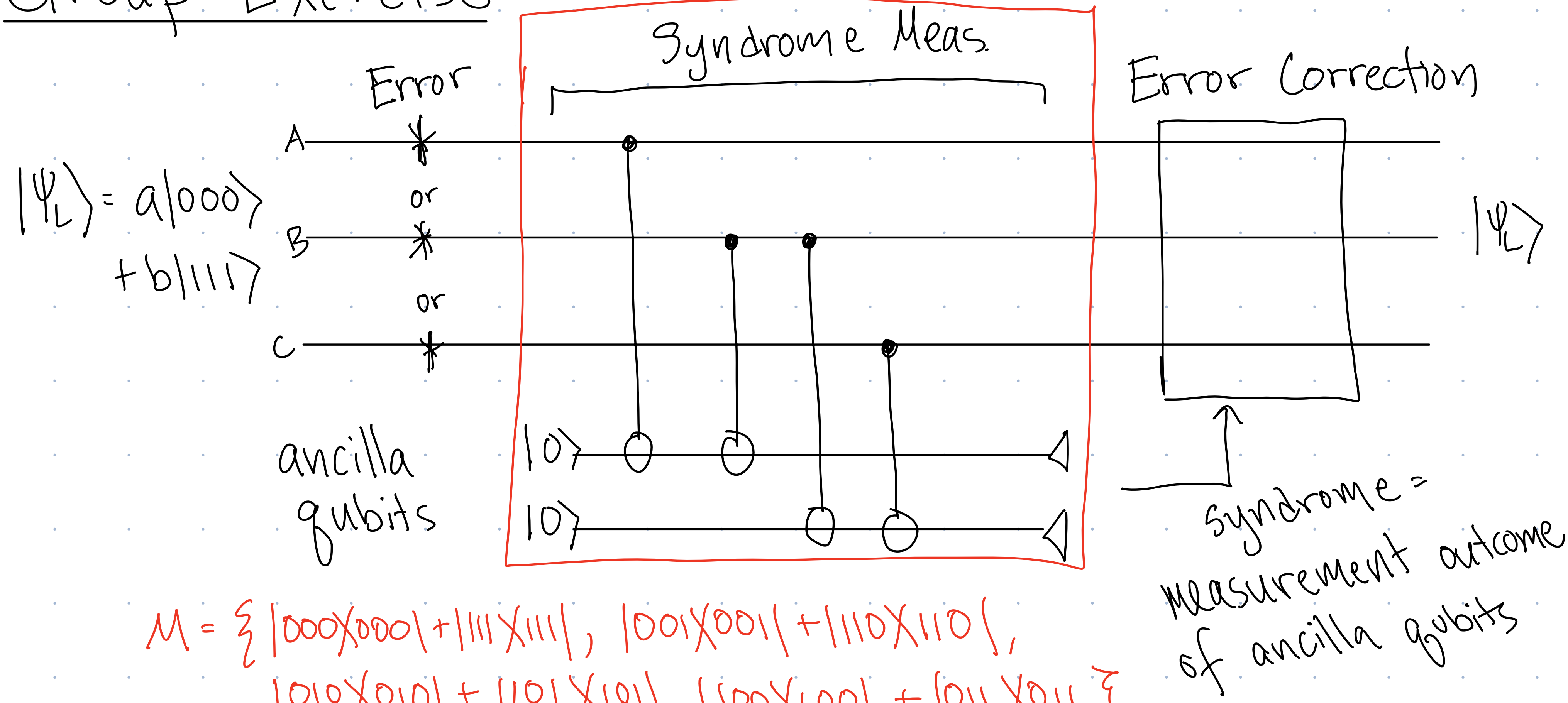
C) P_2

D) P_3

If measure $|\psi\rangle = \frac{1}{\sqrt{6}}|000\rangle + \sqrt{\frac{2}{6}}|001\rangle + \sqrt{\frac{3}{6}}|110\rangle$ with $\mathcal{M}$ and get outcome $P_1 = |001\rangle\langle 001| + |110\rangle\langle 110|$ what is the resultant state?

$$|\psi\rangle \rightarrow \frac{P_i|\psi\rangle}{\sqrt{Pr(i)}} \qquad Pr(i) = \langle\psi|P_i|\psi\rangle$$

Group Exercise



$M = \{ |000\rangle\langle 000| + |111\rangle\langle 111|,\ |001\rangle\langle 001| + |110\rangle\langle 110|,$
$|010\rangle\langle 010| + |101\rangle\langle 101|,\ |100\rangle\langle 100| + |011\rangle\langle 011| \}$

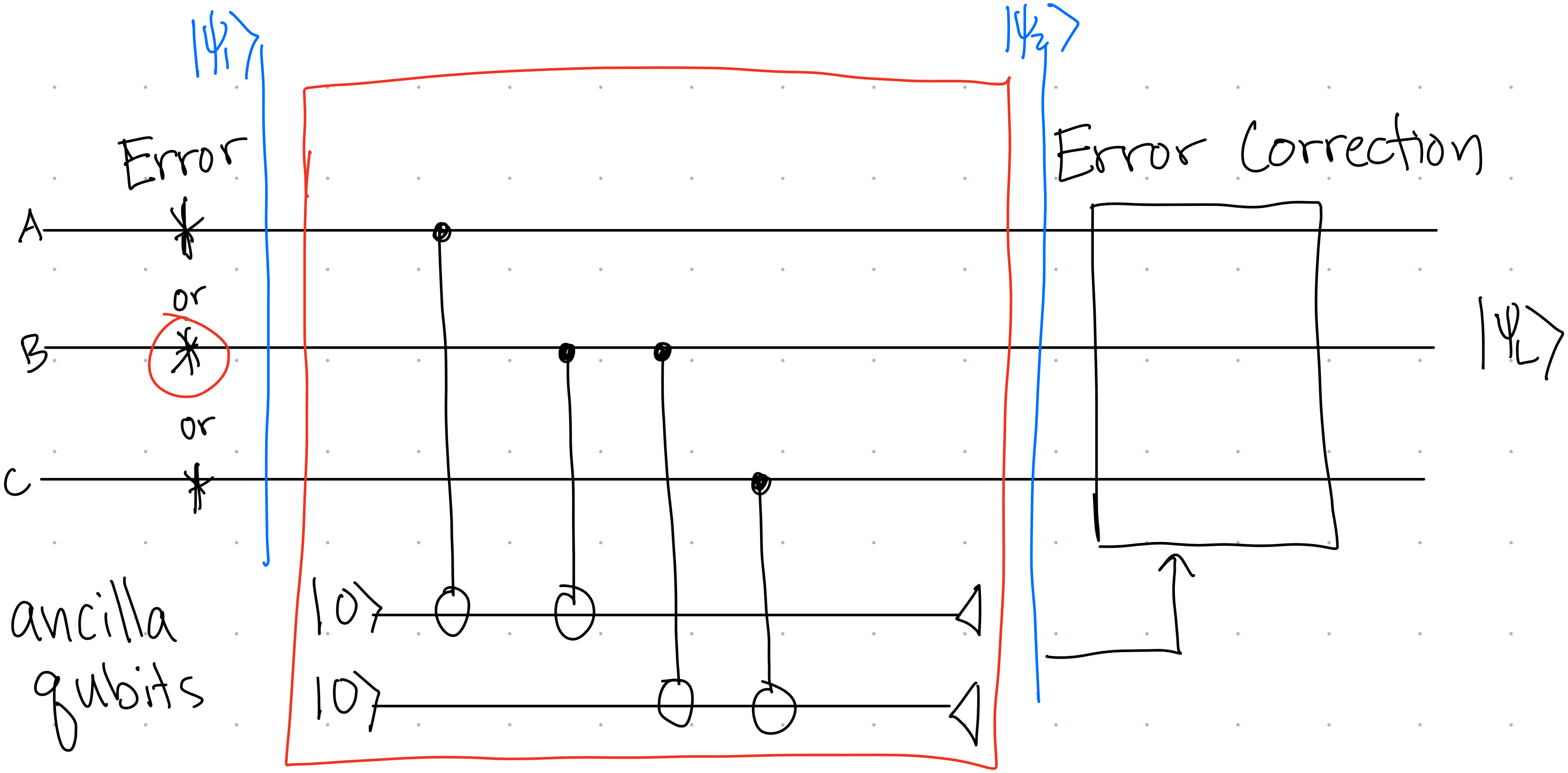
$*$: $R_x(\theta) = \cos\theta\, I + i \sin\theta\, X$

ex: $R_x(\theta)|1\rangle = \cos\theta\, I|1\rangle + i\sin\theta\, X|1\rangle \quad = \cos\theta|1\rangle + i\sin\theta|0\rangle$

1. What are possible outcomes + what does state collapse to?
2. What error correction procedure should be used in each case?

Solution

$$|\psi_L\rangle = a|000\rangle + b|111\rangle$$



$$M = \{ |000\rangle\langle 000| + |111\rangle\langle 111|,\ |001\rangle\langle 001| + |110\rangle\langle 110|,$$
$$|010\rangle\langle 010| + |101\rangle\langle 101|,\ |100\rangle\langle 100| + |011\rangle\langle 011| \}$$

$|\psi_1\rangle =$

$=$

- Outcome

- Outcome

Error on A qubit:

Error on B qubit

Error on C qubit

Big Idea with Error Correction

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Need to Correct More Errors? Bigger Code!

$$|0_L\rangle = \left(|000\rangle_{123} + |111\rangle_{123}\right)\left(|000\rangle_{456} + |111\rangle_{456}\right)\left(|000\rangle_{789} + |111\rangle_{789}\right)$$

$$|1_L\rangle = \left(|000\rangle_{123} - |111\rangle_{123}\right)\left(|000\rangle_{456} - |111\rangle_{456}\right)\left(|000\rangle_{789} - |111\rangle_{789}\right)$$

9-qubit Shor Code

Corrects any single qubit error

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9-qubit Shor Code

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Z_6 error:

$$|0_L\rangle = \left(|000\rangle_{123} + |111\rangle_{123}\right)\left(|000\rangle_{456} + |111\rangle_{456}\right)\left(|000\rangle_{789} + |111\rangle_{789}\right)$$

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Z_1 and Z_4

$$|0_L\rangle = \left(|000\rangle_{123} + |111\rangle_{123}\right)\left(|000\rangle_{456} + |111\rangle_{456}\right)\left(|000\rangle_{789} + |111\rangle_{789}\right)$$

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Which errors can be corrected by Shor's code?

$$|0_L\rangle = \left(|000\rangle_{123} + |111\rangle_{123}\right)\left(|000\rangle_{456} + |111\rangle_{456}\right)\left(|000\rangle_{789} + |111\rangle_{789}\right)$$

$$|1_L\rangle = \left(|000\rangle_{123} - |111\rangle_{123}\right)\left(|000\rangle_{456} - |111\rangle_{456}\right)\left(|000\rangle_{789} - |111\rangle_{789}\right)$$

A) $Z_1 Z_2$

B) $Z_4 Z_8$

C) $Z_1 Z_2 Z_3$

D) $X_1 Z_2$

E) $X_1 X_2$

F) $X_1 X_4$

G) $Y_1 Z_4$

Fault Tolerant Quantum Computing

Shor Code: All single qubit errors, 9 qubits, 1 logical qubit

Concatenation:

$$|0_L\rangle = \left(|000\rangle_{123} + |111\rangle_{123}\right)\left(|000\rangle_{456} + |111\rangle_{456}\right)\left(|000\rangle_{789} + |111\rangle_{789}\right)$$

$$|1_L\rangle = \left(|000\rangle_{123} - |111\rangle_{123}\right)\left(|000\rangle_{456} - |111\rangle_{456}\right)\left(|000\rangle_{789} - |111\rangle_{789}\right)$$

But... more qubits means more chance of error. Can only keep ahead of accumulating errors if error rate is small enough