Error Correction

Learning Goals

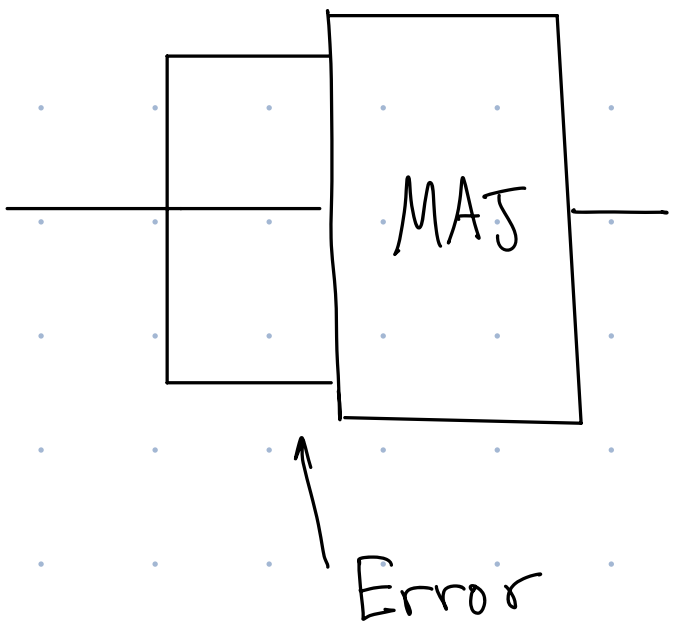
- Describe errors in quantum/classical systems
- Describe repetition codes + why they work
- Analyze how a quantum repetition code corrects errors using projective partial measurements ← [QC4]

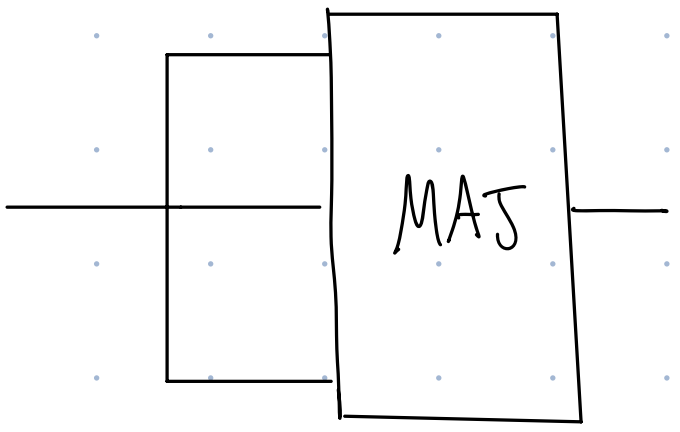
If Q. Computers are so great, why haven't we built one?

- Classical Computers

- Pace maker - vote counting machine - Plane drop
- Super mario glitch

Solution: Repetition Code





- Sources of Quantum Errors
  - control lasers not perfect
    (shape, frequency, focus, intensity)
  - imperfect vacuum
  - Stray magnetic, electric fields
  - Heat (anomalous heating)

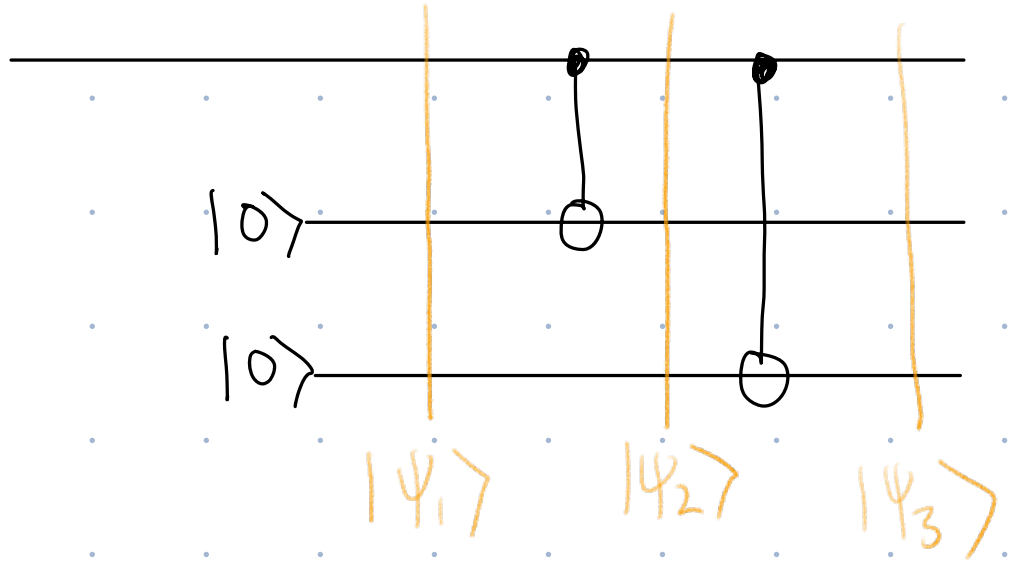
Why won't repetition code work for quantum?

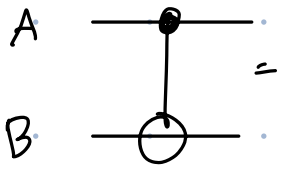
A) MAJ not reversible
B) Doesn't correct phase errors (Z)
C) No fanout because can't copy quant
D) There are uncountably infinitely many errors to deal with

Some scientists think error correction at a large scale is impossible

Shor's Repetition Code

Encoding:





= CNOT

If A = |1⟩,

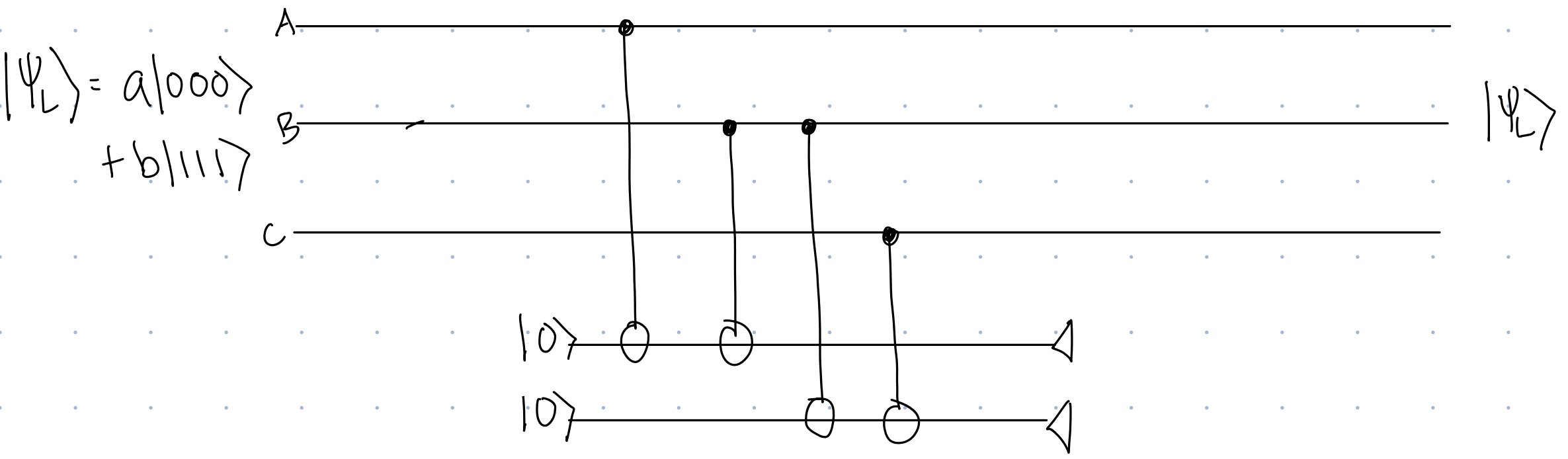
→ X B

$|\psi_1\rangle =$

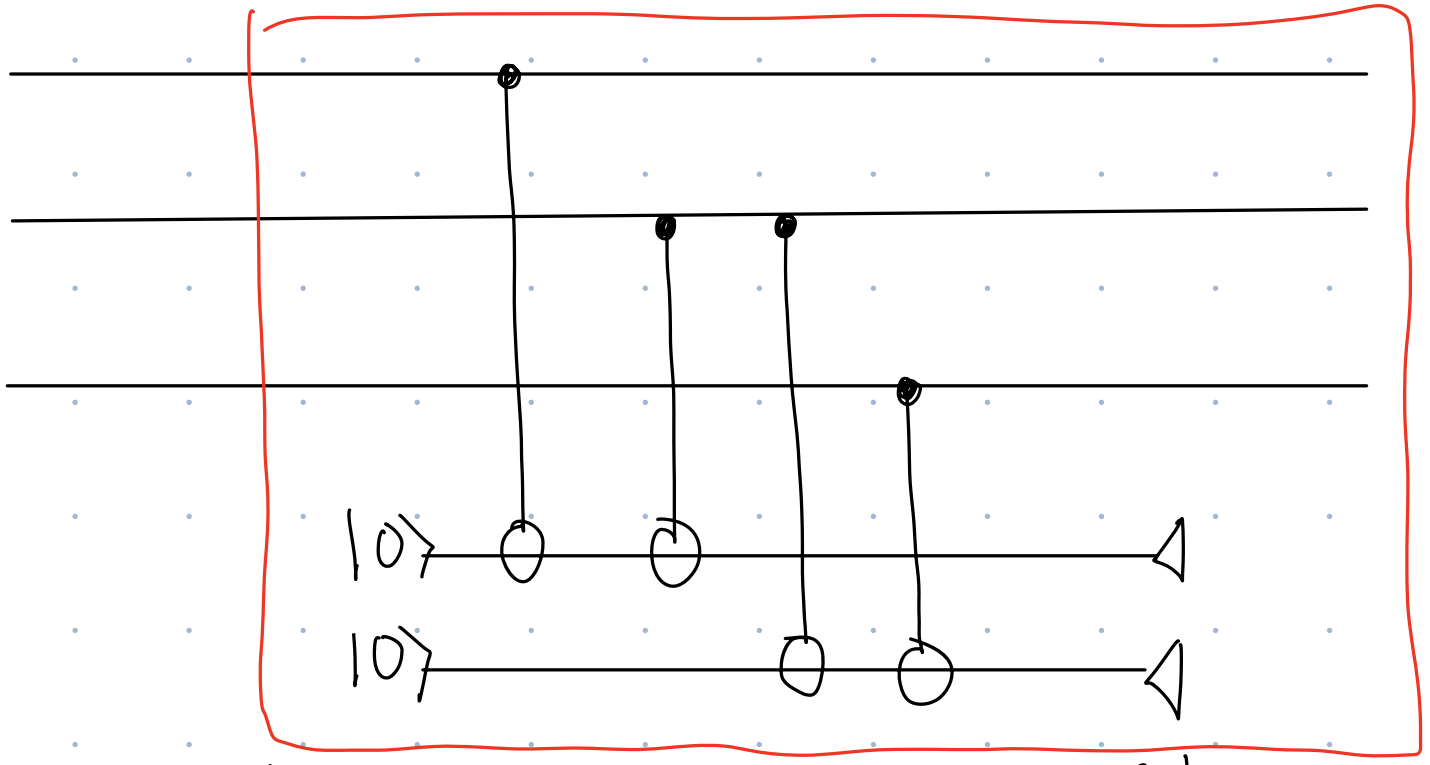
$|\psi_2\rangle$

$|\psi_3\rangle =$

Error Correction Circuit



Syndrome Meas.



Not another partial measurement!

New approach:

Projective Measurement

Described by a
Set of projectors

$P_0 =$

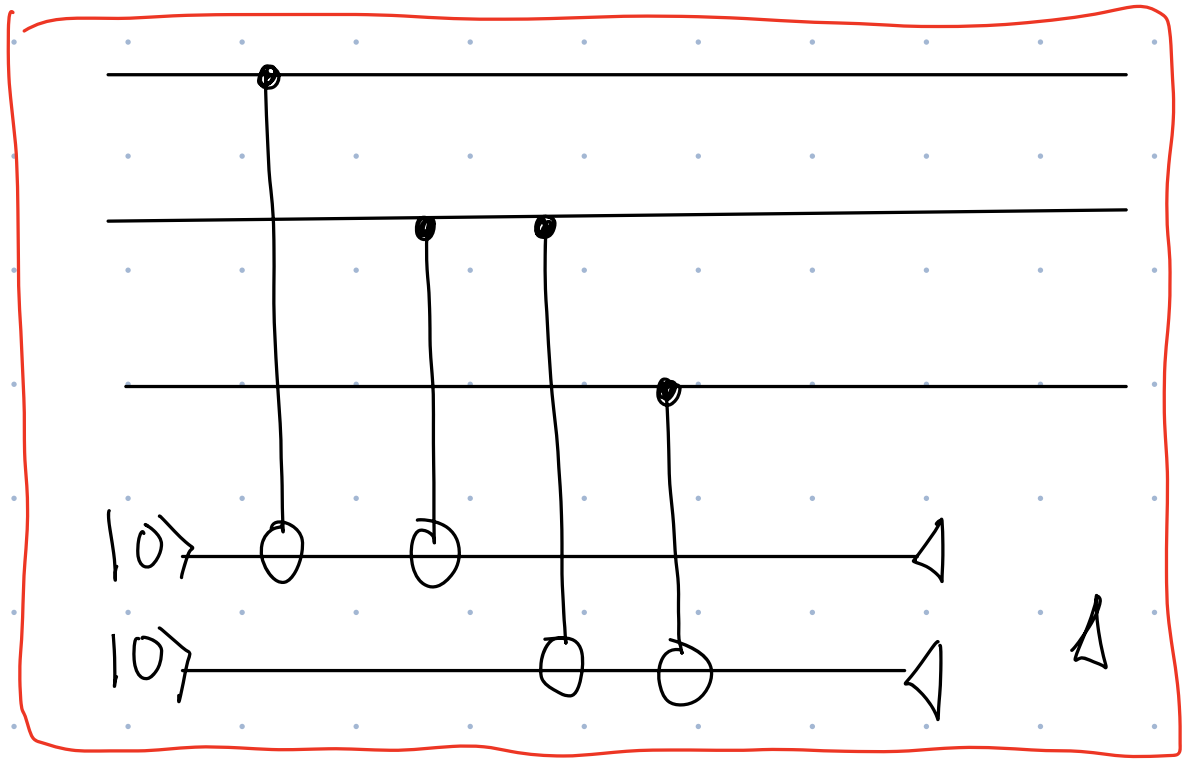
$P_1 =$

$P_2 =$

⋮

techical: $\sum_i P_i = I$

Probability of outcome P_i:
After outcome i, State collapses to



$$M = \{ |000\rangle\langle 000| + |111\rangle\langle 111|,\ |001\rangle\langle 001| + |110\rangle\langle 110|,\ |010\rangle\langle 010| + |101\rangle\langle 101|,\ |100\rangle\langle 100| + |011\rangle\langle 011| \}$$

If measure $|\psi\rangle = \frac{1}{\sqrt{6}}|000\rangle + \sqrt{\frac{2}{6}}|001\rangle + \sqrt{\frac{3}{6}}|110\rangle$ with M, which outcomes are possible? $|\psi\rangle \to \frac{P_i|\psi\rangle}{\sqrt{P_r(i)}}$ $P_r(i) = \langle\psi|P_i|\psi\rangle$

A) P_0

B) P_1

C) P_2

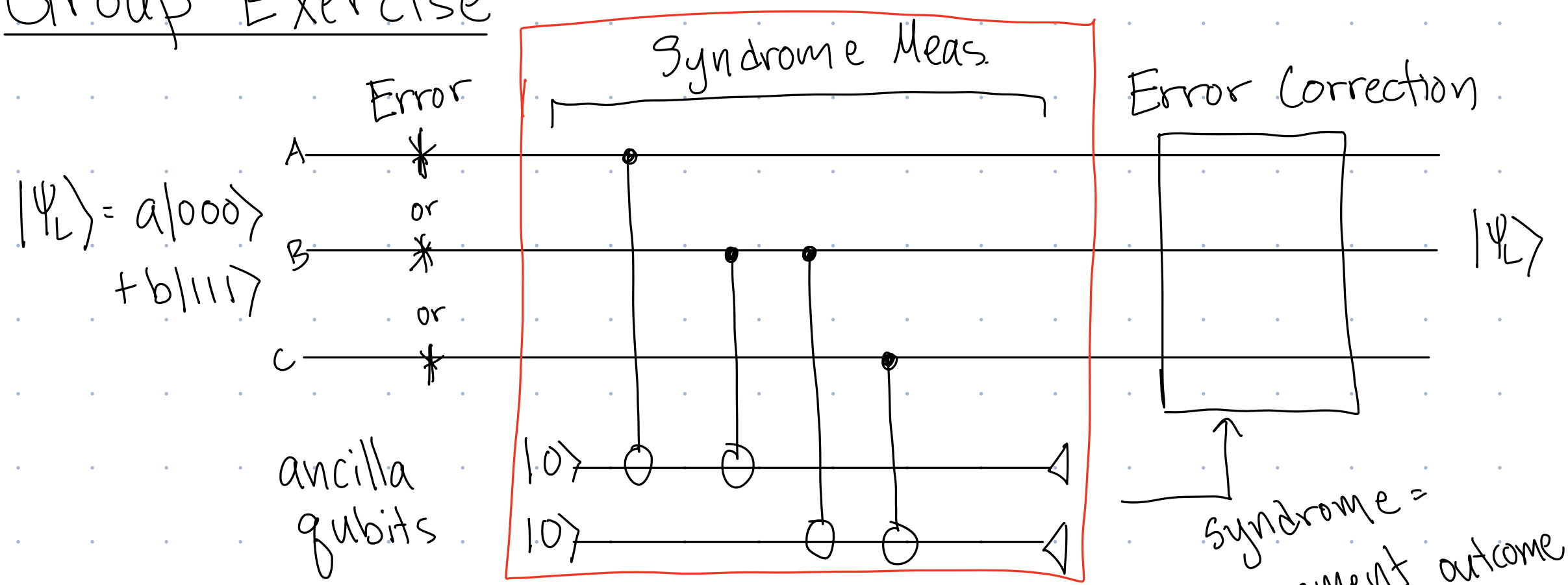
D) P_3

If measure $|\psi\rangle = \frac{1}{\sqrt{6}}|000\rangle + \sqrt{\frac{2}{6}}|001\rangle + \sqrt{\frac{3}{6}}|110\rangle$ with M_1

and get outcome $P_1 = |001\rangle\langle 001| + |110\rangle\langle 110|$ what is

the resultant state? $|\psi\rangle \rightarrow \frac{P_i|\psi\rangle}{\sqrt{Pr(i)}}$ $Pr(i) = \langle\psi|P_i|\psi\rangle$

Group Exercise



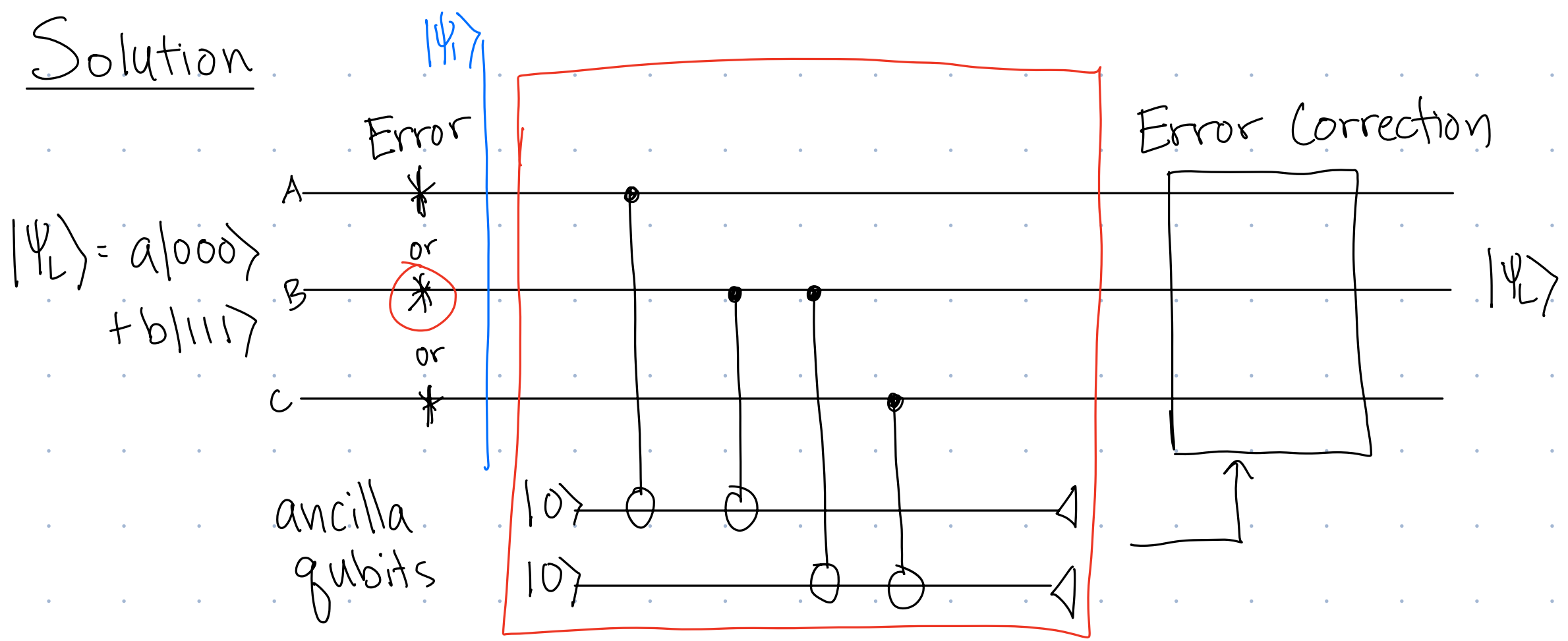
$M = \{|000\rangle\langle 000| + |111\rangle\langle 111|,\ |001\rangle\langle 001| + |110\rangle\langle 110|,$
$|010\rangle\langle 010| + |101\rangle\langle 101|,\ |100\rangle\langle 100| + |011\rangle\langle 011|\}$

*: $R_x(\theta) = \cos\theta I + i\sin\theta X$

ex: $R_x(\theta)|1\rangle = \cos\theta I|1\rangle + i\sin\theta X|1\rangle = \cos\theta|1\rangle + i\sin\theta|0\rangle$

1. What are possible outcomes + what does state collapse to?
2. What error correction procedure should be used in each case?

Solution



$$M = \{|000\rangle\langle 000| + |111\rangle\langle 111|,\ |001\rangle\langle 001| + |110\rangle\langle 110|,$$
$$|010\rangle\langle 010| + |101\rangle\langle 101|,\ |100\rangle\langle 100| + |011\rangle\langle 011|\}$$

$|\psi_i\rangle =$

$=$

- Outcome

- Outcome

To fix:

Error on A qubit:

Error on B qubit

Error on C qubit

Big Idea with Error Correction

Correction Rule: Use lowest weight correction

ex: Get outcome $\{|100X100| + |011X011|\}$ in 3 qubit Shor code, so collapse $\Rightarrow$ $|\psi\rangle \to \alpha|100\rangle + \beta|011\rangle$ (unknown amplitudes)

- If apply $X_1 \Rightarrow$

- If apply $X_2X_3 \Rightarrow$

Need to Correct More Errors? Bigger Code!

$$|0_L\rangle = (|000\rangle_{123} + |111\rangle_{123})(|000\rangle_{456} + |111\rangle_{456})(|000\rangle_{789} + |111\rangle_{789})$$
$$|1_L\rangle = (|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$$

} 9-qubit Shor Code

↑

Corrects any single qubit error

$$|0_L\rangle = (|000\rangle_{123} + |111\rangle_{123})(|000\rangle_{456} + |111\rangle_{456})(|000\rangle_{789} + |111\rangle_{789})$$
$$|1_L\rangle = (|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$$

$$|0_L\rangle = (|000\rangle_{123} + |111\rangle_{123})(|000\rangle_{456} + |111\rangle_{456})(|000\rangle_{789} + |111\rangle_{789})$$
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$$|1_L\rangle = (|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$$

Lowest weight way to return to code space?

Which errors can be corrected by Shor's code?

$$|0_L\rangle = (|000\rangle_{123} + |111\rangle_{123})(|000\rangle_{456} + |111\rangle_{456})(|000\rangle_{789} + |111\rangle_{789})$$

$$|1_L\rangle = (|000\rangle_{123} - |111\rangle_{123})(|000\rangle_{456} - |111\rangle_{456})(|000\rangle_{789} - |111\rangle_{789})$$

A) $Z_1 Z_2$

B) $Z_4 Z_8$

C) $Z_1 Z_2 Z_3$

D) $X_1 Z_2$

E) $X_1 X_2$

F) $X_1 X_4$

G) $Y_1 Z_4$

Fault Tolerant Quantum Computing

Shor Code: All single qubit errors, 9 qubits, 1 logical qubit

Concatenation:

$$|0_L\rangle = (|000\rangle_{123} + |111\rangle_{123})(|000\rangle_{456} + |111\rangle_{456})(|000\rangle_{789} + |111\rangle_{789})$$

$$|1_L\rangle = (|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$$

But... more qubits means more chance of error. Can only keep ahead of accumulating errors if error rate is small enough

Each code has an error threshold they can need to concatenate without accumulating errors.



Papers with more info

https://images.app.goo.gl/vJubz7fLrgNLWdNQ9

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