ENTANGLEMENT + CHSH

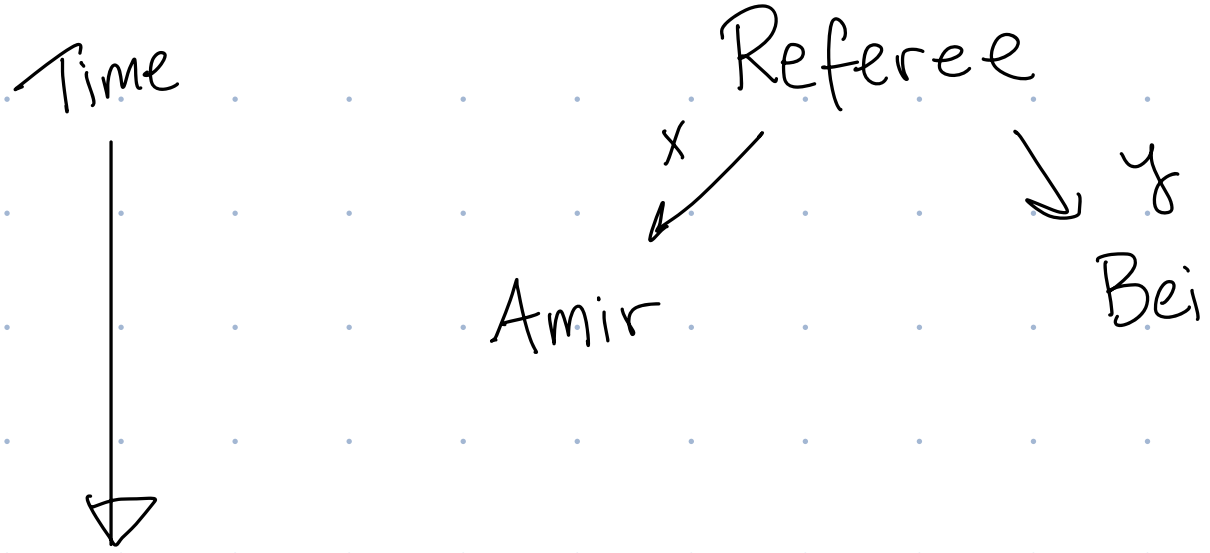
Goals

- Describe entangled states + product states [QI3]
- Determine if a 2-qubit state is entangled
- Describe why entanglement helps us win CHSH
- Analyze 2-qubit systems

Announcements

Exit Tickets

CHSH Game (Clauser, Horne, Shimony, Holt) (Bell Inequality)



Amir + Bei win:

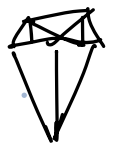
x, y	winning condition

On pset3 show classical strategy can win 75%, if x, y chosen randomly

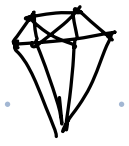
What if we give them each a qubit?

Diamond
nitrogen
vacancy
qubit

Amir



Bei



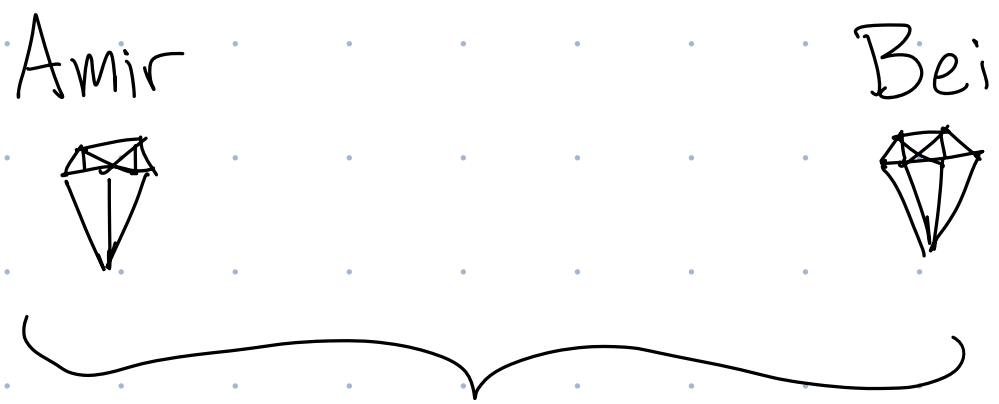
How do we represent the combined state of both qubits?

A: $|0\rangle \otimes |1\rangle$ B: $|0\rangle_A |1\rangle_B$ C: $|01\rangle_{AB}$ D: $|01\rangle$

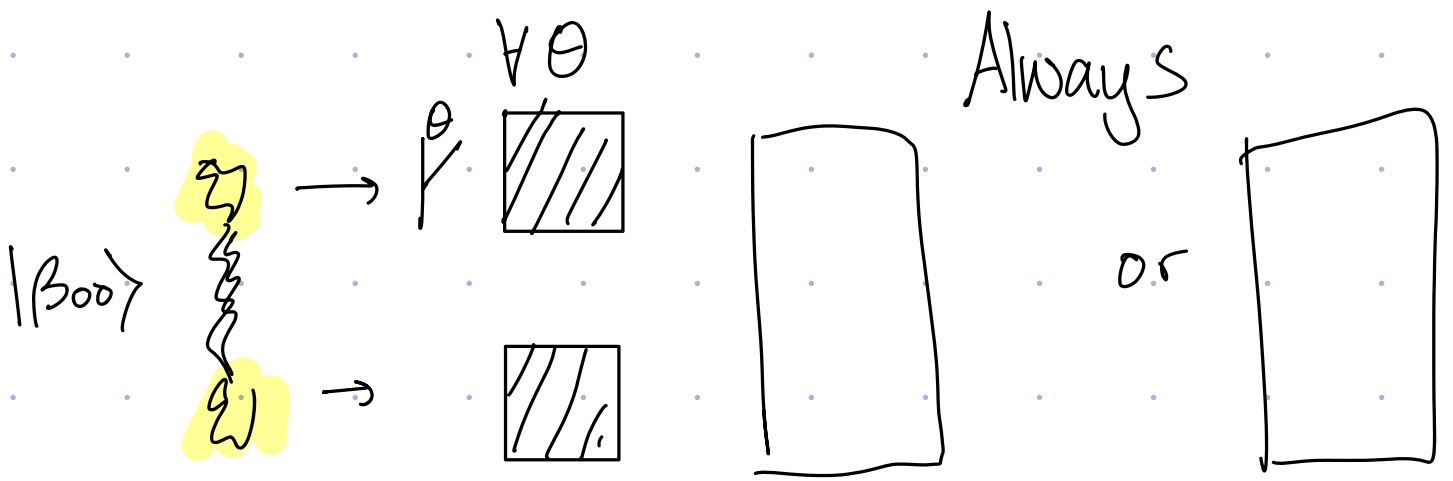
Generic 2 Qubit State:

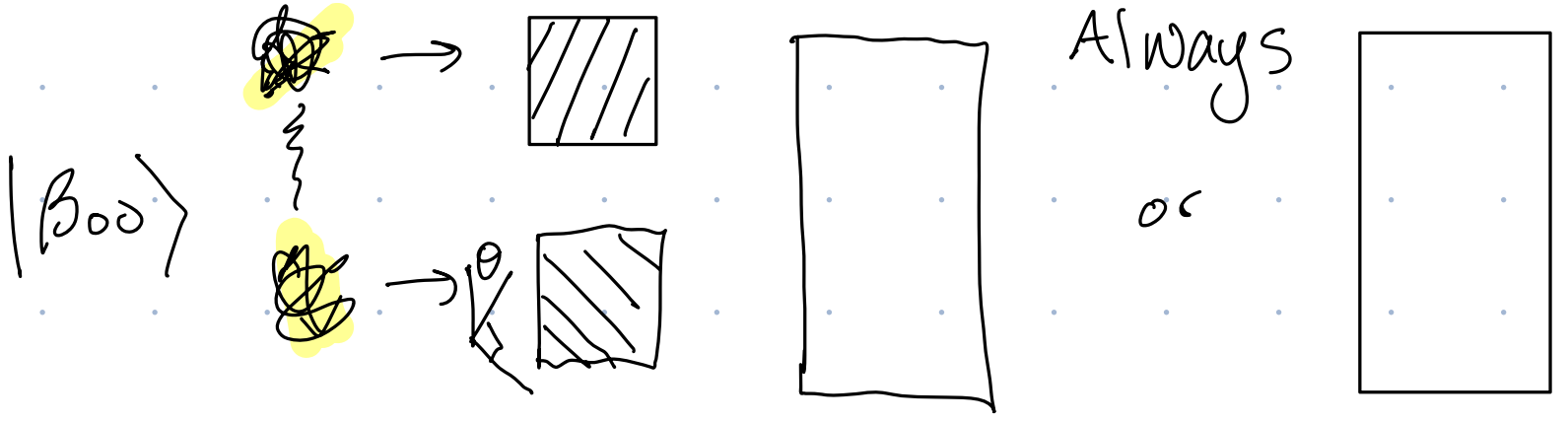
$|\psi\rangle_{AB} =$

Strategy:

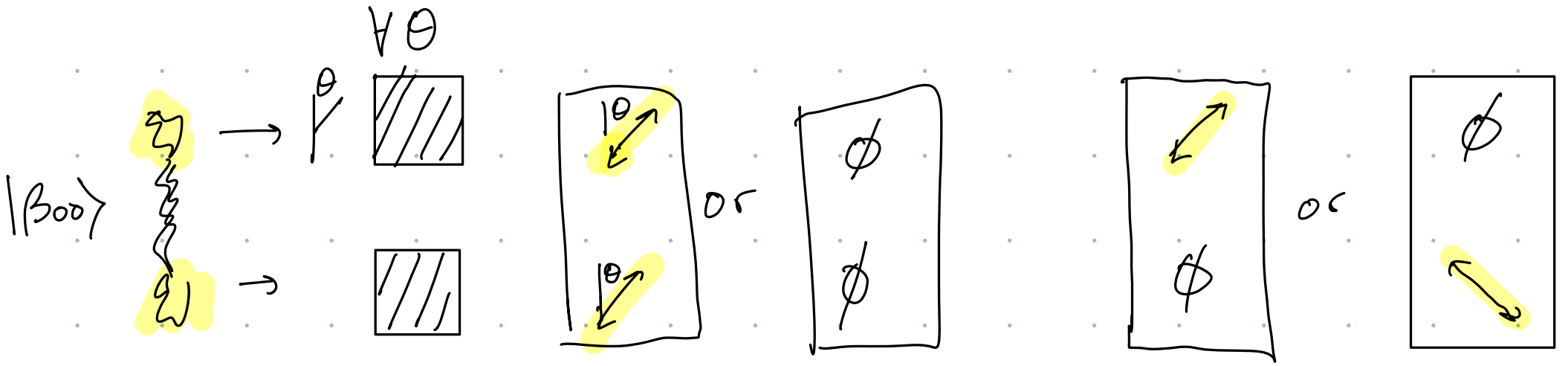


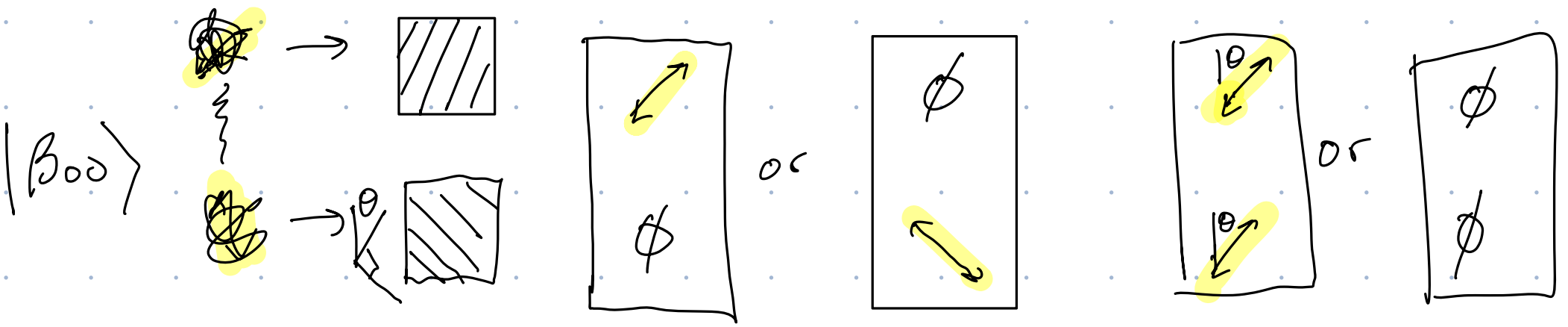
$|\beta_{00}\rangle_{AB} = \frac{1}{\sqrt{2}}|00\rangle_{AB} + \frac{1}{\sqrt{2}}|11\rangle_{AB}$ is <u>Entangled</u>!



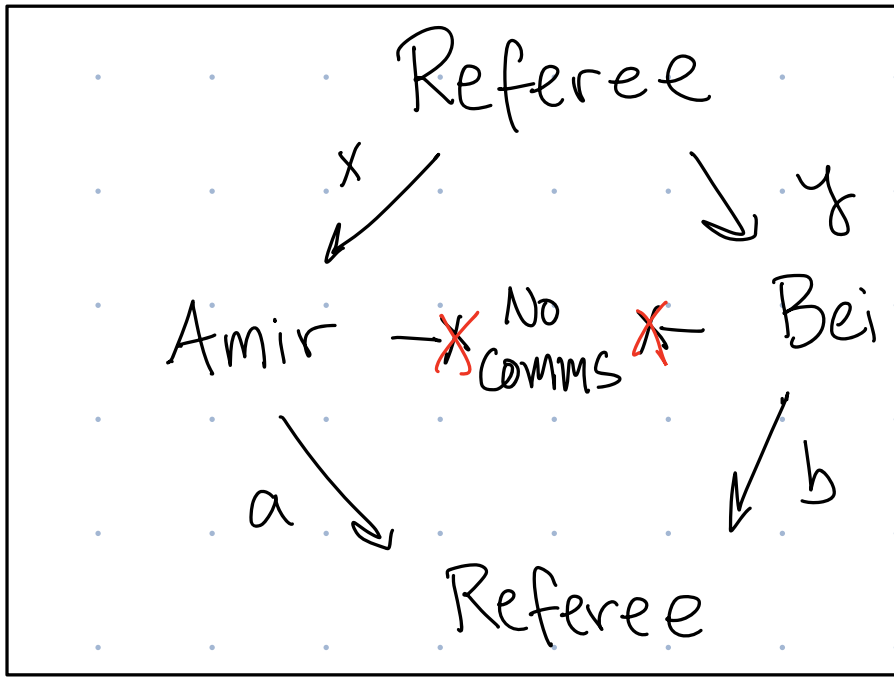


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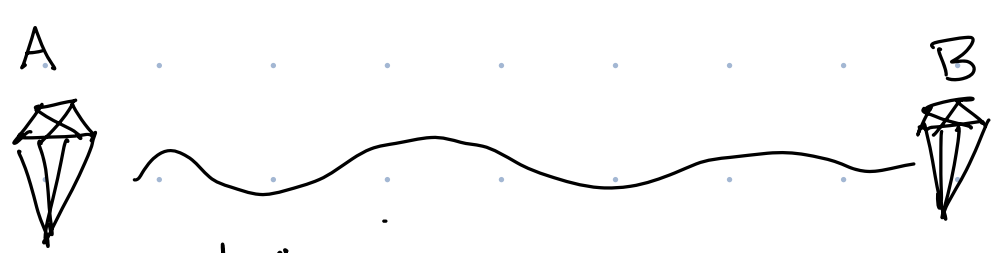
CHSH Game



Amir + Bei win

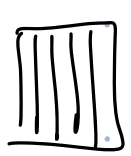
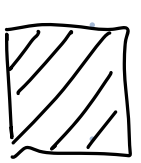
x, y	if
00, 01, 10	$a = b$
11	$a \neq b$

Strategy



$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Success Prob? Why?

Amir Receives	Amir does measurement
$x = 0$	 $\updownarrow$ $a = 0$; ϕ $a = 1$
$x = 1$	 $\pi/4$ $\updownarrow$ $a = 0$; ϕ $a = 1$

Bei Receives	Bei does measurement
$y = 0$	 $\pi/8$ $\updownarrow$ $b = 0$; ϕ $b = 1$
$y = 1$	 $-\pi/8$ $\updownarrow$ $b = 0$; ϕ $b = 1$

Quantum Entanglement

def:

A state $|\psi\rangle_{AB}$ is a product state if $\exists\ |\psi_1\rangle, |\psi_2\rangle$ s.t. $|\psi\rangle_{AB} = |\psi_1\rangle_A |\psi_2\rangle_B$

A state $|\psi\rangle_{AB}$ is entangled if $\nexists\ |\psi_1\rangle, |\psi_2\rangle$ s.t. $|\psi\rangle_{AB} = |\psi_1\rangle_A |\psi_2\rangle_B$

There are valid 2-qubit states that can't be described as A sysem in a state + B system in a state.

Similar to classical correlation ⟹

1

Qubit A

$|\psi_1\rangle_A = a_0|0\rangle + a_1|1\rangle$

Qubit B

$|\psi_2\rangle_B = b_0|0\rangle + b_1|1\rangle$

Combined: $|\psi\rangle_{AB}$ =

Let $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

Prove $|\beta_{00}\rangle$ is entangled.

Pf: Assume for contradiction $|\beta_{00}\rangle$ is not entangled.

> A state $|\psi\rangle_{AB}$ is not entangled if $\exists\, |\psi_1\rangle, |\psi_2\rangle$ s.t. $|\psi\rangle_{AB} = |\psi_1\rangle_A |\psi_2\rangle_B$

If have 2-qubit state $|\psi\rangle_{AB}$ and measure A qubit with $M_A = \{|\alpha_0\rangle, |\alpha_1\rangle\}$ and B qubit with $M_B = \{|\beta_0\rangle, |\beta_1\rangle\}$

Get outcome with prob.

state collapses to

How many measurement outcomes are there?

A) 2 (M_A and M_B) B) 4 ($|\alpha_0\rangle|\beta_0\rangle, |\alpha_0\rangle|\beta_1\rangle, \ldots$)

C) Not enough information

If have 2-qubit state $|\psi\rangle_{AB}$ and measure A qubit with $M_A = \{|\alpha_0\rangle, |\alpha_1\rangle\}$ and B qubit with $M_B = \{|\beta_0\rangle, |\beta_1\rangle\}$

Get outcome with prob.

state collapses to

Suppose Amir + Bei cannot communicate, but share $|\psi\rangle_{AB}$ (unknown state). If Amir measures $M_A = \{|0\rangle, |1\rangle\}$ and Bei measures $M_B = \{|+\rangle, |-\rangle\}$ and they get outcome $|0\rangle_A|+\rangle_B$, what does Amir know about the two qubits after the measurements if no communication?

A) Nothing

B) A is in $|0\rangle$, B unknown

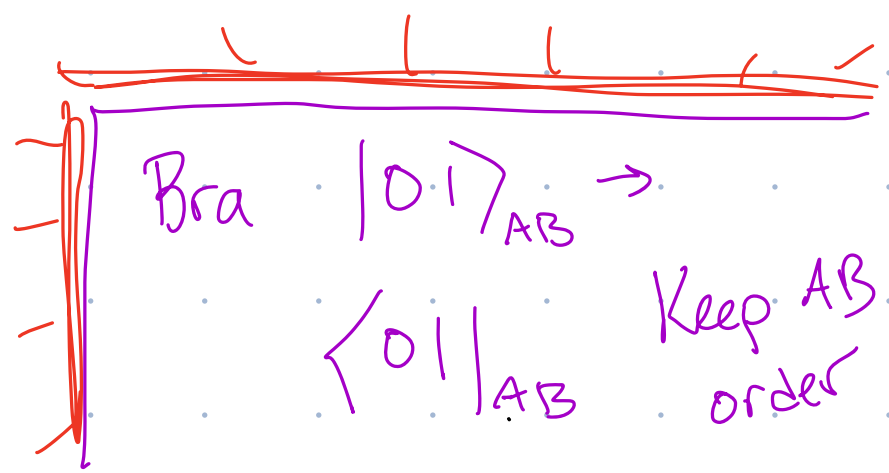
C) A is in $|0\rangle$, B in $|+\rangle$

Example: $|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

$M_A = \{|+\rangle, |-\rangle\}$ $M_B = \{|+\rangle, |-\rangle\}$

Prob $|\langle\alpha_i|_A \langle\beta_j|_B |\psi\rangle_{AB}|^2$

Probability of $|+\rangle_A|-\rangle_B$? $|\alpha_i\rangle_A|\beta_j\rangle_B$



Practice: Start PS3 #4a