Learning Goals

- Design + analyze a quant. algorithm
- Read circuit diagrams
- Describe time + query complexity

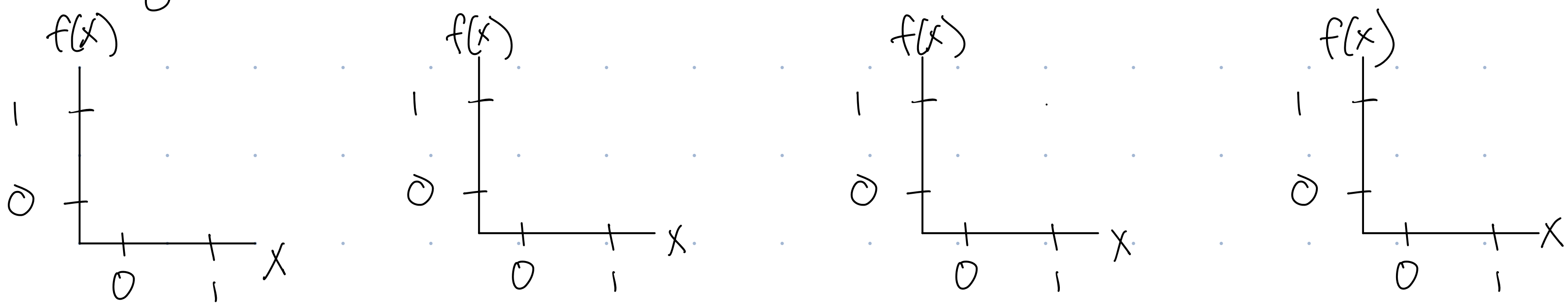
Deutsch's algorithm analyzes a 1-bit function f:

x	$f(x)$

ex: $f(x) = \{$

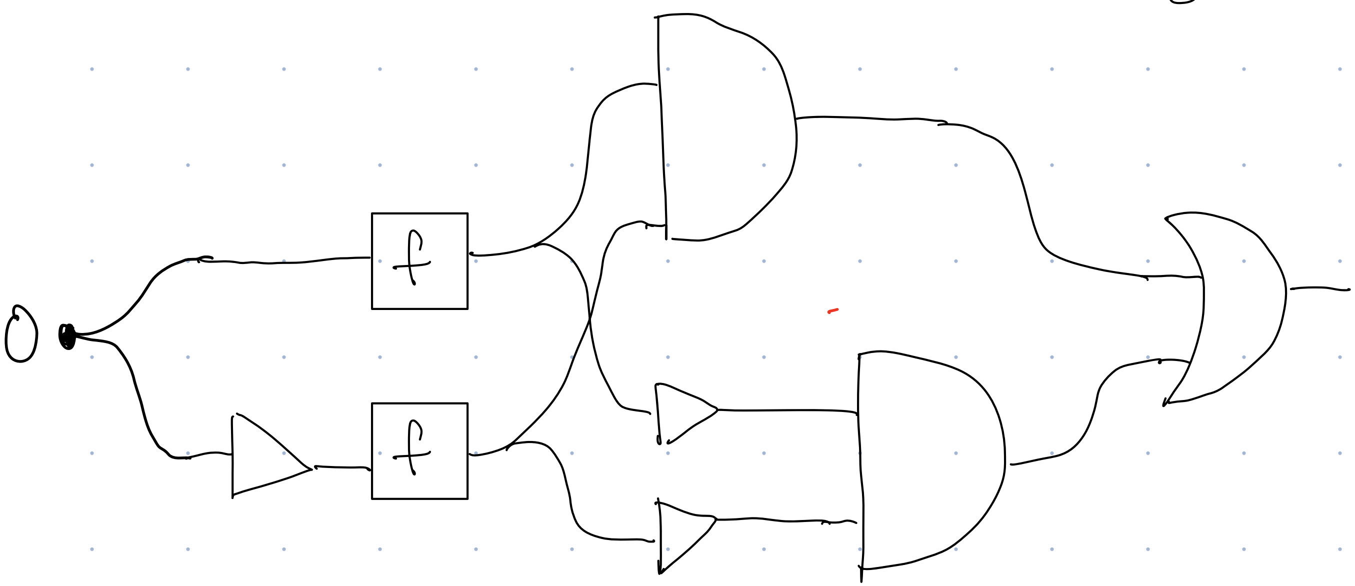
$x = \{$

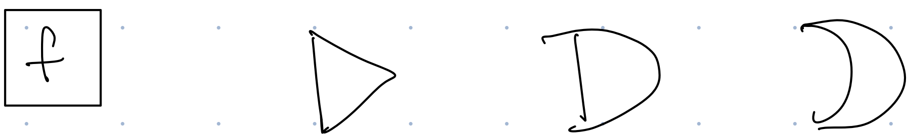
<u>Only 4 1-bit Functions:</u>



Deutsch's Problem:

A Classical Algorithm (circuit diagram)





Query Complexity

Time Complexity =

What is the minimum number of classical queries to f needed to determine flat/sloped

A) 0

B) $\underline{1}$

C) 2

Deutsch's Problem

- Classical Query Complexity:
- Quantum Query Complexity:

Why do we care?!

-
-
-

Quantum Gates (Circuit picture)

Need gate for f. What about:

Explain why this is not an allowed gate.

Instead:

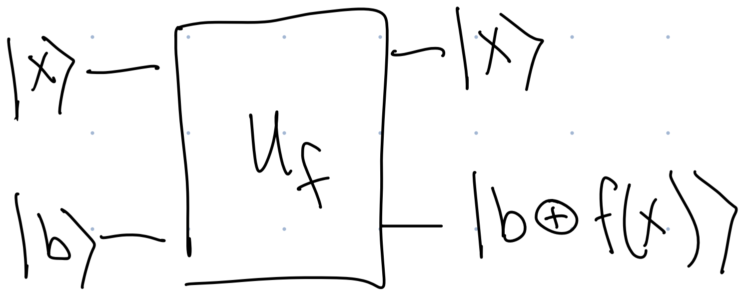
Interpretation:

$|0\rangle|0\rangle \rightarrow$

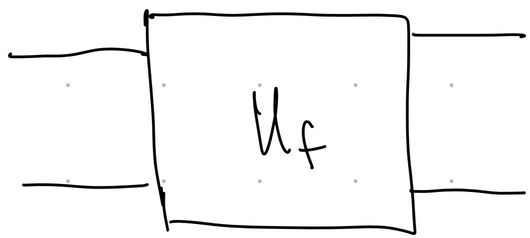
$|0\rangle|1\rangle \rightarrow$

$|1\rangle|0\rangle \rightarrow$

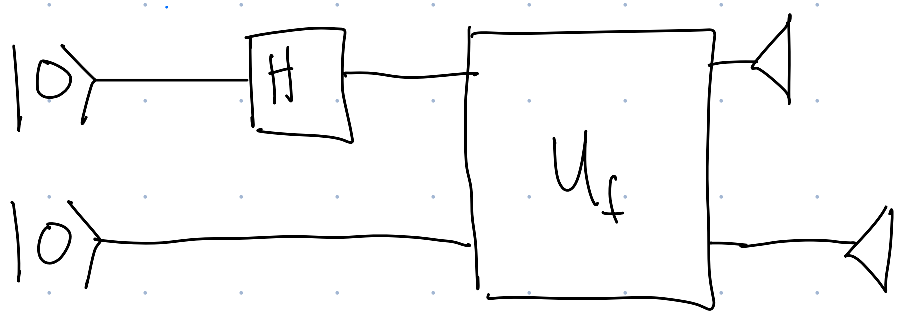
$|1\rangle|1\rangle \rightarrow$



Idea –



Analyzing a Quantum Circuit



$|\psi_1\rangle =$

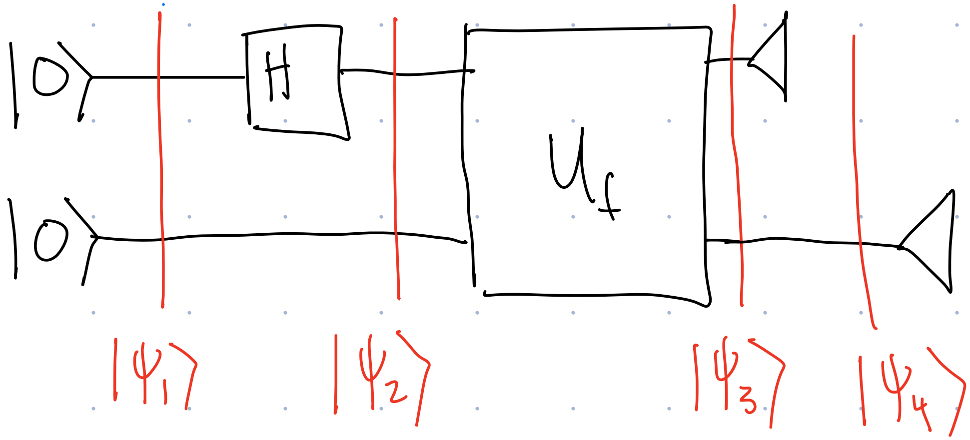
$|\psi_2\rangle =$

$|\psi_3\rangle =$

$=$

$=$

$=$



$$|\psi_3\rangle = \frac{1}{\sqrt{2}} |0\rangle|f(0)\rangle + \frac{1}{\sqrt{2}}|1\rangle|f(1)\rangle$$

Partial Measurement:

Outcome $|0\rangle$

- Prob

- Collapse

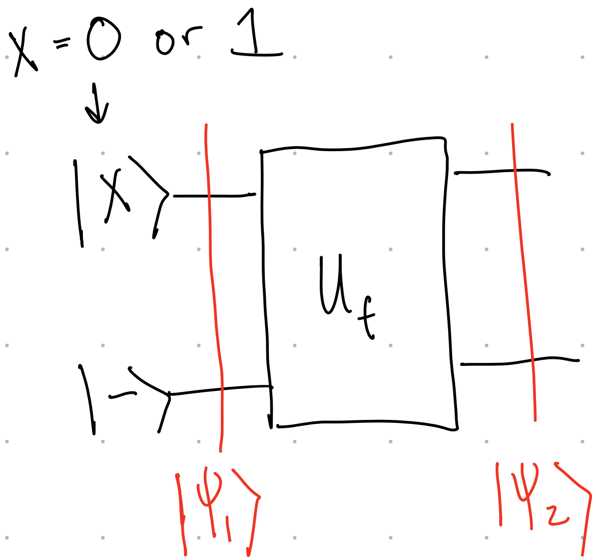
Outcome $|1\rangle$

- Prob

- Collapse

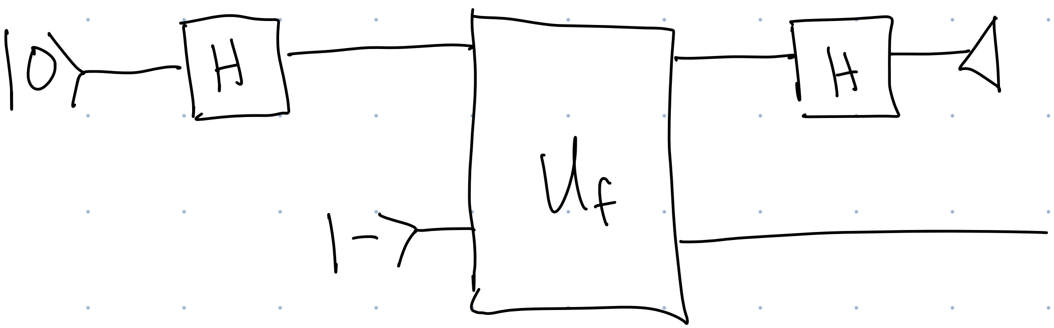
Mini-Hype Lesson

Group Exercise:



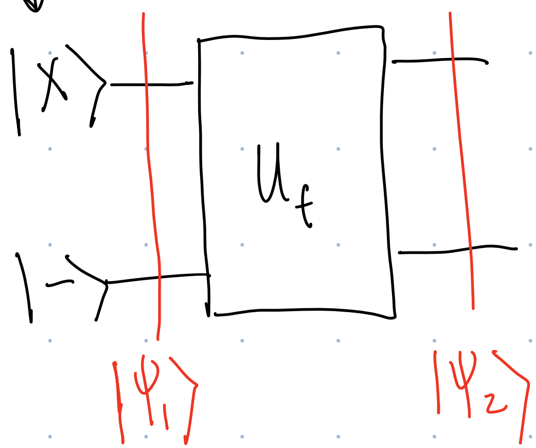
Show that $|\psi_2\rangle = (-1)^{f(x)} |x\rangle |-\rangle$

Analyze outcome of this circuit:



Solution

$x = 0$ or 1

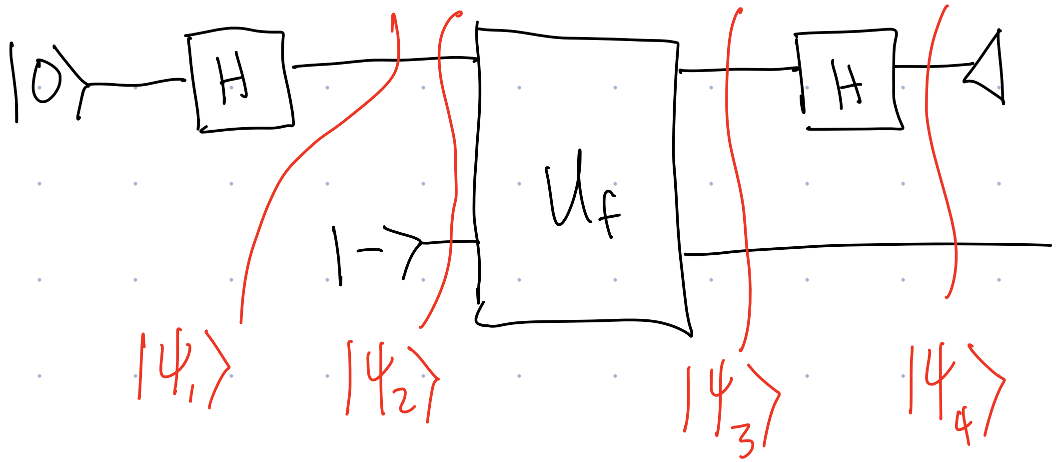


Show that $|\psi_2\rangle = (-1)^{f(x)} |x\rangle |-\rangle$

New rule for U_f:

Phase Kickback

U_f :



$|\psi_1\rangle =$

$|\psi_2\rangle =$

$|\psi_3\rangle =$

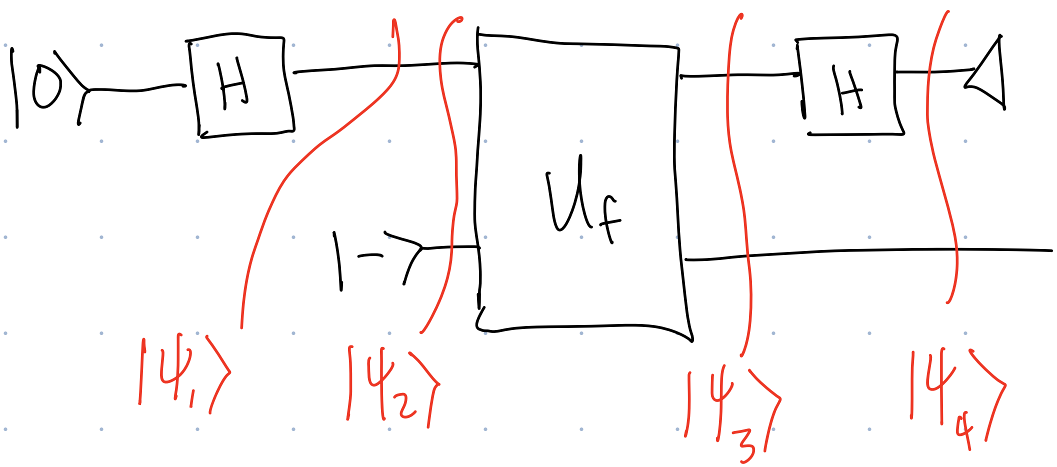
$=$

$=$

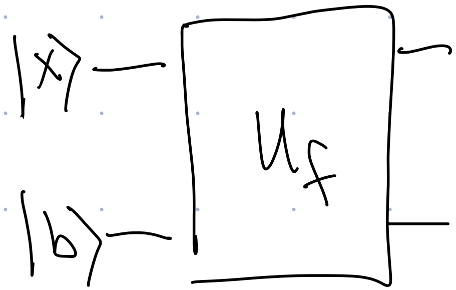
$=$

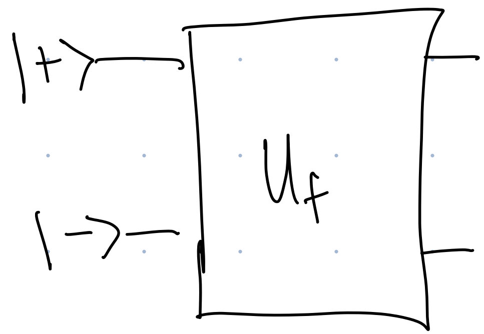
$=$

Deutsch's Alg:



Phase Kickback





|+⟩ vs |−⟩