Deutsch's Algorithm

Learning Goals

- Design + analyze a quant. algorithm [QC2]
- Read circuit diagrams
- Describe time + query complexity

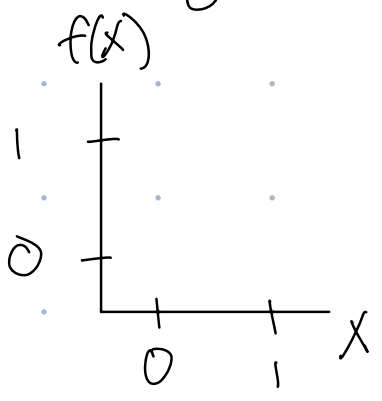
Deutsch's algorithm analyzes a 1-bit function f:

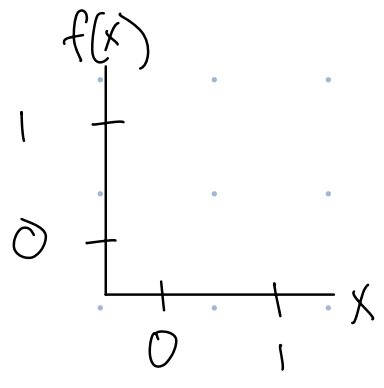
x	f(x)

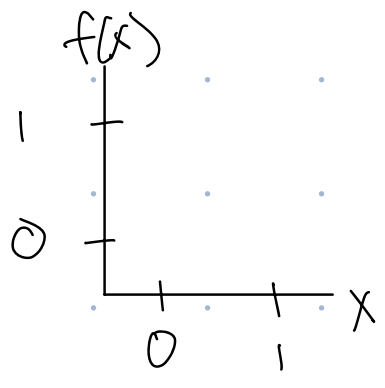
ex: $f(x) = \begin{cases} 1 & \text{if it will rain at time } x \\ 0 & \text{if it will not rain at time } x \end{cases}$

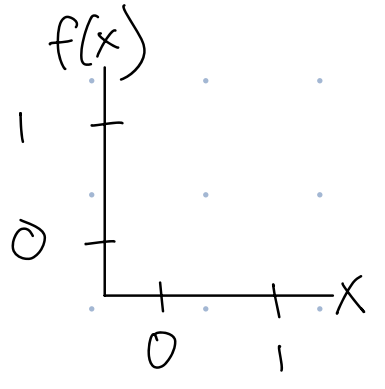
$x = \begin{cases} 0 \rightarrow \text{day} \\ 1 \rightarrow \text{night} \end{cases}$

Only 4 1-bit Functions:



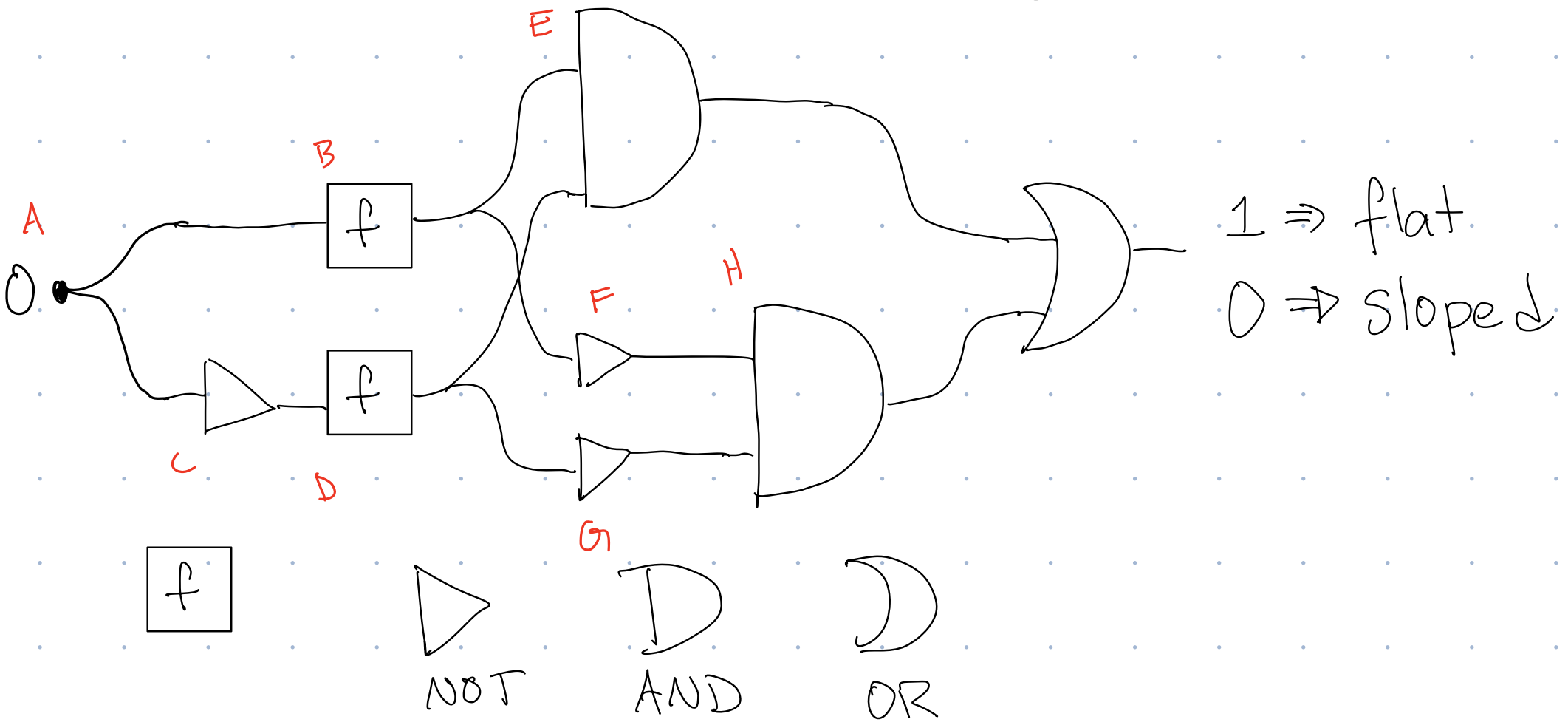






Deutsch's Problem:

A Classical Algorithm (circuit diagram)



↳

Query Complexity =

Time Complexity =

We would like to determine if f is flat or sloped using as few queries as possible.

What is the minimum number of classical queries to f needed to determine flat/sloped

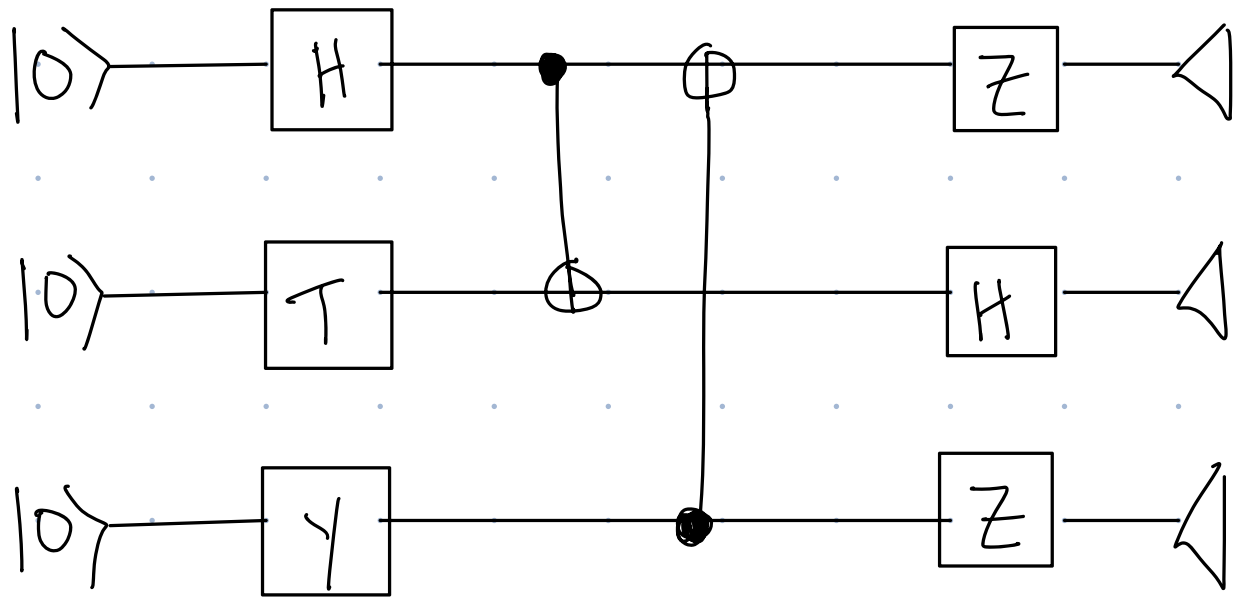
A) 0 B) 1 C) 2

Deutsch's Problem

- Classical Query Complexity:
- Quantum Query Complexity:

Why do we care?!

Quantum Circuit



Need gate for f. What about:

$$|0\rangle \rightarrow |f(0)\rangle$$
$$|1\rangle \rightarrow |f(1)\rangle$$

Explain why this is not an allowed gate.

* State → State
* Reversible

Instead:

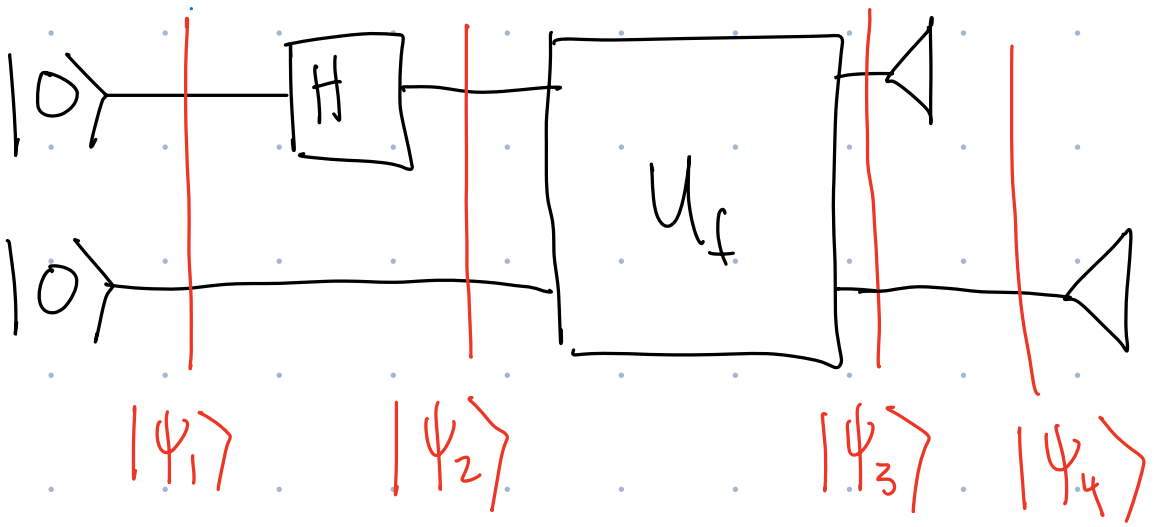
Analyzing a Quantum Circuit



$$|\psi_1\rangle = |0\rangle \otimes |0\rangle$$

$$|\psi_2\rangle =$$

$$|\psi_3\rangle =$$



Partial Measurement:

Outcome $|0\rangle$

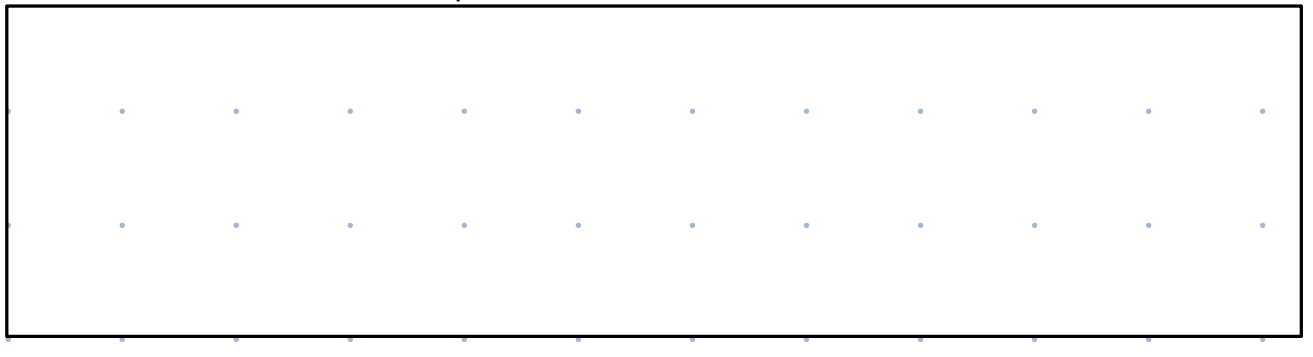
Outcome $|1\rangle$

Mini-Hype Lesson

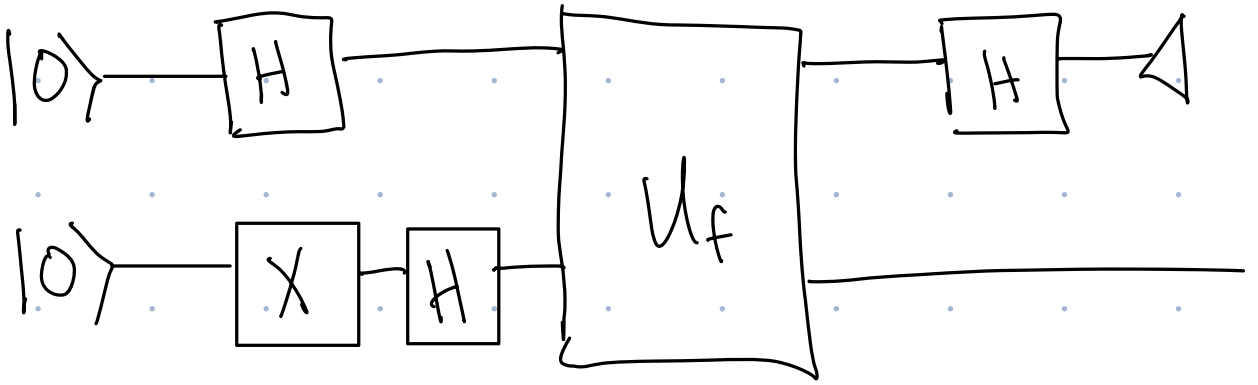
You need more than superposition to get a quantum advantage.

New rule for U_f's behavior:

New Rule



① Analyze outcome of this circuit using this rule

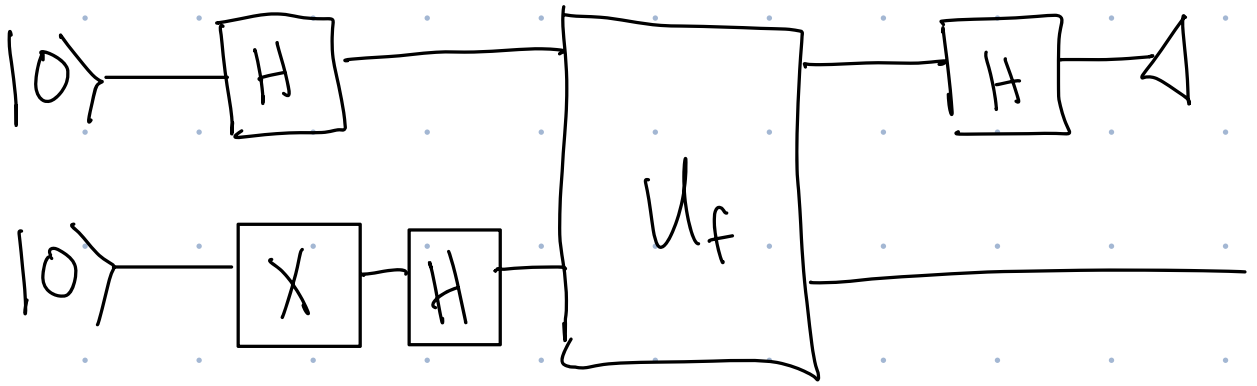


② Use old rule to derive new rule

Old Rule

$x, b \in \{0,1\}$:

$$U_f|x\rangle|b\rangle = |x\rangle|b \oplus f(x)\rangle$$



$|\psi_1\rangle =$

Phase Kickback

$$U_f : |x\rangle|-\rangle \rightarrow (-1)^{f(x)}|x\rangle|-\rangle$$

Phase Kickback



