ENTANGLEMENT + CHSH

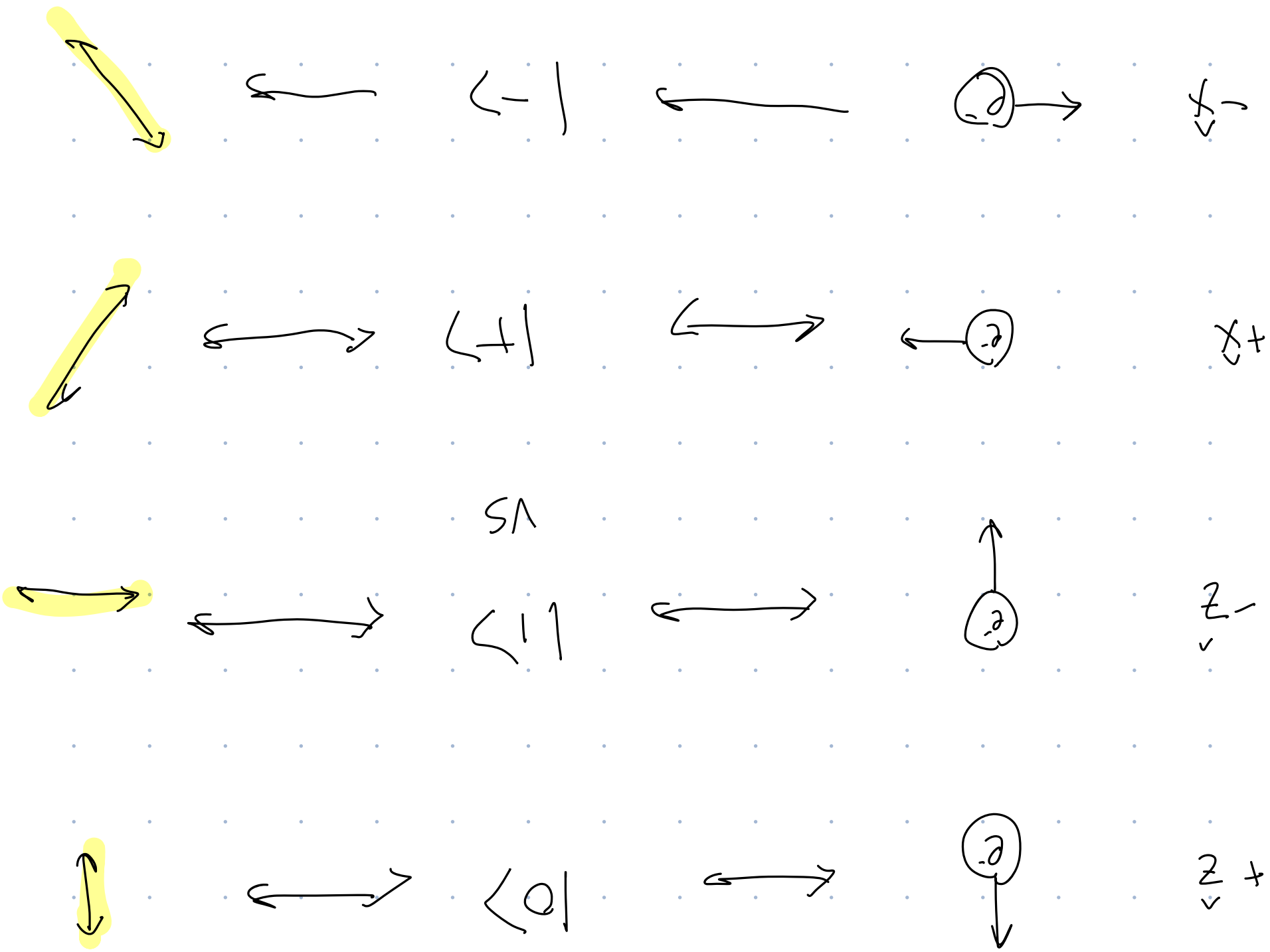
Goals

- Describe entangled states + product states [QI3]
- Determine if a 2-qubit state is entangled
- Describe why entanglement helps us win CHSH
- Analyze 2-qubit systems

Announcements

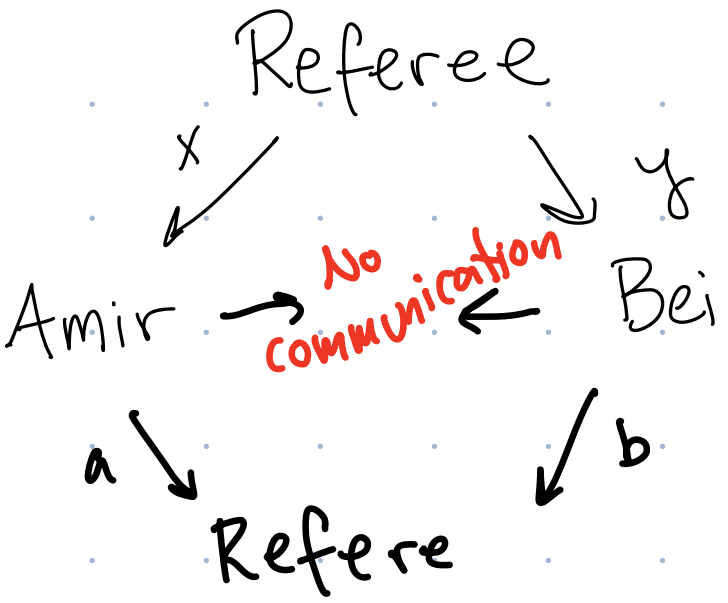
Exit Tickets

- How are photons entangled physically?
- How are photons correlated w/o physical conxn? Simultaneity?


SA
x-
x+
z-
z+

CHSH Game (Clauser, Horne, Shimony, Holt) (Bell Inequality)





$x, y \in \{0,1\}$

$a, b \in \{0,1\}$

Amir + Bei win:

x,y	winning condition
00, 01, 10	$a = b$
11	$a \neq b$

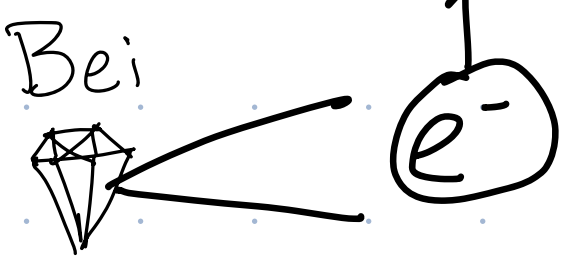
On pset3 show classical strategy can win 75%, if x, y chosen randomly

What if we give them each a qubit?

Diamond
nitrogen
vacancy
qubit

~1 sec
coherence







How do we represent the combined state of both qubits?

A: $|0\rangle \otimes |1\rangle$ B: $|0\rangle_A |1\rangle_B$ C: $|01\rangle_{AB}$ D: $|01\rangle$

"Tensor Product"

Bra-Ket = "Inner Product" → Number

$\langle 0| \rightarrow \leftarrow |1\rangle \Rightarrow \langle 0|1\rangle = 0$

"Tensor Product" = Bigger ket

$|0\rangle \rightarrow \leftarrow |1\rangle = |01\rangle$



Generic 2 Qubit State:

$$|\psi\rangle_{AB} = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$$

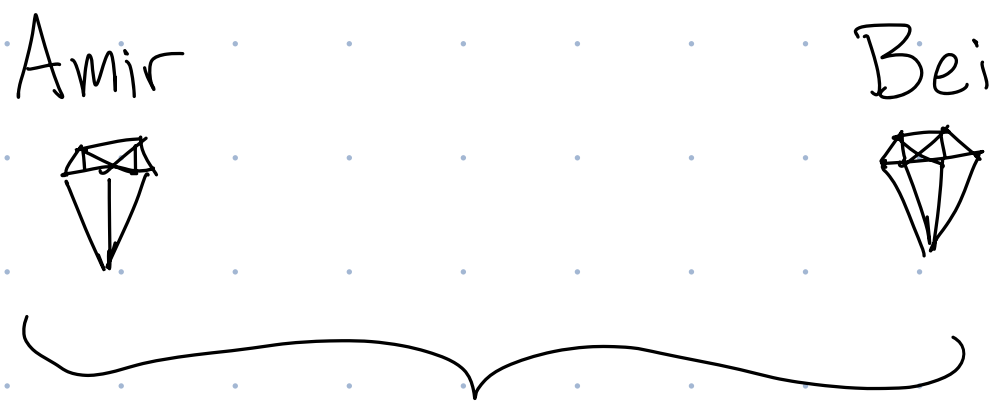
standard basis states

"amplitudes"

$$\text{s.t.} \sum_{i \in \{0,1\}^2} |a_i|^2 = 1$$

normalization

Strategy:

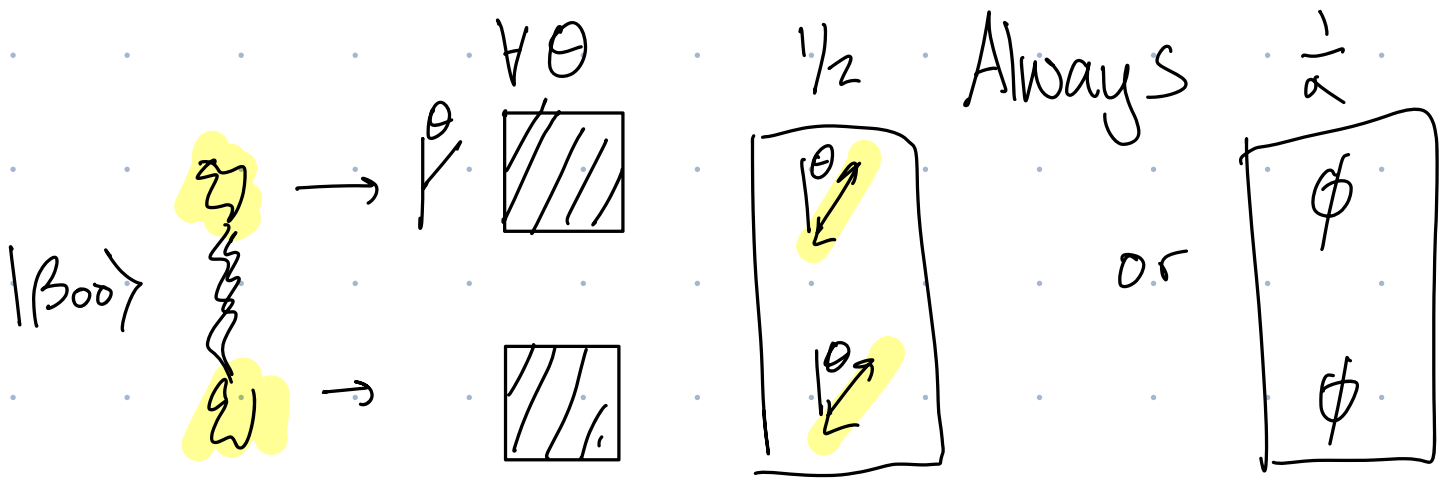


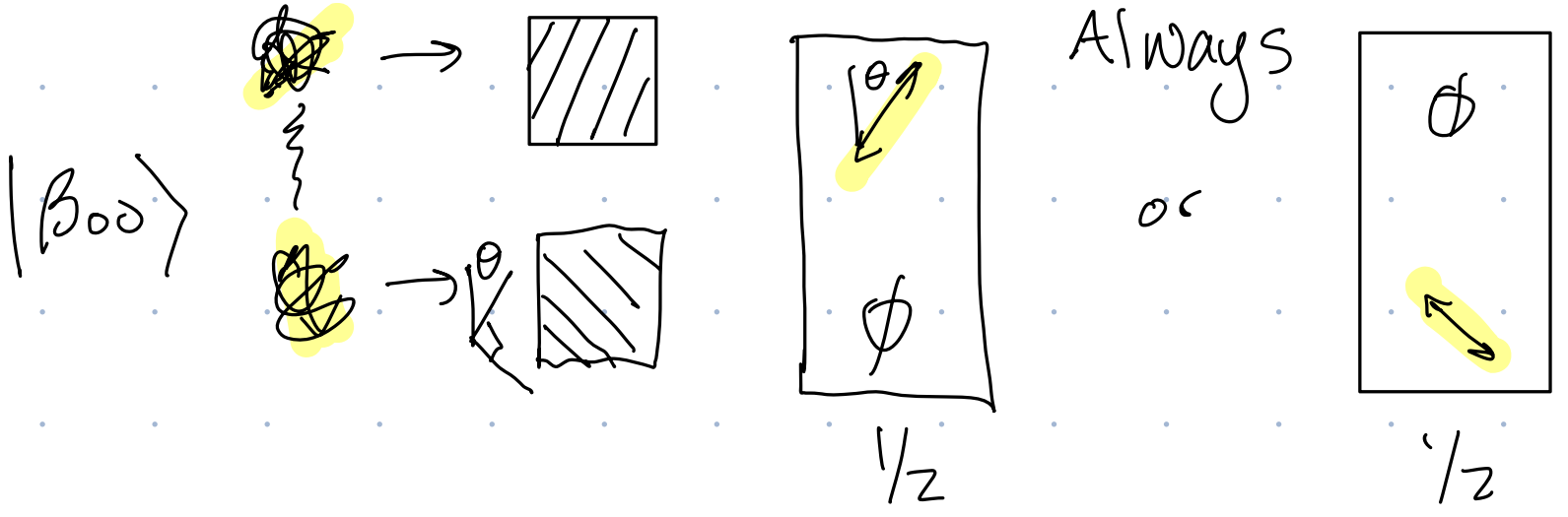
$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}|00\rangle_{AB} + \frac{1}{\sqrt{2}}|11\rangle_{AB}$$

$$a_{01} = a_{10} = 0$$

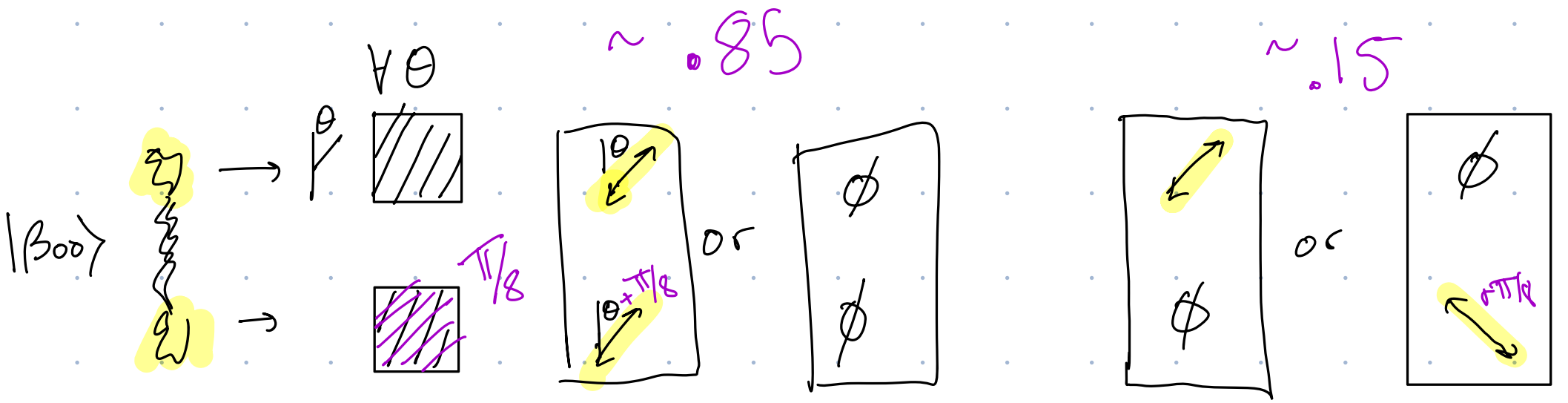
$$a_{00} = a_{11} = \frac{1}{\sqrt{2}}$$

$|\beta_{00}\rangle_{AB} = \frac{1}{\sqrt{2}}|00\rangle_{AB} + \frac{1}{\sqrt{2}}|11\rangle_{AB}$ is <u>Entangled</u>!



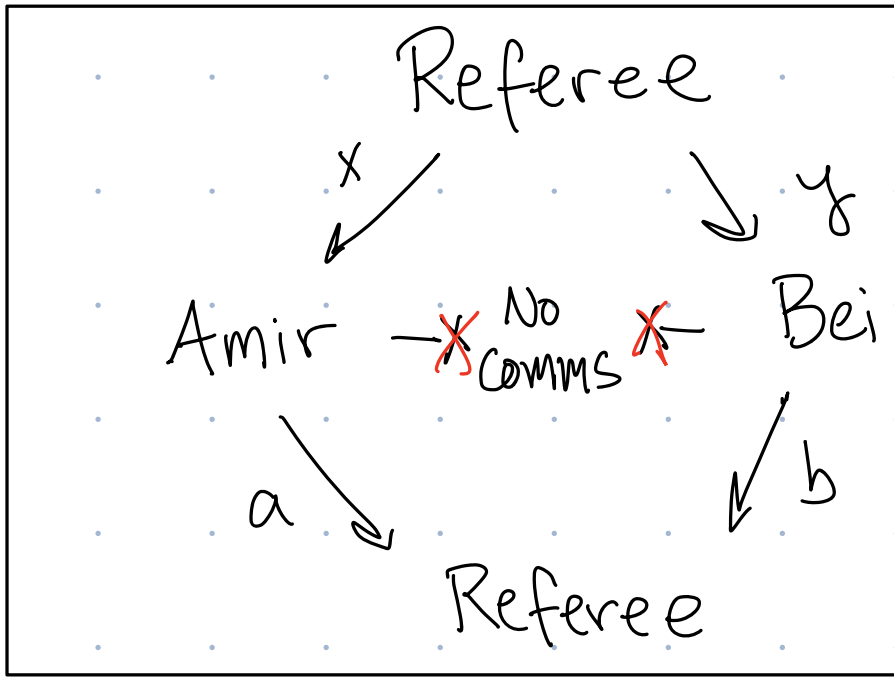


$|\beta_{00}\rangle_{AB} = \frac{1}{\sqrt{2}}|00\rangle_{AB} + \frac{1}{\sqrt{2}}|11\rangle_{AB}$ is <u>Entangled!</u>





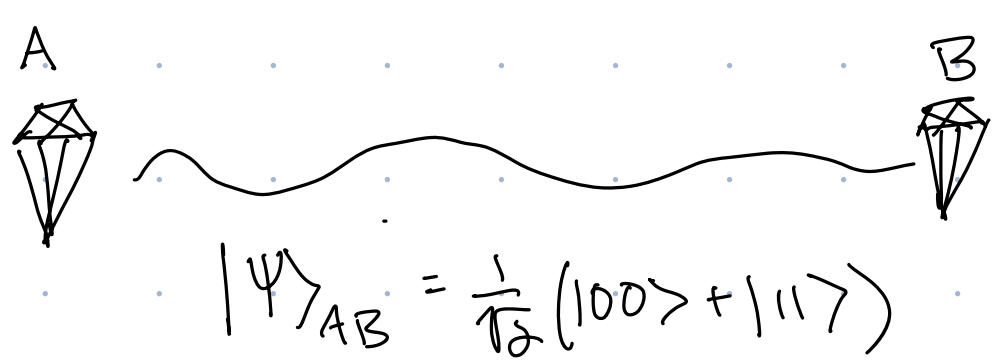
CHSH Game



Amir + Bei win

x, y	if
00, 01, 10	$a = b$
11	$a \neq b$

Strategy

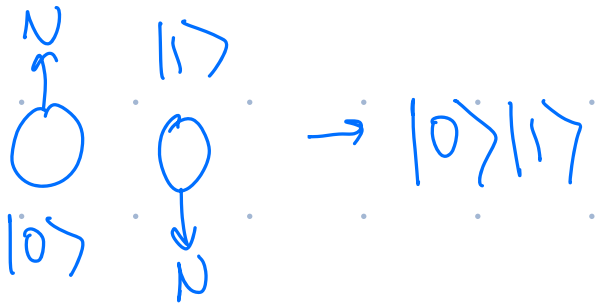


$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Success Prob?
Why?

Amir Receives	Amir does measurement	Bei Receives	Bei does measurement
$x = 0$	0: $a = 0$; ϕ: $a = 1$	$y = 0$	$\pi/8$: 0: $b = 0$; ϕ: $b = 1$
$x = 1$	$\pi/4$: 0: $a = 0$; ϕ: $a = 1$	$y = 1$	$-\pi/8$: 0: $b = 0$; ϕ: $b = 1$

Quantum Entanglement



def:

A state $|\psi\rangle_{AB}$ is a product state if $\exists\ |\psi_1\rangle, |\psi_2\rangle$ s.t. $|\psi\rangle_{AB} = |\psi_1\rangle_A |\psi_2\rangle_B$

A state $|\psi\rangle_{AB}$ is entangled if $\nexists\ |\psi_1\rangle, |\psi_2\rangle$ s.t. $|\psi\rangle_{AB} = |\psi_1\rangle_A |\psi_2\rangle_B$

There are valid 2-qubit states that can't be described as A system in a state + B system in a state.

Similar to classical correlation ⟹

$P(00) = \frac{1}{2}$

$P(11) = \frac{1}{2}$

Qubit A

$|\psi_1\rangle_A = a_0|0\rangle + a_1|1\rangle$

Qubit B

$|\psi_2\rangle_B = b_0|0\rangle + b_1|1\rangle$

Combined: $|\psi\rangle_{AB} = (a_0|0\rangle + a_1|1\rangle) \otimes (b_0|0\rangle + b_1|1\rangle)$

$= a_0|0\rangle \otimes b_0|0\rangle$

$a_0 b_0 |0\rangle \otimes |0\rangle$

$= a_0 b_0 |00\rangle + a_0 b_1 |01\rangle + a_1 b_0 |10\rangle + a_1 b_1 |11\rangle$

Let $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle + 0|01\rangle + 0|10\rangle$

Prove $|\beta_{00}\rangle$ is entangled.

$\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$ $\quad$ $0 \cdot 0 = 0$

Pf: Assume for contradiction $|\beta_{00}\rangle$ is not entangled.

A state $|\psi\rangle_{AB}$ is not entangled if $\exists\, |\psi_1\rangle, |\psi_2\rangle$ s.t. $|\psi\rangle_{AB} = |\psi_1\rangle_A |\psi_2\rangle_B$

$|\psi_1\rangle_A = a_0|0\rangle + a_1|1\rangle$ $\qquad$ $|\psi_2\rangle_B = b_0|0\rangle + b_1|1\rangle$

Combined: $|\psi\rangle_{AB} = (a_0|0\rangle + a_1|1\rangle) \otimes (b_0|0\rangle + b_1|1\rangle)$

$= a_0b_0|00\rangle + a_0b_1|01\rangle + a_1b_0|10\rangle + a_1b_1|11\rangle$

$a_0b_0\, a_1b_1 \quad \simeq \quad a_0b_1\, a_1b_0$

If have 2-qubit state $|\psi\rangle_{AB}$ and measure A qubit with $M_A = \{|\alpha_0\rangle, |\alpha_1\rangle\}$ and B qubit with $M_B = \{|\beta_0\rangle, |\beta_1\rangle\}$

Get outcome $|\alpha_i\rangle_A |\beta_j\rangle_B$ with prob. $|\langle\alpha_i|_A \langle\beta_j|_B |\psi\rangle_{AB}|^2$

state collapses to

$$|\alpha_i\rangle_A |\beta_j\rangle_B$$

How many measurement outcomes are there?

A) 2 (M_A and M_B) B) 4 ($|\alpha_0\rangle|\beta_0\rangle, |\alpha_0\rangle|\beta_1\rangle, \ldots$)

C) Not enough information

If have 2-qubit state $|\psi\rangle_{AB}$ and measure A qubit with $M_A = \{|\alpha_0\rangle, |\alpha_1\rangle\}$ and B qubit with $M_B = \{|\beta_0\rangle, |\beta_1\rangle\}$

Get outcome with prob.

state collapses to

Suppose Amir + Bei cannot communicate, but share $|\psi\rangle_{AB}$ (unknown state). If Amir measures $M_A = \{|0\rangle, |1\rangle\}$ and Bei measures $M_B = \{|+\rangle, |-\rangle\}$ and they get outcome $|0\rangle_A|+\rangle_B$, what does Amir know about the two qubits after the measurements if no communication?

A) Nothing

B) A is in $|0\rangle$, B unknown

C) A is in $|0\rangle$, B in $|+\rangle$

Example: $|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

Prob = $\left|\langle\alpha_i|_A \langle\beta_j|_B |\psi\rangle_{AB}\right|^2$

$x=0$ ⇓ $M_A = \{|0\rangle, |1\rangle\}$

$y=0$ ⇓ $M_B = \{\cos\frac{\pi}{8}|0\rangle + \sin\frac{\pi}{8}|1\rangle,\ \sin\frac{\pi}{8}|0\rangle - \cos\frac{\pi}{8}|1\rangle\}$

Probability of $|0\rangle_A(\cos\frac{\pi}{8}|0\rangle + \sin\frac{\pi}{8}|1\rangle)_B$

Inner Product ⇒ $(\langle 0|_A \otimes)(\cos\frac{\pi}{8}\langle 0|_B + \sin\frac{\pi}{8}\langle 1|_B)(\frac{1}{\sqrt{2}}|00\rangle_{AB} + \frac{1}{\sqrt{2}}|11\rangle_{AB})$

KEEP AB ORDER $= (\cos\frac{\pi}{8}\langle 00|_{AB} + \sin\frac{\pi}{8}\langle 01|_{AB})(\frac{1}{\sqrt{2}}|00\rangle_{AB} + \frac{1}{\sqrt{2}}|11\rangle_{AB})$

$= \frac{\cos\frac{\pi}{8}}{\sqrt{2}}\langle 00|00\rangle + \frac{\sin\frac{\pi}{8}}{\sqrt{2}}\langle 01|00\rangle + \frac{\cos\frac{\pi}{8}}{\sqrt{2}}\langle 00|11\rangle + \frac{\sin\frac{\pi}{8}}{\sqrt{2}}\langle 01|11\rangle$ ($\langle 00|00\rangle = 1$)

Rule: $\langle x|y\rangle = \begin{cases} 1 & \text{if } x=y \\ 0 & \text{else} \end{cases}$

(x, y binary strings of the same length)

$= \frac{\cos\frac{\pi}{8}}{\sqrt{2}}$

Prob: $\frac{1}{2}\cos^2\frac{\pi}{8} = .4\ldots$

Bra $|01\rangle_{AB} \rightarrow \langle 01|_{AB}$ Keep AB order

Practice: Start PS3 #4a