How did quantum help us win CHSH?

$$|\beta_{00}\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |11\rangle_{AB}\right) = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

has a special property: entangled

<u>def:</u>

A state $|\psi\rangle_{AB}$ is a product state if $\exists\ |\psi_1\rangle, |\psi_2\rangle$ s.t.

$$|\psi\rangle_{AB} = |\psi_1\rangle|\psi_2\rangle$$

A state $|\psi\rangle_{AB}$ is entangled if $\nexists\ |\psi_1\rangle, |\psi_2\rangle$ s.t.

$$|\psi\rangle_{AB} = |\psi_1\rangle|\psi_2\rangle$$

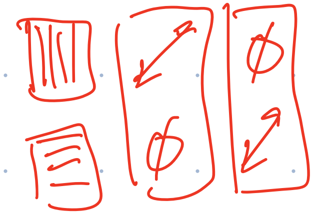
There are valid 2-qubit states that can't be described as A system in a state + B system in a state.

Similar to classical correlation $\Longrightarrow$ $\Pr(00) = 1/2$, $\Pr(11) = 1/2$

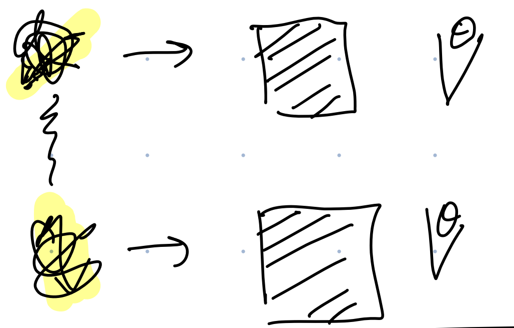
Value of bit 1?

$$\frac{1}{\sqrt{2}}|\uparrow\downarrow\rangle - \frac{1}{\sqrt{2}}|\downarrow\uparrow\rangle$$

Property of $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}|00\rangle_{AB} + \frac{1}{\sqrt{2}}|11\rangle_{AB}$



$|\beta_{00}\rangle$



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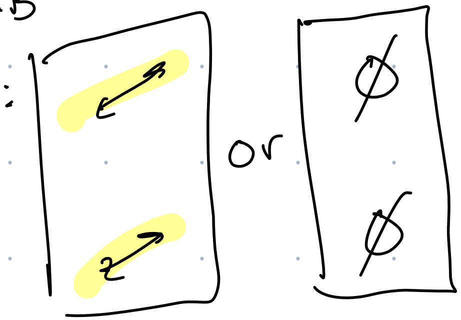
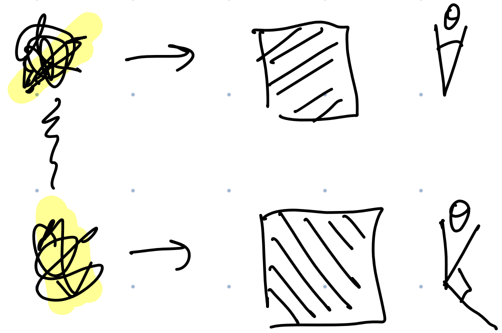


$|\beta_{00}\rangle$



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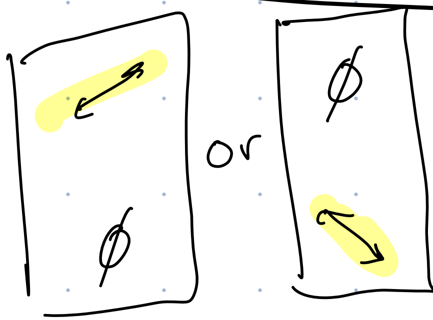
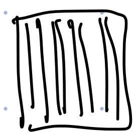


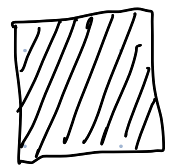
Amir

Bei

$x=0 \quad y=0$

RR or QG

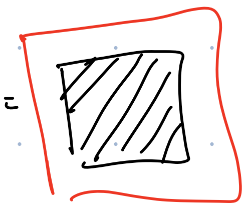
$x=0 \rightarrow M(0) =$ 

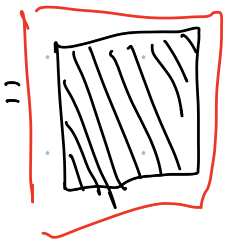
$y=0 \rightarrow M\left(\frac{\pi}{8}\right) =$ 

$x=0 \quad y=1$

RR $\quad$ QG

$x=1 \quad y=0$

$x=1 \rightarrow M\left(\frac{\pi}{4}\right) =$ 

$y=1 \rightarrow M\left(\frac{-\pi}{8}\right) =$ 

$x \wedge y = 0 \Rightarrow$ want $a \oplus b = 0 \rightarrow a = b$ correlated

$x \wedge y = 1 \Rightarrow$ want $a \oplus b = 1 \rightarrow a \neq b$ anticorrelated

0 1

01

Recall

Qubit A

$|\psi_1\rangle_A = a_0|0\rangle + a_1|1\rangle$

Qubit B

$|\psi_2\rangle_B = b_0|0\rangle + b_1|1\rangle$

Combined: $|\psi\rangle_{AB} = |\psi_1\rangle_A \otimes |\psi_2\rangle_B = |\psi_1\rangle_A |\psi_2\rangle_B$

Stick kets together to combine

$= (a_0|0\rangle + a_1|1\rangle)(b_0|0\rangle + b_1|1\rangle)$

Distribute ⊗ (tensor) multiplication

(Keep ket order!)

$= a_0b_0|0\rangle|0\rangle + a_0b_1|0\rangle|1\rangle + a_1b_0|1\rangle|0\rangle + a_1b_1|11\rangle$

$= a_0b_0|00\rangle + a_0b_1|01\rangle + a_1b_0|10\rangle + a_1b_1|11\rangle$

00, 01, 10, 11 ⇒ 2 bits! 2 qubits

Generic 2 Qubit State:

$|\psi\rangle_{AB} = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$

amplitudes

standard basis states

s.t. $\underbrace{\sum_{i \in \{0,1\}^2} |a_i|^2 = 1}_{\text{normalization}}$

Vector in $\mathbb{C}^4$

$$\begin{pmatrix} a_{00} \\ a_{01} \\ a_{10} \\ a_{11} \end{pmatrix}$$

Let $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

Prove $|\beta_{00}\rangle$ is entangled.

- Names, pronouns
- best thing watched/listened to

Pf:

A state $|\psi\rangle_{AB}$ is entangled if $\nexists\ |\psi_1\rangle, |\psi_2\rangle$ s.t. $|\psi\rangle_{AB} = |\psi_1\rangle_A |\psi_2\rangle_B$

Assume for contradiction $|\beta_{00}\rangle$ is not entangled. So it must be product. Thus $|\beta_{00}\rangle = |\psi_1\rangle|\psi_2\rangle$

$|\psi_1\rangle_A = a_0|0\rangle + a_1|1\rangle$ $\qquad$ $|\psi_2\rangle_B = b_0|0\rangle + b_1|1\rangle$

$$(a_0|0\rangle + a_1|1\rangle)(b_0|0\rangle + b_1|1\rangle) = \cancel{a_0b_0|00\rangle} + \cancel{a_1b_0|10\rangle}$$
$$+ a_0b_1|01\rangle + \cancel{a_1b_1|11\rangle}$$
$$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

$a_1b_0 = 0$

$a_1 = 0$ or $b_0 = 0$

More Practice:

- $|\psi\rangle = \sqrt{\frac{3}{10}}\, i|00\rangle + \frac{1}{\sqrt{10}}|01\rangle_{AB} - \boxed{\sqrt{\frac{2}{10}}|10\rangle_{AB}} - i\sqrt{\frac{4}{10}}|11\rangle_{AB}$

$M_A = \{|0\rangle, |1\rangle\}$ $M_B = \{|0\rangle, |1\rangle\}$ $\rightarrow M_{AB} = \{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

What outcomes are possible + what is probability of each outcome?

$|00\rangle \rightarrow |\langle 00|\psi\rangle|^2 = \left|\sqrt{\frac{3}{10}}\, i\right|^2 = \sqrt{\frac{3}{10}}\, i \sqrt{\frac{3}{10}}(-i) = \frac{3}{10}$

$|10\rangle \rightarrow |\langle 10|\psi\rangle|^2 = \left|-\sqrt{\frac{2}{10}}\right|^2 = \frac{2}{10}$

- $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB}$ $M_A = \{|+\rangle, |-\rangle\}$ $M_B = \{|+\rangle, |-\rangle\}$

What is the probability of $|+\rangle|-\rangle$