

Learning Goals

- Design + analyze a quant. algorithm
- Read circuit diagrams
- Describe time + query complexity

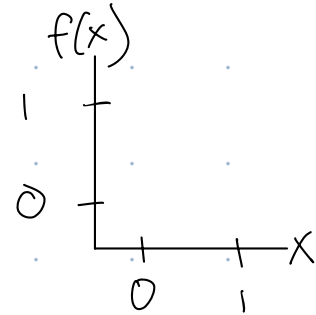
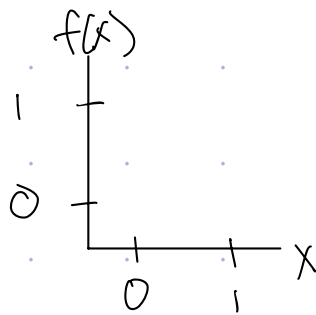
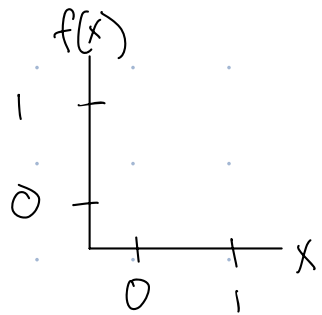
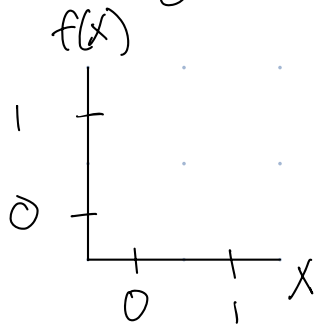
Deutsch's algorithm analyzes a 1-bit function f :

x	$f(x)$
0	
1	

ex: $f(x) = \begin{cases} 1 & \text{if it will rain at time } x \\ 0 & \text{if it will not rain at time } x \end{cases}$

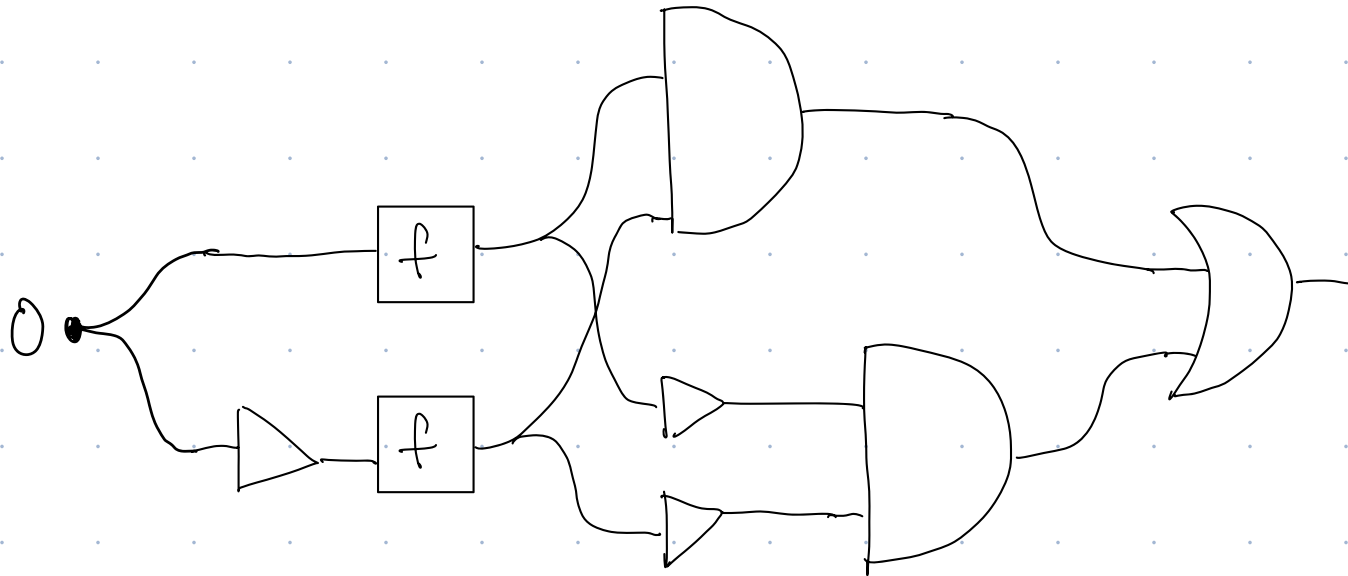
$x = \begin{cases} 0 & \rightarrow \text{day} \\ 1 & \rightarrow \text{night} \end{cases}$

Only 4 1-bit Functions:

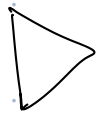


Deutsch's Problem: Given query access to f , determine if flat or sloped

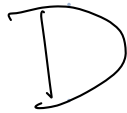
A Classical Algorithm (circuit diagram)



"query to f "



NOT



AND



OR

Query Complexity

Time Complexity

We would like to determine if f is flat or sloped using as few queries as possible.

What is the minimum number of classical queries to f needed to determine flat/sloped

A) 0

B) 1

C) 2

Deutsch's Problem

- Classical Query Complexity:
- Quantum Query Complexity:

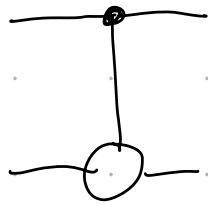
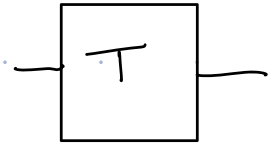
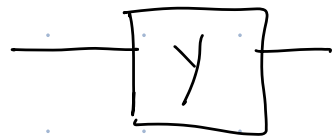
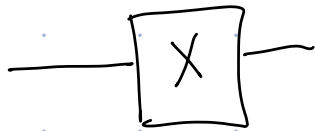
Why do we care?!

Quantum Gates (Circuit picture)

- time $L \rightarrow R$
- wires are qubit's
- Gates + measurements are shapes

$|0\rangle$ • —

Initialize each qubit
 $|0\rangle$ (usually)



CNOT



or



Measure in standard basis
 $M = \{ |0\rangle, |1\rangle \}$

Need gate for f . What about:

$$|0\rangle \rightarrow |f(0)\rangle$$

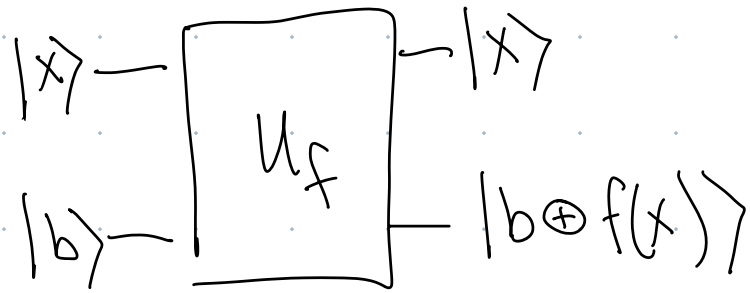
$$|1\rangle \rightarrow |f(1)\rangle$$

Explain why this is not an allowed gate.

* State $\rightarrow$ State

* Reversible

Instead:



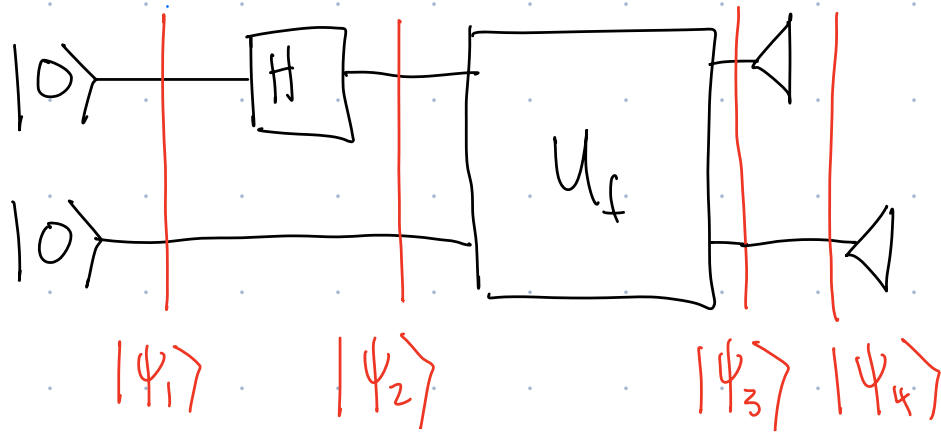
If $x=0$ get info about $f(0)$

If $x=1$ get info about $f(1)$

Idea - **Superposition!**



Analyzing a Quantum Circuit



$$|\psi_1\rangle = |0\rangle|0\rangle$$

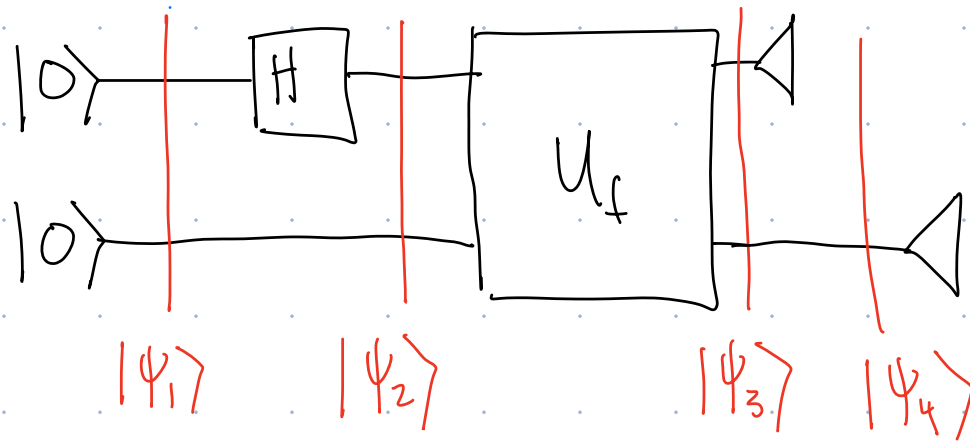
$$|\psi_2\rangle =$$

$$|\psi_3\rangle =$$

=

=

=



Partial Measurement:

Outcome $|0\rangle$

- Prob

- Collapse

Outcome $|1\rangle$

- Prob

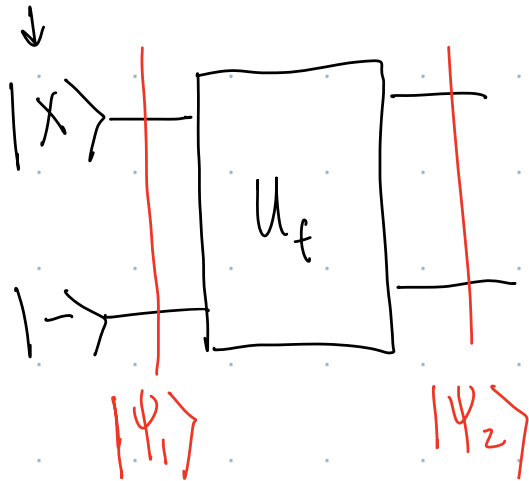
- Collapse

Mini-Hype Lesson

You need more than superposition to get a quantum advantage.

Group Exercise:

$x = 0 \text{ or } 1$

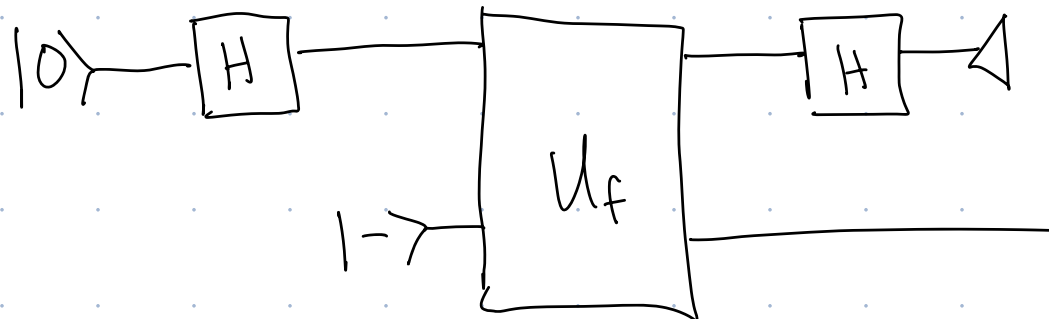


Show that $|\psi_2\rangle = (-1)^{f(x)} |x\rangle |-\rangle$

If $f(x) = 0$

$\downarrow$ $f(x) = 1$

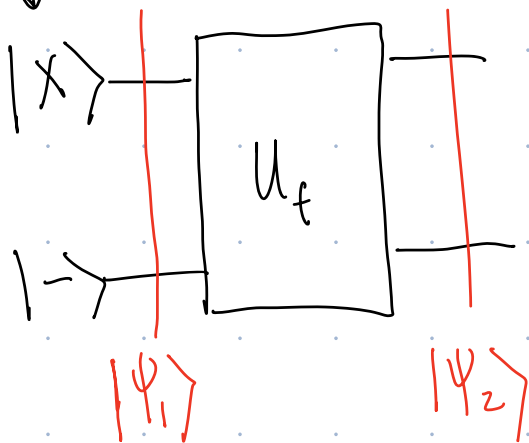
Analyze outcome of this circuit:



Solution

$x = 0$ or 1

↓



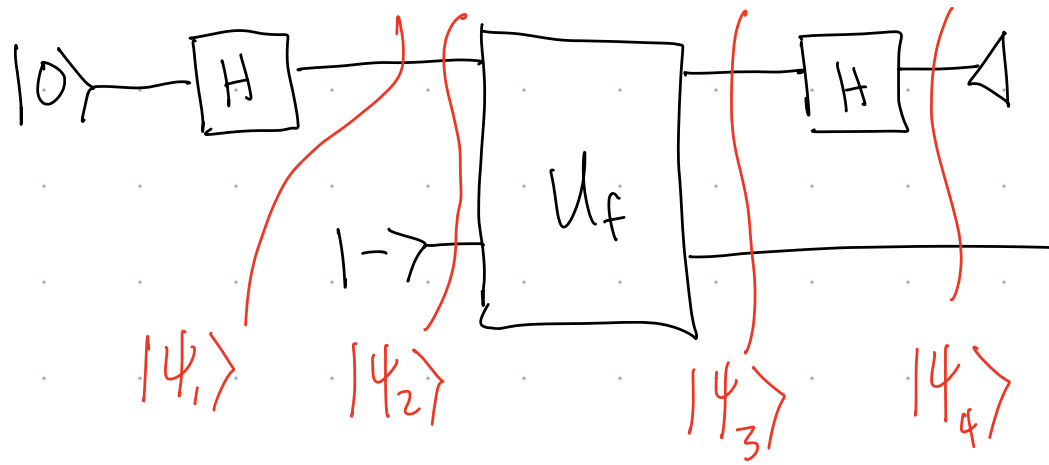
Show that $|\psi_2\rangle = (-1)^{f(x)} |x\rangle |-\rangle$

$$|\psi_2\rangle = U_f |x\rangle |-\rangle$$

New rule for U_f :

Phase Kickback

$$U_f : |x\rangle|-\rangle \rightarrow (-1)^{f(x)}|x\rangle|-\rangle$$



$$|\psi_1\rangle =$$

$$|\psi_2\rangle =$$

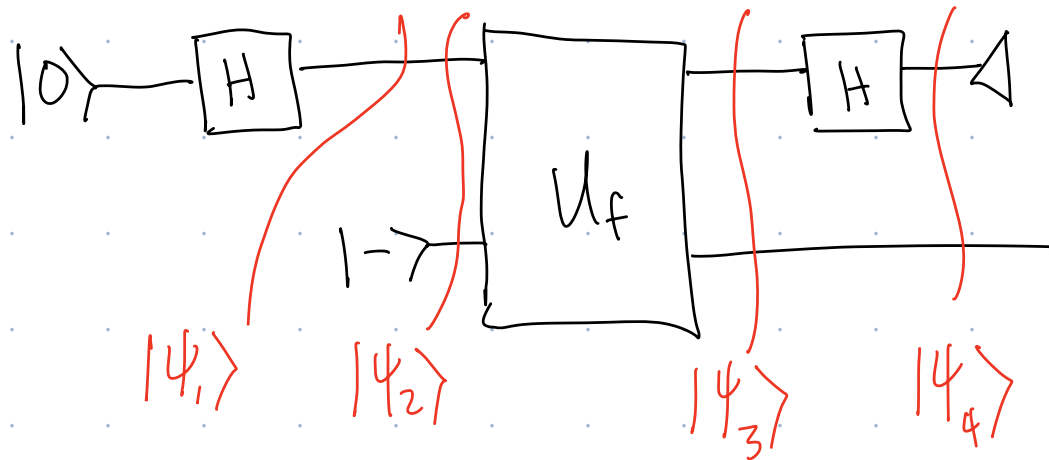
$$|\psi_3\rangle =$$

=

=

=

Deutsch's Alg:



Phase Kickback

