BOMB DETECTION

Learning Goals

- Describe quantum operations
- Describe partial q. measurements
- Use rules of q. ops + partial meas. to analyze interferometer scenarios

QI3

Feels Hard/Confusing ...

- Come discuss @ OH
- Meet up @ go/CSHelp
- Practice Practice Practice

COMPUTER SCIENCE

DEPARTMENT SOCIAL

10/9 | 12:20-1:20 | 75SHS 102

COME CHAT AND CATCH UP
WITH YOUR FACULTY AND PEERS

BRING YOURSELF, BRING A FRIEND,
BRING YOUR APPETITE

DELICIOUS FOOD PROVIDED BY
THE TASTE OF INDIA!

Exit Tickets

If a state has only $|00\rangle$ and $|11\rangle$ or only $|01\rangle$ and $|10\rangle$ it is an entangled state?

A) Yes B) No C) Depends

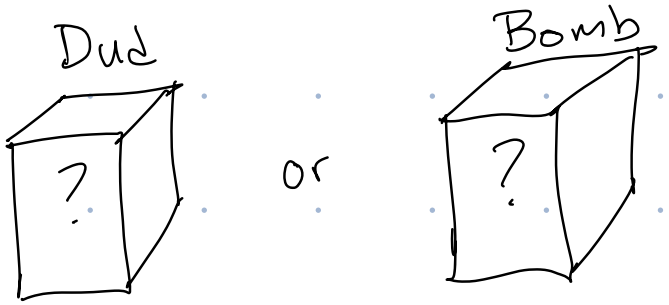
Entanglement Measure: $H(|\psi\rangle_{AB}) \in [0, 1]$

0: No entanglement

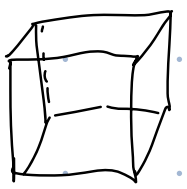
1: Max entanglement

- If have 2 states (4 qubits) each with $H = \frac{1}{2}$ ⇒ can combine into 1 state with $H = 1$, 1 state with $H = 0$
- More entanglement
  - Gives higher prob. of winning CHSH + other similar protocols
  - Needed for exponential time advantage

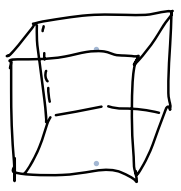
Problem: Quantum Bomb Detection (Elitzur-Vaidman)



Dud: photon

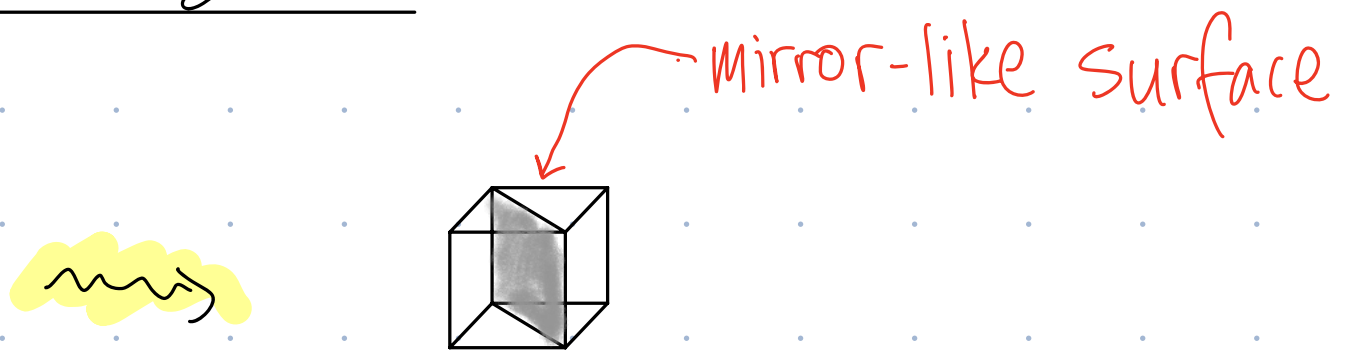


Bomb: photon



Want to give friend a rainbow bomb not a dud. How??

Beam Splitter Demo



The beamsplitter transmits _____(i)_____ polarized light and reflects _____(ii)_____ polarized light

A. (i) vertical
   (ii) horizontal

B. (i) horizontal
   (ii) vertical

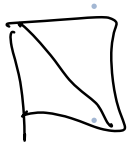
C. (i) vertical
   (ii) diagonal

Quantum Gate

Operation that changes q. state without collapse (no info extracted)

Can describe gate via action on standard basis states

Beam-Splitter



$|0\rangle_P |H\rangle_D \rightarrow$

$|1\rangle_P |H\rangle_D \rightarrow$

$|0\rangle_P |V\rangle_D \rightarrow$

$|1\rangle_P |V\rangle_D \rightarrow$

Mirror \

$|H\rangle_D \rightarrow$

$|V\rangle_D \rightarrow$

Wave plate

(0°)

Only affects polarization

(45°)

$|0\rangle_P \rightarrow$

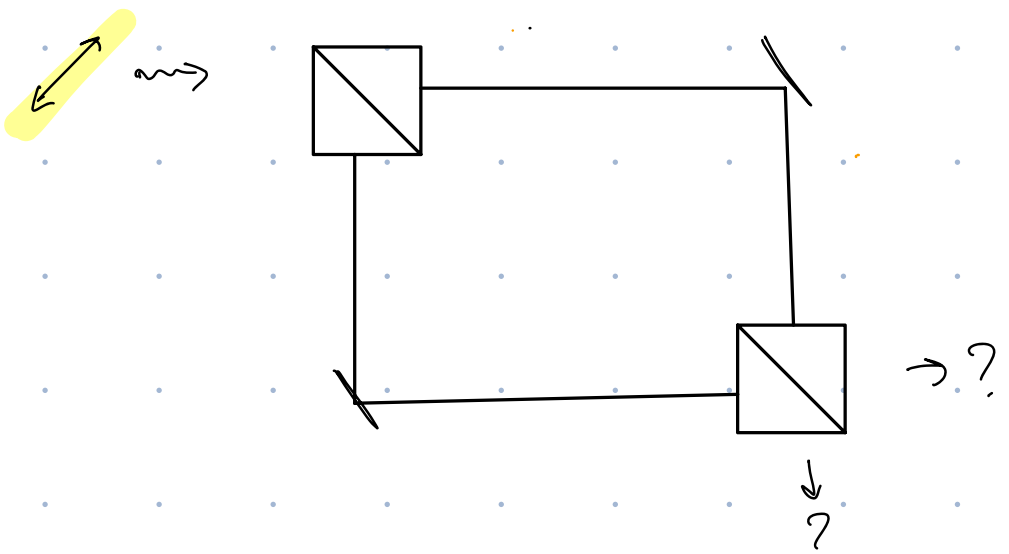
$|1\rangle_P \rightarrow$

(90°)

$|0\rangle_P \rightarrow$

$|1\rangle_P \rightarrow$

Interferometer





$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$

$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$

$|0\rangle_P|V\rangle_D \rightarrow |0\rangle_P|V\rangle_D$

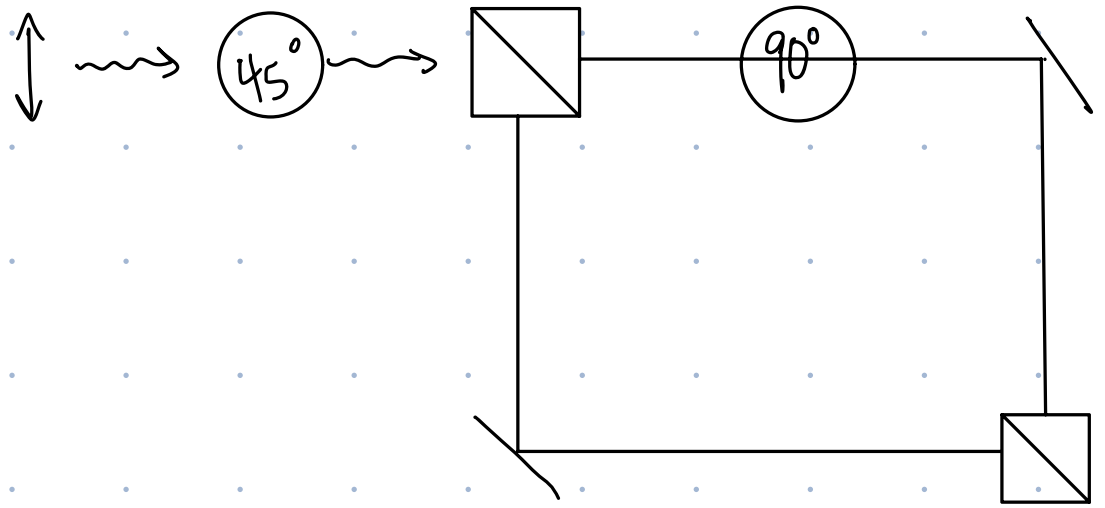
$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$

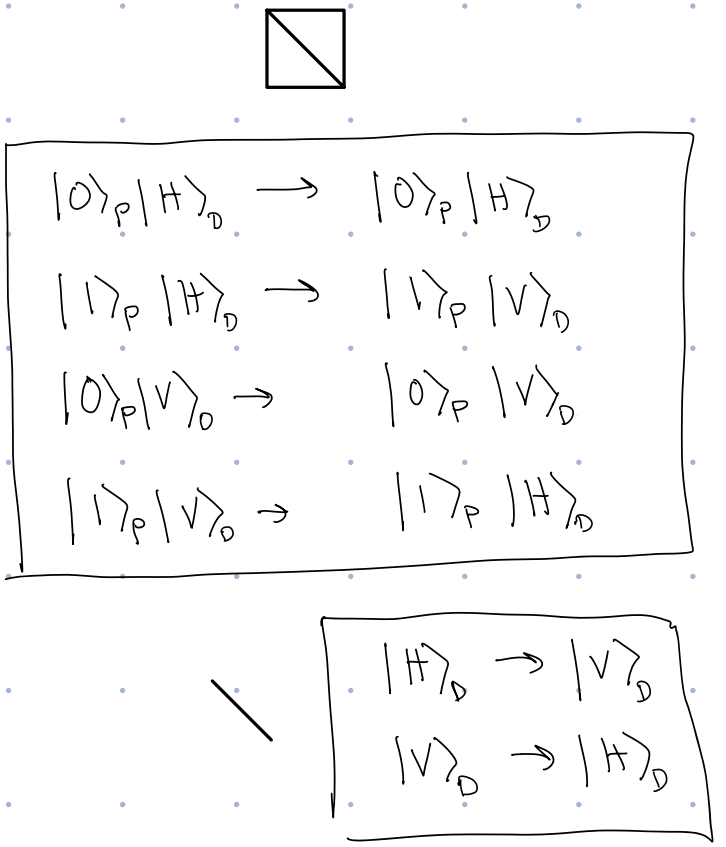


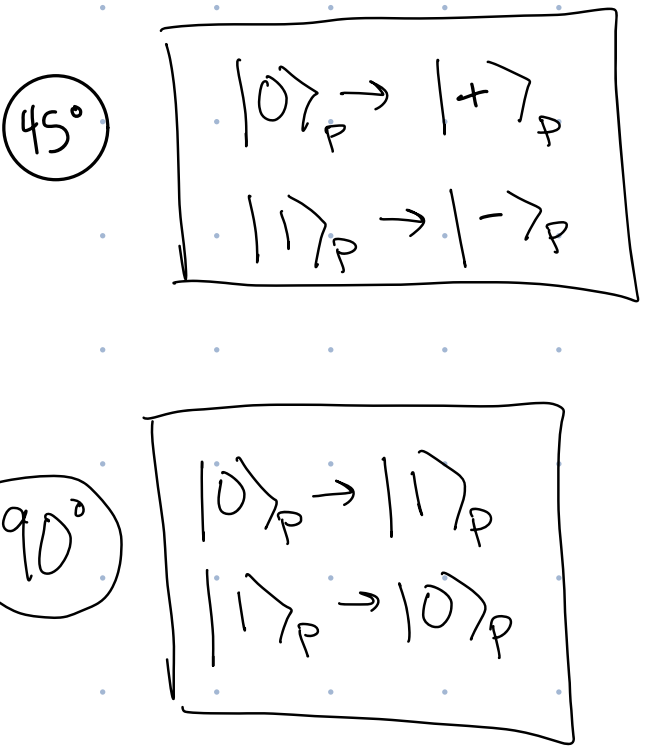
$|H\rangle_D \rightarrow |V\rangle_D$

$|V\rangle_D \rightarrow |H\rangle_D$

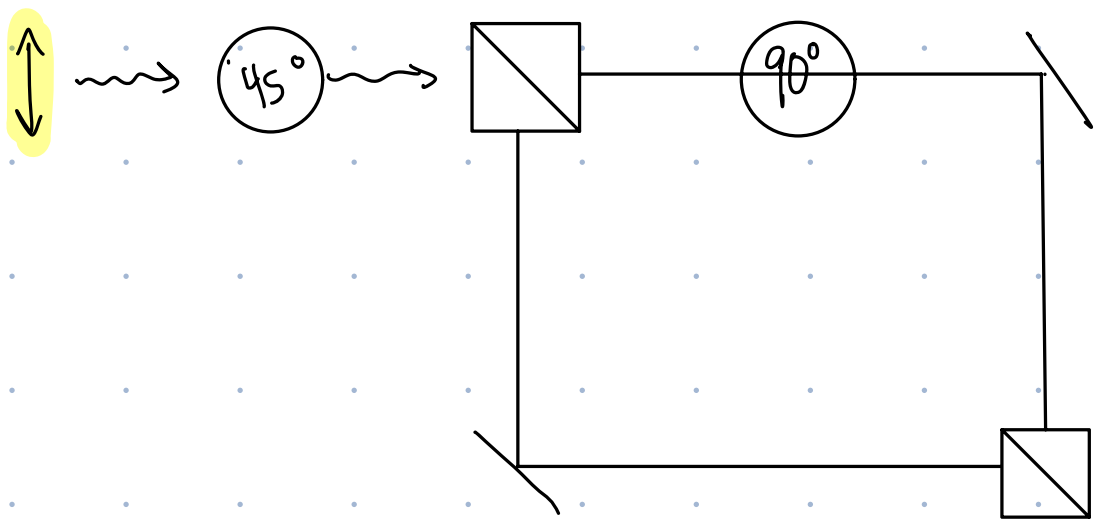
Group Practice

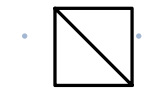






Group Practice





$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$

$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$

$|0\rangle_P|V\rangle_D \rightarrow |0\rangle_P|V\rangle_D$

$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$



$|H\rangle_D \rightarrow |V\rangle_D$

$|V\rangle_D \rightarrow |H\rangle_D$

45°

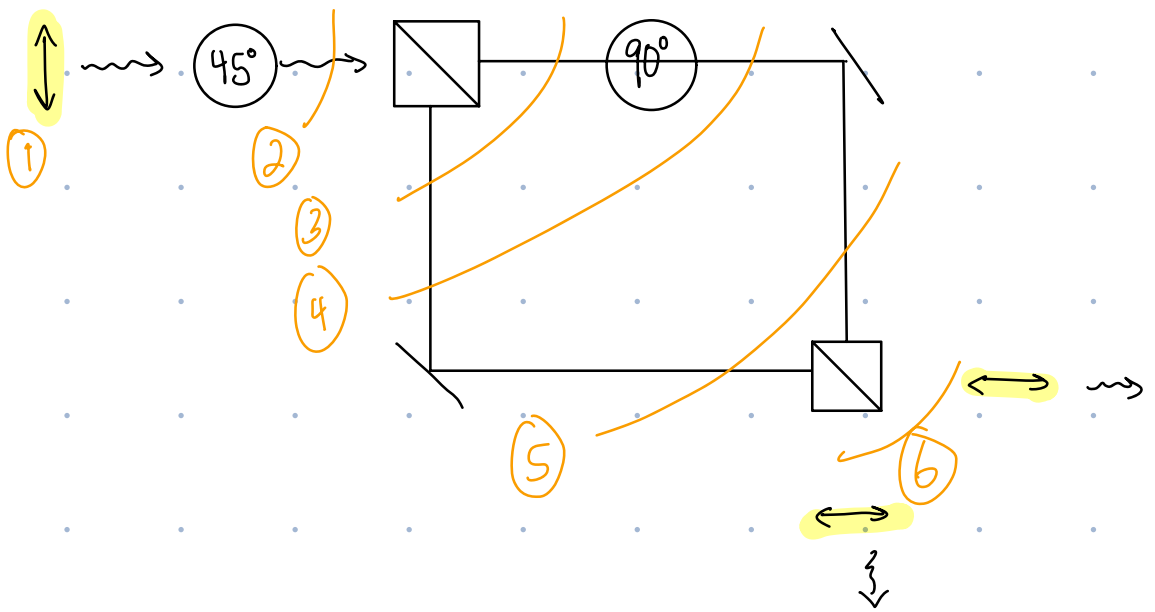
$|0\rangle_P \rightarrow |-\rangle_P$

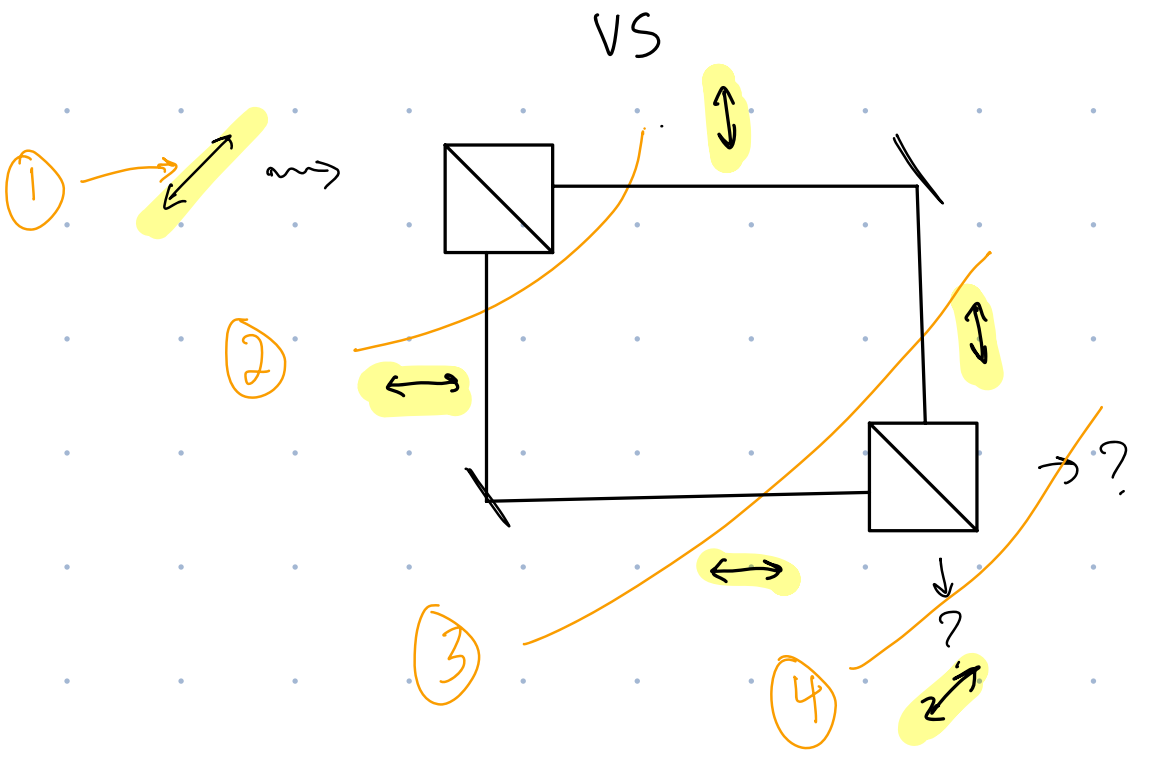
$|1\rangle_P \rightarrow |+\rangle_P$

90°

$|0\rangle_P \rightarrow |1\rangle_P$

$|1\rangle_P \rightarrow |0\rangle_P$



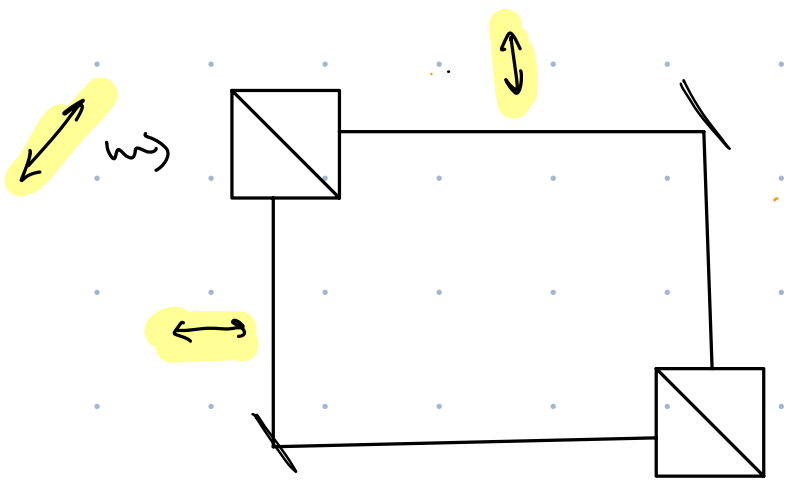


Big ideas:

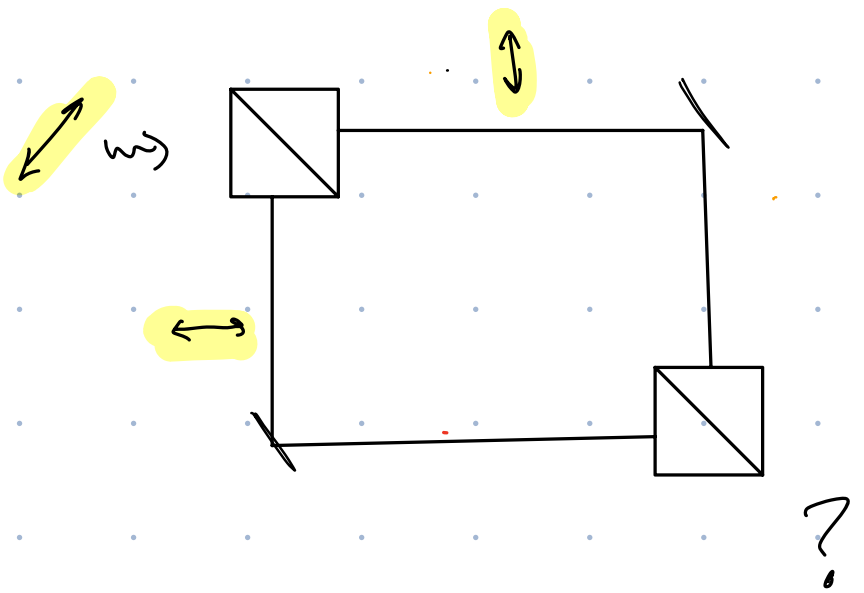
- One photon takes both paths (superposition) $|H\rangle_D + |V\rangle_D$
- Path and polarization are entangled
- Objects in arm affect output photon
- ~~D~~

Put potential bomb in one path!

Bomb Test



VS



Partial Q. Measurement (in standard basis)

- State to be measured: $|\psi\rangle_{AB}$
- Want to measure qubit A in $M_A = \{|0\rangle, |1\rangle\}$ No measurement of B.
- To determine outcome:
  - Write $|\psi\rangle_{AB}$ in standard basis:

$$|\psi\rangle_{AB} = a_{00}|00\rangle_{AB} + a_{01}|01\rangle_{AB} + a_{10}|10\rangle_{AB} + a_{11}|11\rangle_{AB}$$

  - Rewrite: Factor/group A kets:

$$|\psi\rangle_{AB} =$$

  - Calculate outcome probabilities/collapse state
    - Outcome $|0\rangle$
      - Probability:
      - Collapse state:

- Outcome 17
  - Probability:
  - Collapse state:

Example:

$$\frac{1}{\sqrt{10}}|00\rangle + \sqrt{\frac{4}{10}}|01\rangle + \frac{1}{\sqrt{10}}|10\rangle - i\sqrt{\frac{4}{10}}|11\rangle$$

Measure 2nd qubit with $M = \{|0\rangle, |1\rangle\}$

Prob of $|0\rangle$:

State collapses to

Prob of $|1\rangle$:

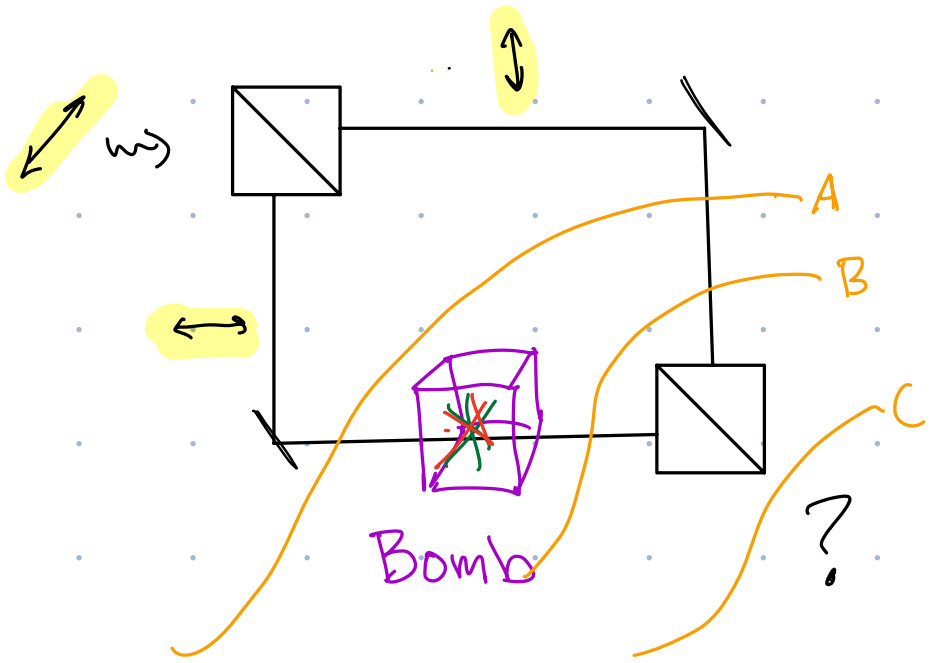
State collapses to

Back to the Bomb

Our two qubits are polarization + path (direction) $\left(|+\rangle_P |H\rangle_D\right)$

Is the bomb measuring the polarization qubit or path qubit?

A: Polarization B: Path (direction)



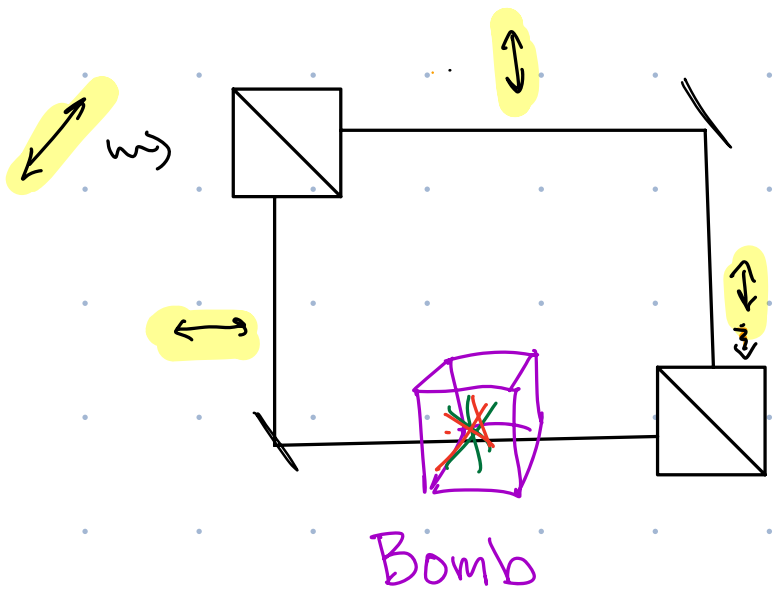
- Outcome $|H\rangle_D$: Prob

  Collapses →

- Outcome $|V\rangle_D$: Prob

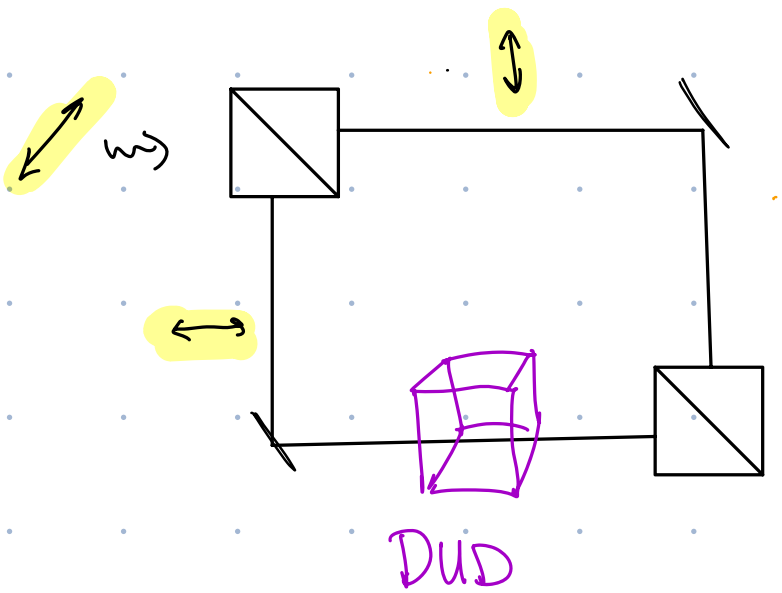
  Collapses →

If no explosion



Bomb

VS.

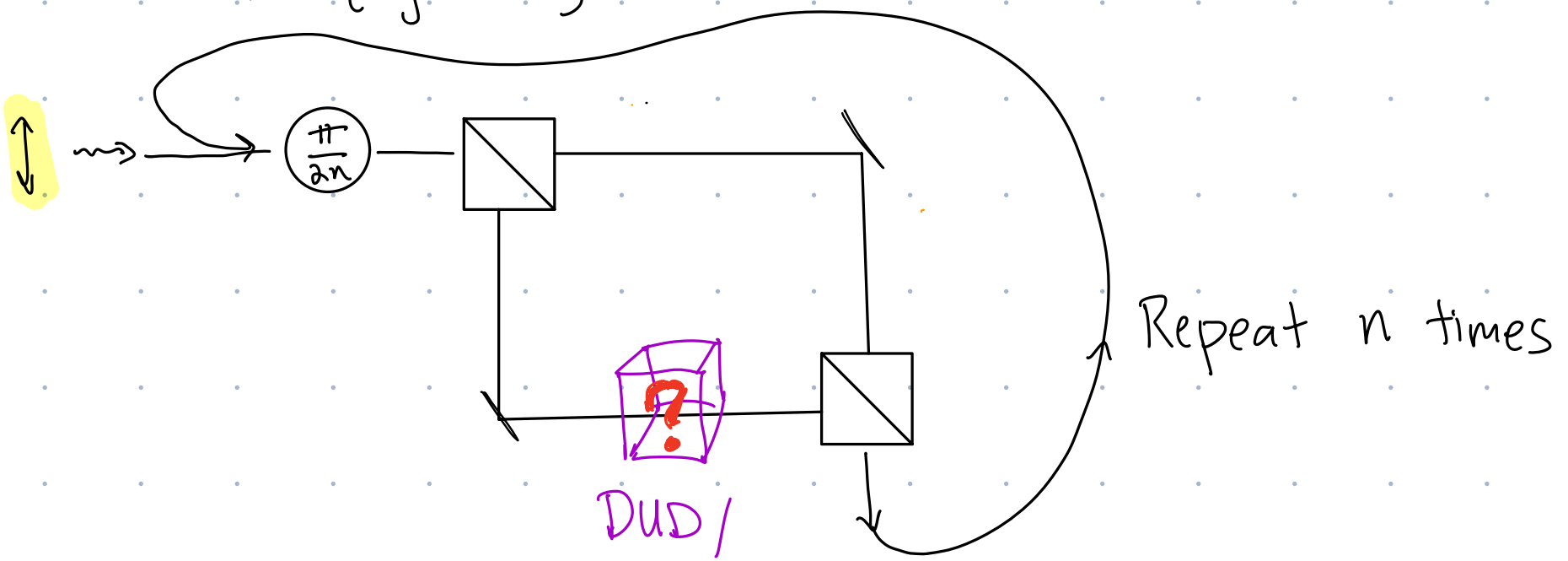


DUD

Can we do better?

Group Work: Analyze the following set up.

Choose $n \gg 1$. (Eg. 1000)



$\frac{\pi}{2n}|0\rangle \rightarrow \cos\left(\frac{\pi}{2n}\right)|0\rangle + \sin\left(\frac{\pi}{2n}\right)|1\rangle$

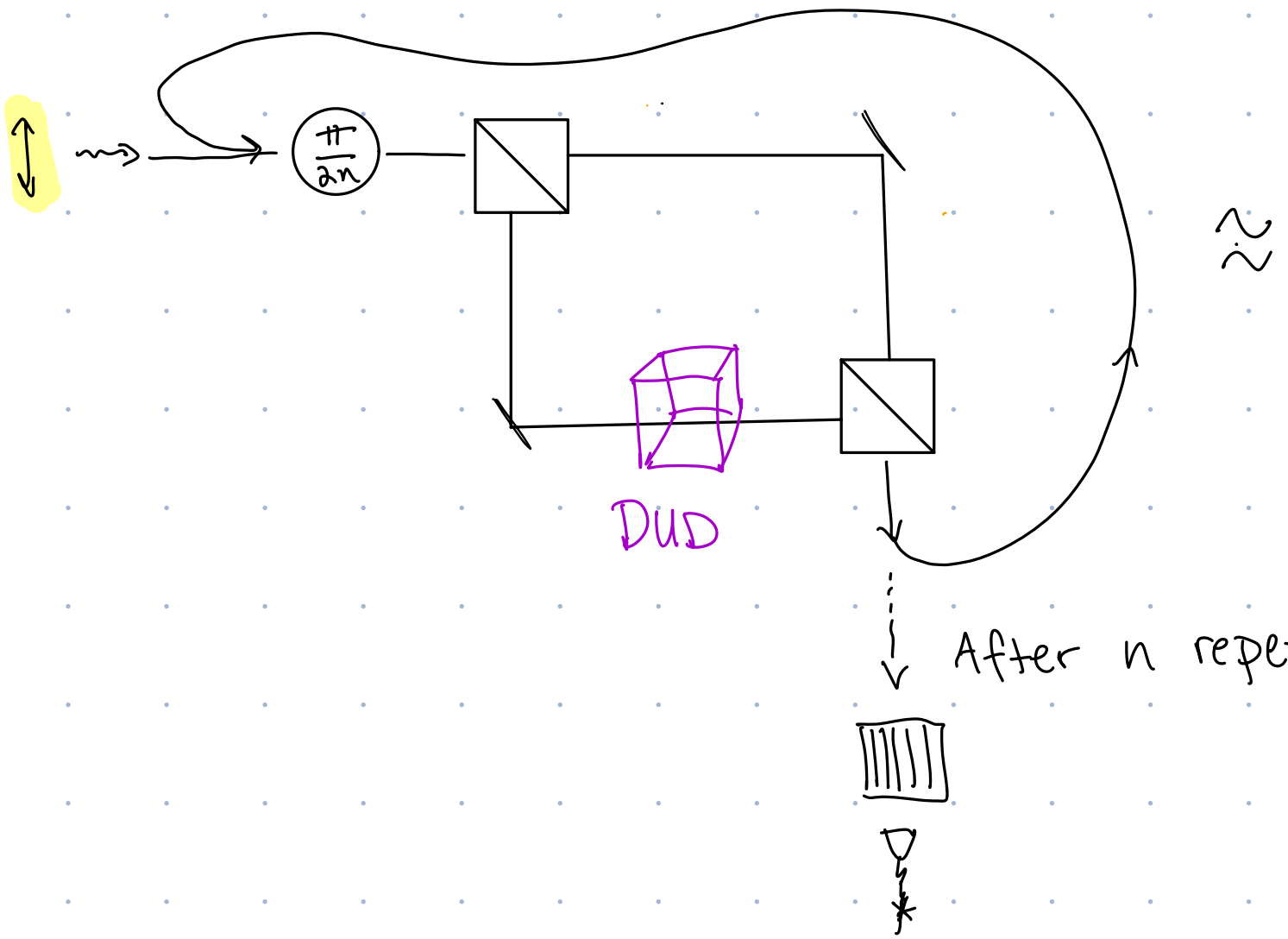
$\frac{\pi}{2n}$ x n times = 90°

For small x

- $\cos(x) \approx 1 - \frac{x^2}{2}$
- $(1-x)^k \approx 1 - kx$

1) What happens if dud?
2) If bomb, probability of NO explosion
3) If bomb but no explosion, probability of detection?

1) If Dud:



≈

After n repetitions

If Bomb:

