ENTANGLEMENT + CHSH

Goals

- Describe entangled states + product states
- Determine if a 2-qubit state is entangled
- Describe why entanglement helps us win CHSH
- Analyze 2-qubit systems

Announcements

- In class exam Thursday QI1, QI2 (15 min, no notes)
- Review where to find notes on Canvas

Exit Tickets

Exit Tickets

- How many qubit states are there?

A) 2 B) 4 C) COUNTABLY ∞ D UNCOUNTABLY ∞

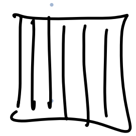
- Does this represent a qubit state?

$$|\psi\rangle = \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle$$

A) YES B) NO C) Maybe D) IDK

- When $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ vs $\cos\theta|0\rangle + \sin\theta|1\rangle$?

- Does orientation/order of $|0\rangle$, $\langle 1|$ matter?

- Bra physical? Bra name?
-  vs $\{|0\rangle, |1\rangle\}$
- No sines/cosines on exam (or will provide)

CHSH Game (Clauser, Horne, Shimony, Holt) (Bell Inequality)

Time

Referee

$\in \{0,1\}$

Amir

Bei

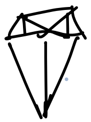
Amir + Bei win:

x,y	winning condition
00, 01, 10	
11	

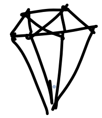
What if we give them each a qubit?

Diamond nitrogen vacancy qubit

Amir



Bei



How do we represent the combined state of both qubits?

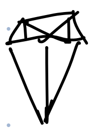
A: $|0\rangle \otimes |1\rangle$ B: $|0\rangle_A |1\rangle_B$ C: $|01\rangle_{AB}$ D: $|01\rangle$

Generic 2 Qubit State:

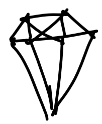
$|\psi\rangle_{AB} =$

Strategy:

Amir



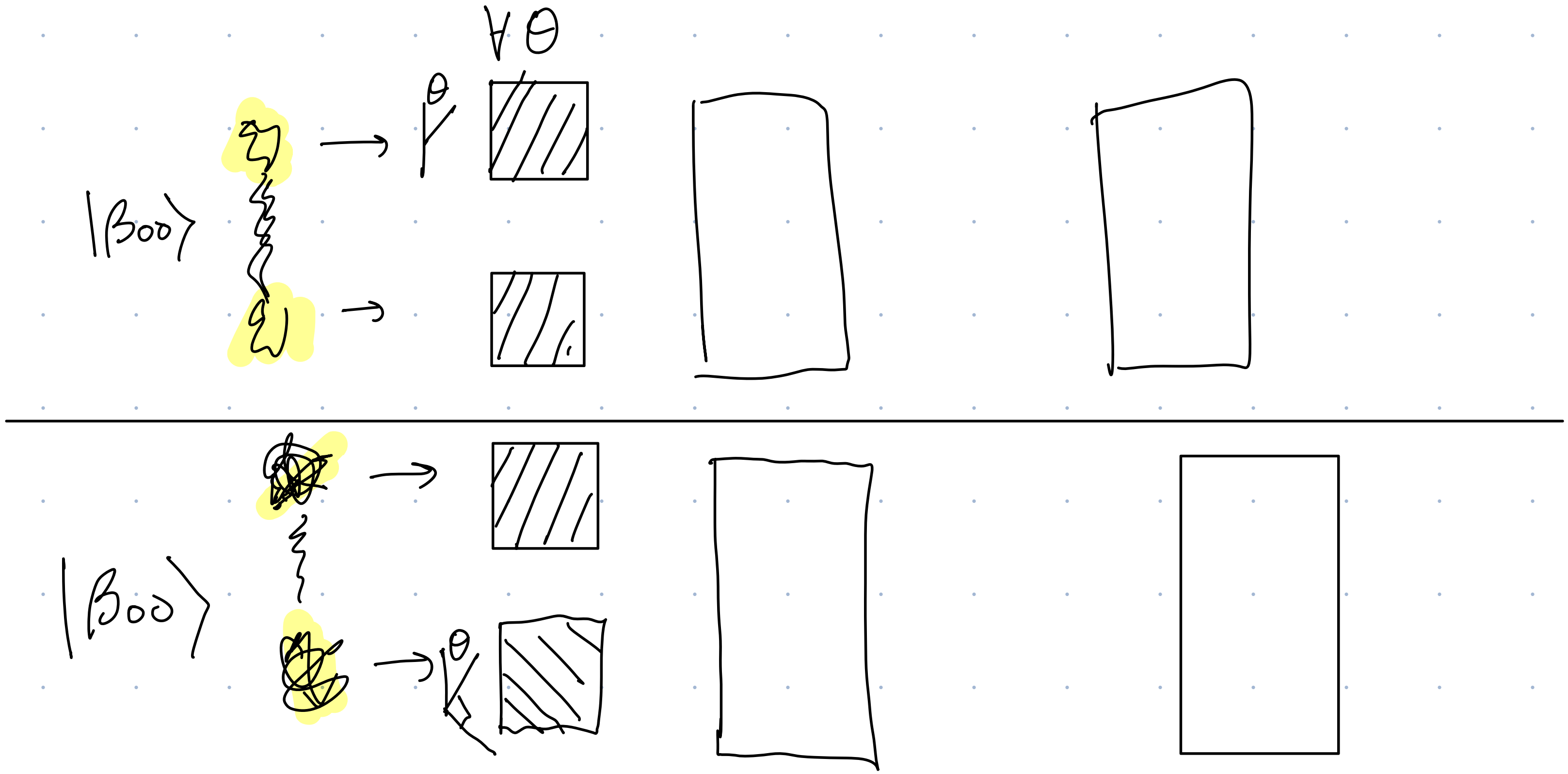
Bei



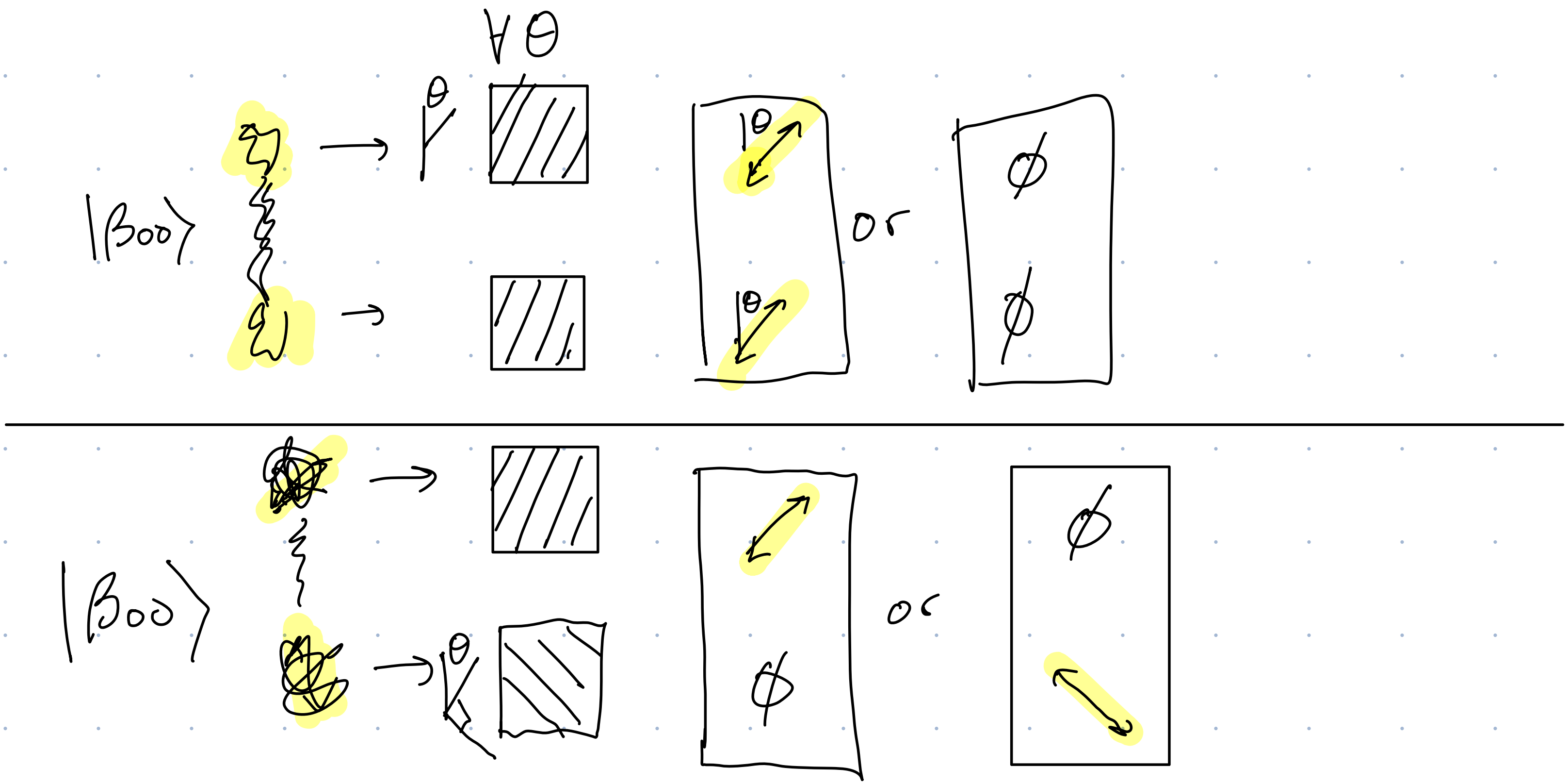
$a_{00} = a_{11} =$

$a_{01} = a_{10} =$

$\frac{1}{\sqrt{2}}|00\rangle_{AB} + \frac{1}{\sqrt{2}}|11\rangle_{AB}$ is <u>Entangled!</u>



$\frac{1}{\sqrt{2}}|00\rangle_{AB} + \frac{1}{\sqrt{2}}|11\rangle_{AB}$ is Entangled!



CHSH Game

Referee

x ↙ ↘ y

Amir → ~~✗~~ No comms ~~✗~~ ← Bei

a ↘ ↙ b

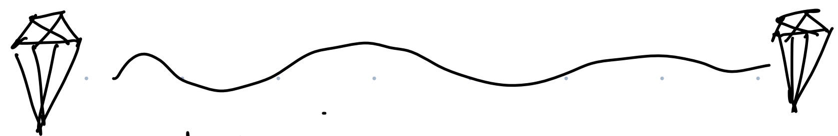
Referee

Amir + Bei win

x,y	if
00, 01, 10	$a=b$
11	$a\neq b$

Strategy

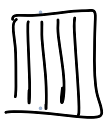
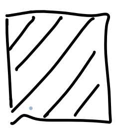
A

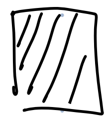
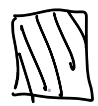


B

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$$

Success Prob? Why?

Amir Receives	Amir does measurement
$x=0$	 ↗ ↕ $a=0$; ↘ ϕ $a=1$
$x=1$	 $\pi/4$ ↗ ⤢ $a=0$; ↘ ϕ $a=1$

Bei Receives	Bei does measurement
$y=0$	 $\pi/8$ ↗ ⤢ $b=0$; ↘ ϕ $b=1$
$y=1$	 $-\pi/8$ ↗ ⤡ $b=0$; ↘ ϕ $b=1$

Quantum Entanglement

<u>def:</u>

A state $|\psi\rangle_{AB}$ is a product state if $\exists\ |\psi_1\rangle, |\psi_2\rangle$ s.t. $|\psi\rangle_{AB} = |\psi_1\rangle_A |\psi_2\rangle_B$

A state $|\psi\rangle_{AB}$ is entangled if $\nexists\ |\psi_1\rangle, |\psi_2\rangle$ s.t. $|\psi\rangle_{AB} = |\psi_1\rangle_A |\psi_2\rangle_B$

There are valid 2-qubit states that can't be described as A system in a state + B system in a state.

Similar to classical correlation $\Longrightarrow$

Qubit A

$|\psi_1\rangle_A = a_0|0\rangle + a_1|1\rangle$

Qubit B

$|\psi_2\rangle_B = b_0|0\rangle + b_1|1\rangle$

Combined: $|\psi\rangle_{AB}$

Let $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

Prove $|\beta_{00}\rangle$ is entangled.

Pf: Assume for contradiction $|\beta_{00}\rangle$ is not entangled.

$\boxed{\text{A state } |\psi\rangle_{AB} \text{ is not entangled if } \exists\, |\psi_1\rangle, |\psi_2\rangle \text{ s.t. } |\psi\rangle_{AB} = |\psi_1\rangle_A |\psi_2\rangle_B}$

1 If have 2-qubit state $|\psi\rangle_{AB}$ and measure A qubit with $M_A = \{|\alpha_0\rangle, |\alpha_1\rangle\}$ and B qubit with

$M_B = \{|\beta_0\rangle, |\beta_1\rangle\}$

3 Get outcome with prob.

state collapses to

2 How many measurement outcomes are there?

A) 2 (M_A and M_B)

B) 4 ($|\alpha_0\rangle|\beta_0\rangle, |\alpha_0\rangle|\beta_1\rangle, \ldots$)

C) Not enough information

① If have 2-qubit state $|\psi\rangle_{AB}$ and measure A qubit with $M_A = \{|\alpha_0\rangle, |\alpha_1\rangle\}$ and B qubit with $M_B = \{|\beta_0\rangle, |\beta_1\rangle\}$

$$\left|\langle\psi|_{AB}\,|\alpha_i\rangle_A\,|\beta_j\rangle_B\right|^2$$
$$\shortparallel$$

③ Get outcome $|\alpha_i\rangle_A|\beta_j\rangle_B$ with prob. $\left|\langle\alpha_i|_A\langle\beta_j|_B\,|\psi\rangle_{AB}\right|^2$, state collapses to $|\alpha_i\rangle_A|\beta_j\rangle_B$

④ Suppose Amir + Bei cannot communicate, but share $|\psi\rangle_{AB}$ (unknown state). If Amir measures $M_A = \{|0\rangle, |1\rangle\}$ and Bei measures $M_B = \{|+\rangle, |-\rangle\}$ and they get outcome $|0\rangle_A|+\rangle_B$, what does Amir know about the two qubits after the measurements if no communication?

A) Nothing

B) A is in $|0\rangle$, B unknown

C) A is in $|0\rangle$, B in $|+\rangle$

Example: $|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

$\left| \, {}_{A}\langle\alpha_i| \, {}_{B}\langle\beta_j| \, |\psi\rangle_{AB} \right|^2$

$M_A = \{|+\rangle, |-\rangle\}$ $M_B = \{|+\rangle, |-\rangle\}$

Probability of $|+\rangle_A |-\rangle_B$?

Practice: Start PS3 # 4a