BOMB DETECTION

Learning Goals

- Describe quantum operations
- Describe partial q. measurements
- Use rules of q. ops + partial meas. to analyze interferometer scenarios

QI4

Feels Hard/Confusing ...

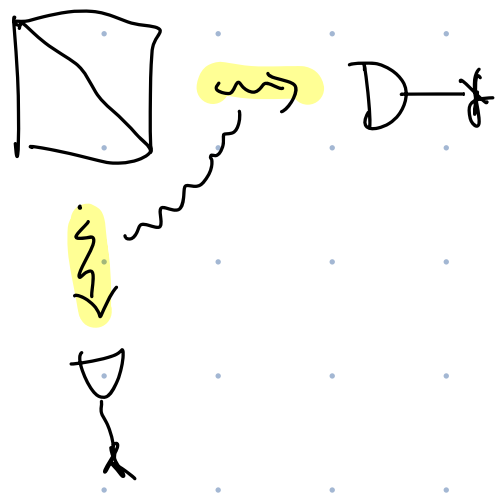
- Come discuss @ OH
- Meet up @ go/CSHelp
- Practice Practice Practice

Entangled or Not Entangled?

- $|+\rangle_P |H\rangle_D$ — X
- $\frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D - \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D$ — X $|-\rangle_P|H\rangle_D$
- $\frac{1}{\sqrt{2}}|0\rangle_P|1\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|0\rangle_D$ — ✓

If a photon is in 2 places at once is it

- Entangled ?



- In superposition ✓
- Both ?

$|0\rangle_P\left(\frac{1}{\sqrt{2}}|H\rangle_D + \frac{1}{\sqrt{2}}|V\rangle_D\right)$

Exactly one ~~detector~~ lights up.

Why?

COMPUTER SCIENCE

DEPARTMENT SOCIAL

10/9 | 12:20-1:20 | 75SHS 102

COME CHAT AND CATCH UP
WITH YOUR FACULTY AND PEERS

BRING YOURSELF, BRING A FRIEND,
BRING YOUR APPETITE

DELICIOUS FOOD PROVIDED BY
THE TASTE OF INDIA!

Exit Tickets

$|00\rangle = |0\rangle|0\rangle$ $\quad a|00\rangle + d|11\rangle$

If a state has only $|00\rangle$ and $|11\rangle$ or only $|01\rangle$ and $|10\rangle$ is it an entangled state?

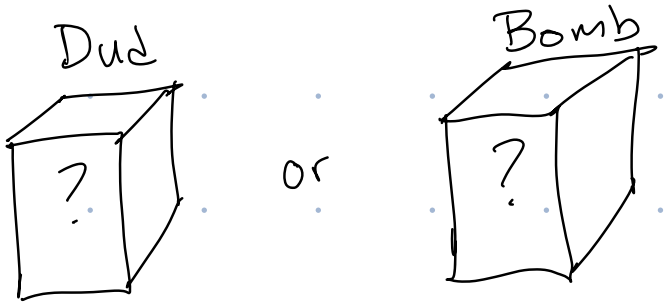
A) Yes B) No C) Depends

Entanglement Measure: $H(|\psi\rangle_{AB}) \in [0, 1]$

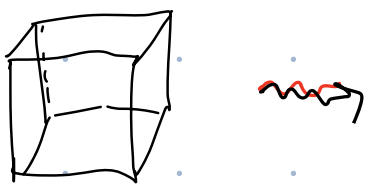
No entanglement — Max entanglement

- If have 2 states (4 qubits) each with $H = 1/2$ ⇒ can combine into 1 state with $H = 1$, 1 state with $H = 0$
- More entanglement
  - Gives higher prob. of winning CHSH + other similar protocols
  - Needed for exponential time advantage

Problem: Quantum Bomb Detection (Elitzur-Vaidman)



Dud: photon

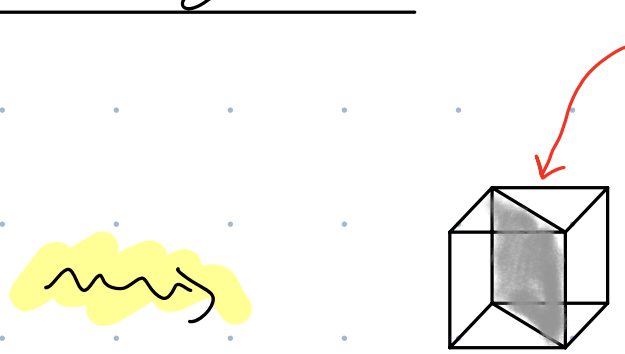


Bomb: photon



Want to give friend a unexploded rainbow bomb not a dud. How??

Beam Splitter Demo



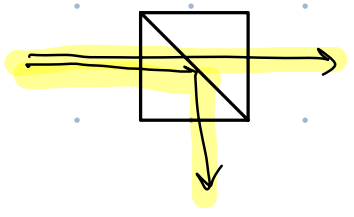
The beamsplitter transmits _____(i)_____ polarized light and reflects _____(ii)_____ polarized light

A. (i) vertical
(ii) horizontal

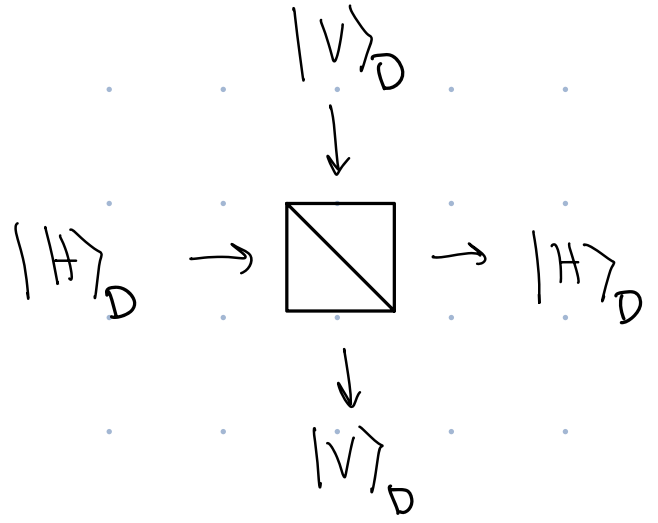
B. (i) horizontal
(ii) vertical

C. (i) vertical
(ii) diagonal

New Tool: Beamsplitter



- Transmits $|0\rangle_P$ (vertically polarized light)
- Reflects $|1\rangle_P$ (horiz. " ")



"P" for polarization
"D" for direction

$\{|V\rangle_D, |H\rangle_D\}$ standard basis states

Behavior:

$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$ ↕ → ▱ → ↕

$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$ ↔ → ▱ ↓ ↔

$|0\rangle_P|V\rangle_D \rightarrow |0\rangle_P|V\rangle_D$ ↓ ▱

$|1\rangle_P|V\rangle_D \rightarrow$ ↓ ▱ →

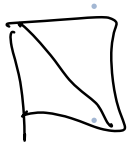
A) $|0\rangle_P|H\rangle_D$ B) $|0\rangle_P|V\rangle_D$ C) $|1\rangle_P|H\rangle_D$ D) $|1\rangle_P|V\rangle_D$

Quantum Gate

Operation that changes q. state without collapse (no info extracted)

Can describe gate via action on standard basis states

Beam-Splitter



$$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$$

$$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$$

$$|0\rangle_P|V\rangle_D \rightarrow |0\rangle_P|V\rangle_D$$

$$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$$

Mirror \

$$|H\rangle_D \rightarrow |V\rangle_D$$

$$|V\rangle_D \rightarrow |H\rangle_D$$

Does not the "P" qubit

Wave plate

(θ)

Only affects polarization

(45°)

$$|0\rangle_P \rightarrow |+\rangle_P$$

$$|1\rangle_P \rightarrow |-\rangle_P$$

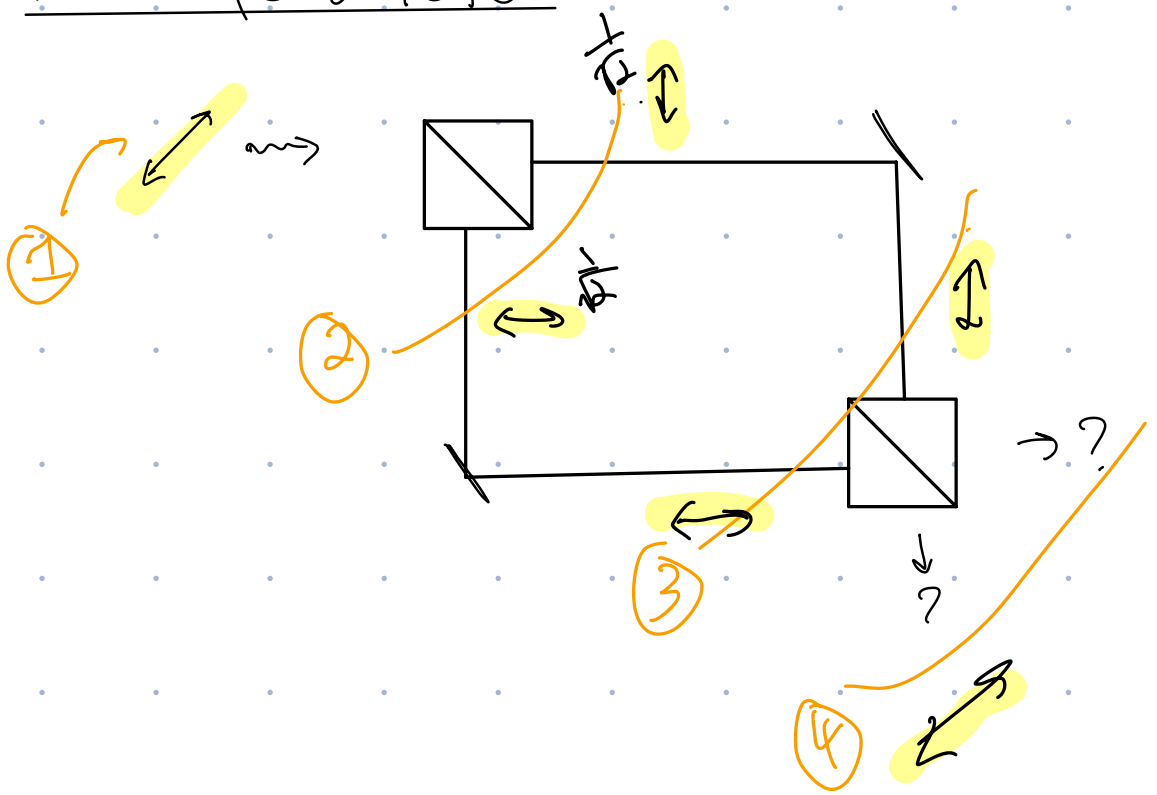
(90°)

$$|0\rangle_P \rightarrow |1\rangle_P$$

$$|1\rangle_P \rightarrow |0\rangle_P$$

Interferometer

Q13





$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$

$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$

$|0\rangle_P|V\rangle_D \rightarrow |0\rangle_P|V\rangle_D$

$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$



$|H\rangle_D \rightarrow |V\rangle_D$

$|V\rangle_D \rightarrow |H\rangle_D$

$|\psi_1\rangle = |+\rangle_P|H\rangle_D$

$|\psi_2\rangle = \square\,|+\rangle_P|H\rangle_D = \square\left(\frac{1}{\sqrt{2}}|0\rangle_P + \frac{1}{\sqrt{2}}|1\rangle_P\right)|H\rangle_D =$ Distributed $|+\rangle$

$= \square\left(\frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D\right) = \square\,\frac{1}{\sqrt{2}}|0\rangle|H\rangle + \square\,\frac{1}{\sqrt{2}}|1\rangle|H\rangle$ Commuted amplitudes

Distributed Gate $= \frac{1}{\sqrt{2}}\,\square\,|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}\,\square\,|1\rangle_P|H\rangle_D = \frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D$

$|\psi_3\rangle = \diagdown\left(\frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D\right) = \diagdown\,\frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \diagdown\,\frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D$

$= \frac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D$

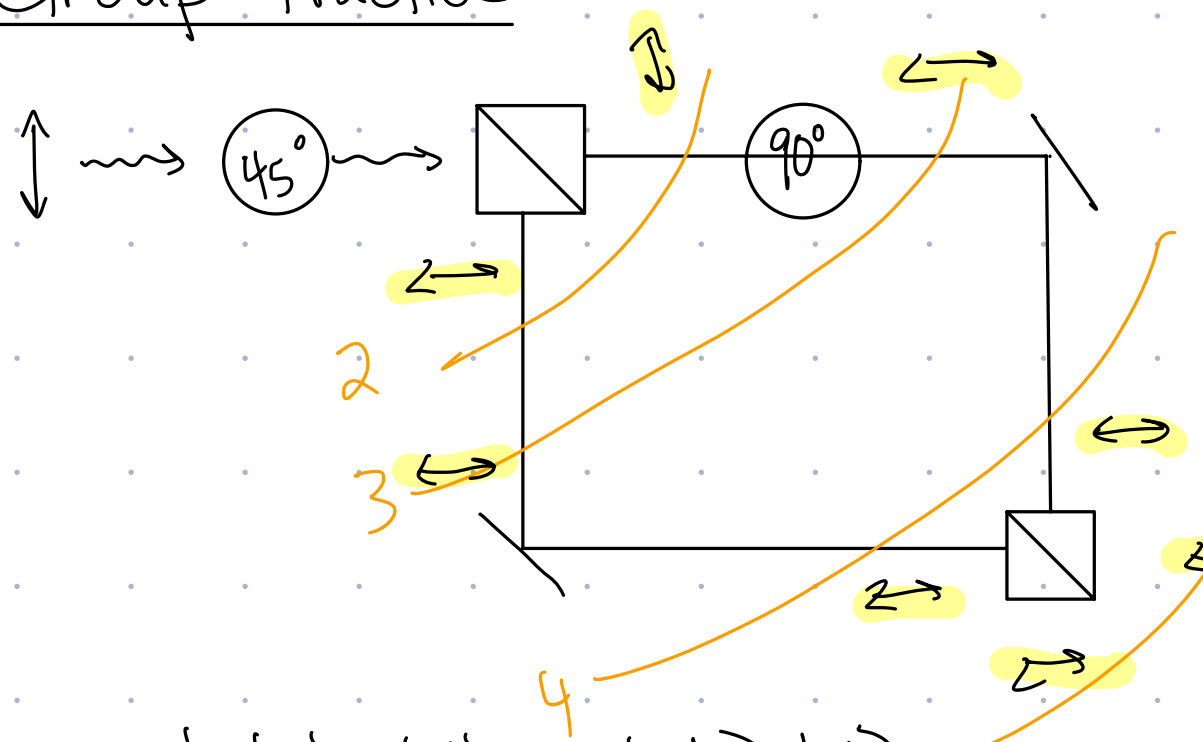
$$|\psi_4\rangle = \square \left(\frac{1}{\sqrt{2}} |0\rangle_P |V\rangle_D + \frac{1}{\sqrt{2}} |1\rangle_P |H\rangle_D \right)$$

$$= \frac{1}{\sqrt{2}} |0\rangle_P |V\rangle_D + \frac{1}{\sqrt{2}} |1\rangle_P |V\rangle_D$$

$$= \left(\frac{1}{\sqrt{2}} |0\rangle_P + \frac{1}{\sqrt{2}} |1\rangle_P \right) |V\rangle_D$$

$$= |+\rangle_P |V\rangle_D$$

Group Practice



$$|\psi_2\rangle = \frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D$$

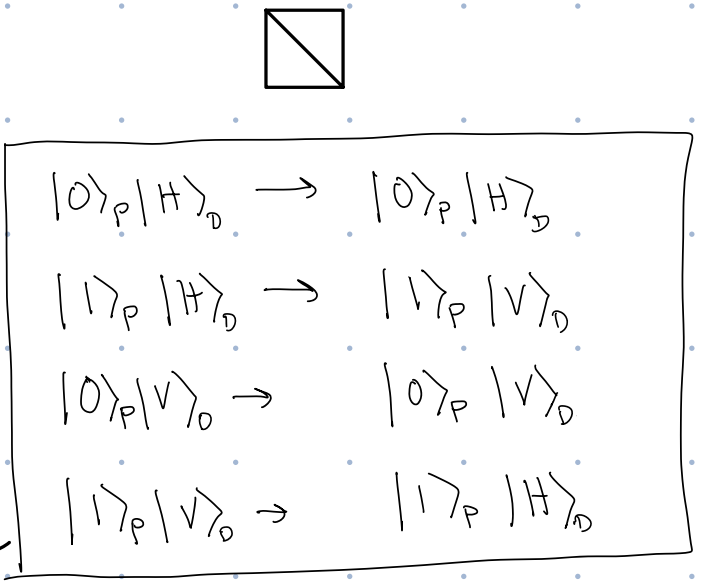
$$|\psi_3\rangle = \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D$$

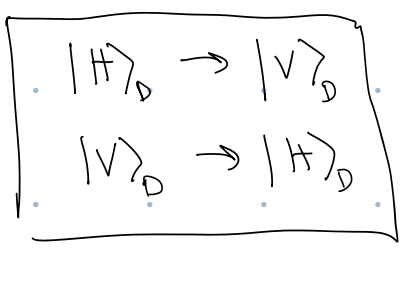
$$|\psi_4\rangle = \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D$$

$$|\psi_5\rangle = \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D$$

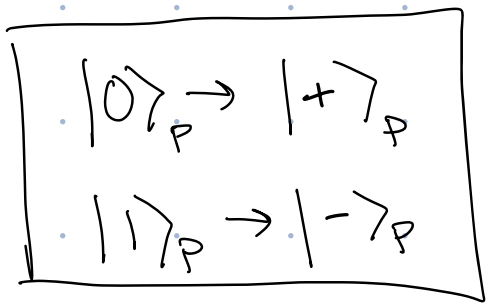
$$= |1\rangle_P\left(\frac{1}{\sqrt{2}}|H\rangle_D + \frac{1}{\sqrt{2}}|V\rangle\right)$$

"Horiz. polarized photon in a superposition of traveling horizontally + vertically"

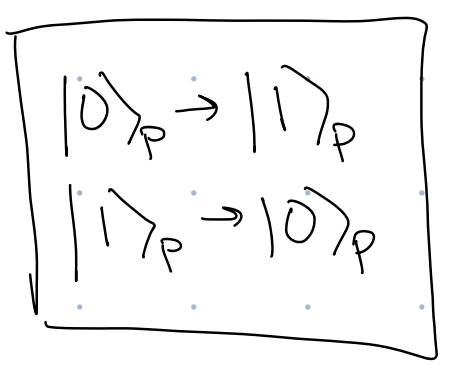




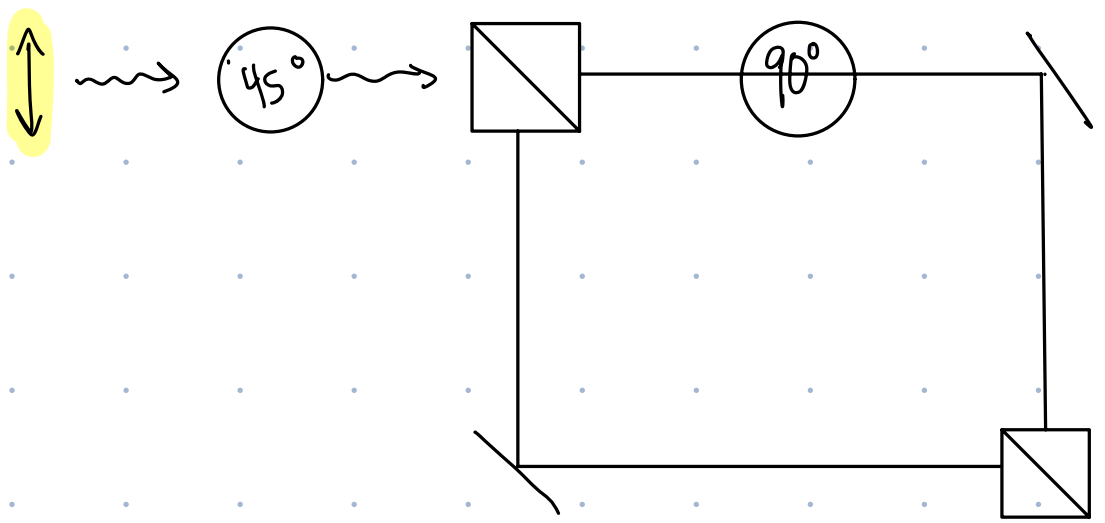
45°

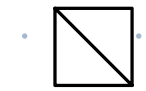


90°



Group Practice





$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$

$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$

$|0\rangle_P|V\rangle_D \rightarrow |0\rangle_P|V\rangle_D$

$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$



$|H\rangle_D \rightarrow |V\rangle_D$

$|V\rangle_D \rightarrow |H\rangle_D$

45°

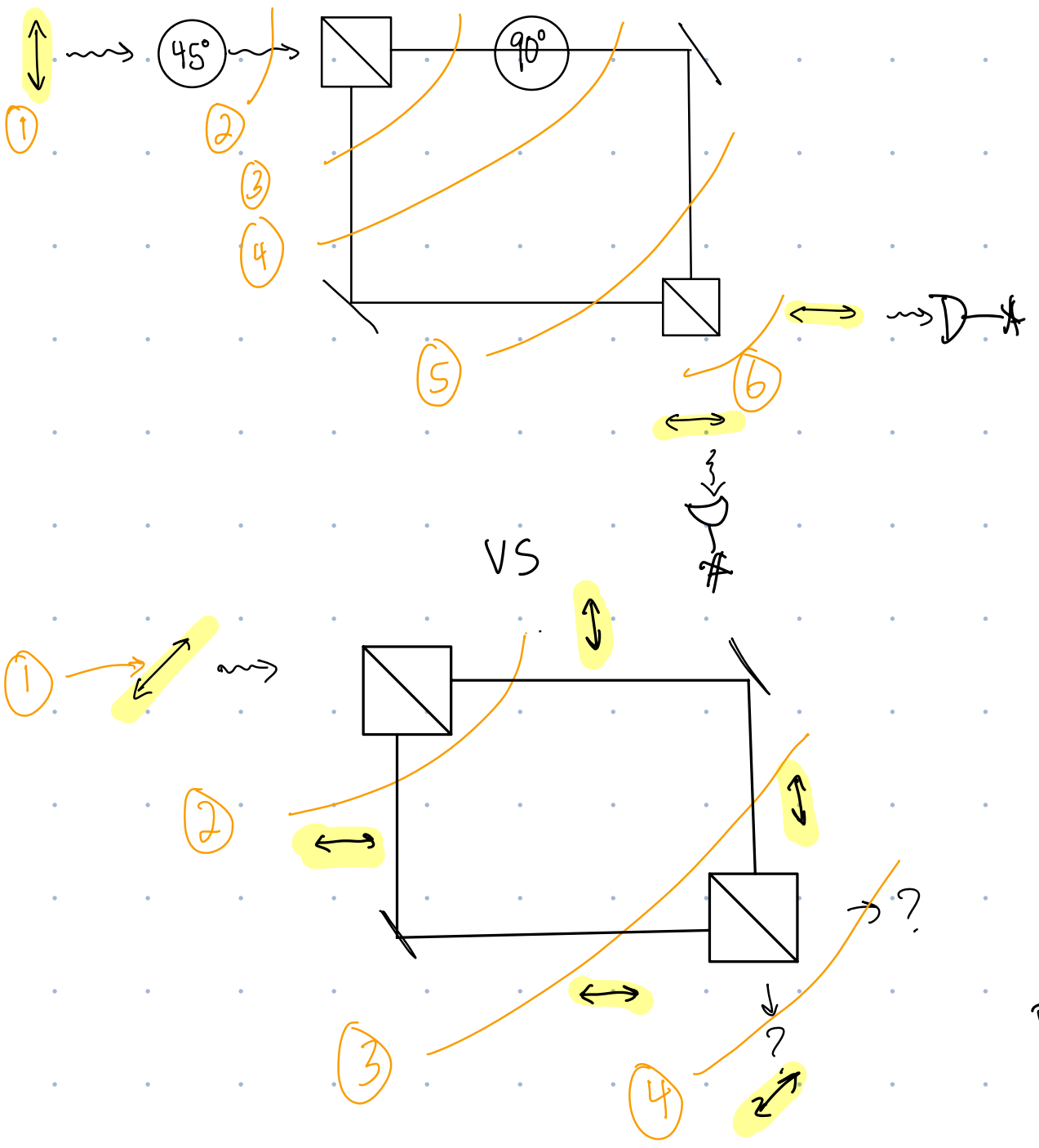
$|0\rangle_P \rightarrow |-\rangle_P$

$|1\rangle_P \rightarrow |+\rangle_P$

90°

$|0\rangle_P \rightarrow |1\rangle_P$

$|1\rangle_P \rightarrow |0\rangle_P$

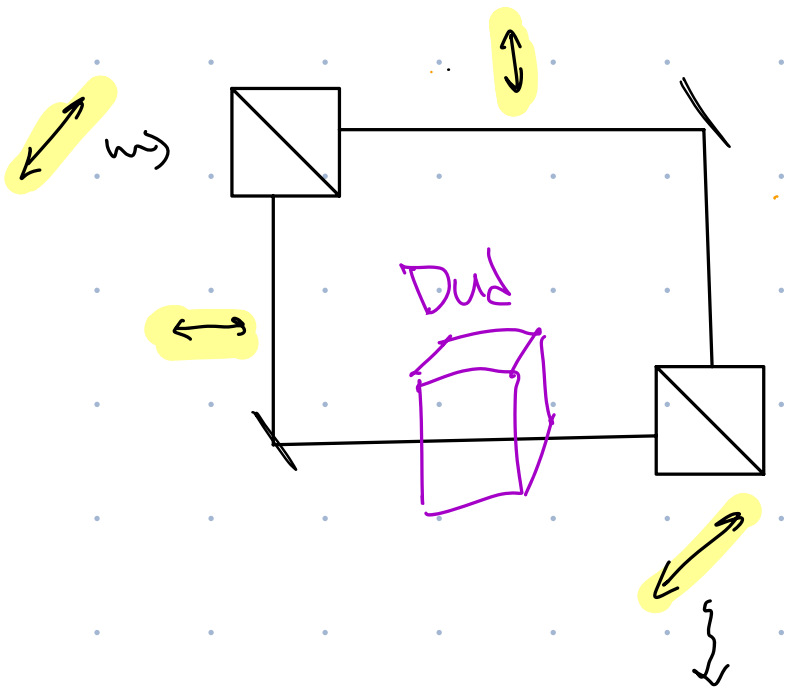


Big ideas:

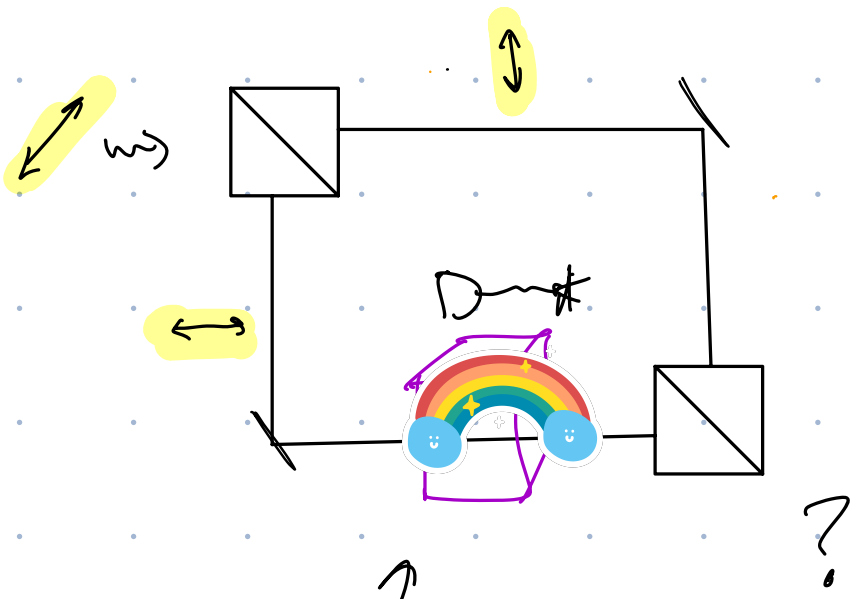
- One photon takes both paths (superposition) $|H\rangle_D + |V\rangle_D$
- Path and polarization are entangled
- Objects in arm affect output photon
- D

Put potential bomb in one path!

Bomb Test



vs



Partial Measurement

Partial Q. Measurement (in standard basis)

- State to be measured: $|\psi\rangle_{AB}$
- Want to measure qubit A in $M_A = \{|0\rangle, |1\rangle\}$ No measurement of B.
- To determine outcome:
  - Write $|\psi\rangle_{AB}$ in standard basis: $\left(|+\rangle \to \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)$

$$|\psi\rangle_{AB} = a_{00}|00\rangle_{AB} + a_{01}|01\rangle_{AB} + a_{10}|10\rangle_{AB} + a_{11}|11\rangle_{AB}$$

  - Rewrite: Factor/group A kets:

$$|\psi\rangle_{AB} = |0\rangle_A\left(a_{00}|0\rangle_B + a_{01}|1\rangle_B\right) + |1\rangle_A\left(a_{10}|0\rangle_B + a_{11}|1\rangle_B\right)$$

  - Calculate outcome probabilities/collapse state
    - Outcome $|0\rangle$
      - Probability: $|a_{00}|^2 + |a_{01}|^2 = P(|0\rangle_A)$
      - Collapse state: $\frac{1}{\sqrt{P(|0\rangle_A)}}|0\rangle_A\left(a_{00}|0\rangle_B + a_{01}|1\rangle_B\right)$

- Outcome $|1\rangle$
  - Probability: $|a_{10}|^2 + |a_{11}|^2 = P(|1\rangle_A)$
  - Collapse state: $\frac{1}{\sqrt{P(|1\rangle_A)}} |1\rangle_A (a_{10}|0\rangle_B + a_{11}|1\rangle_B)$.

Example:

$$0|00\rangle + \sqrt{\frac{4}{10}}|01\rangle + \sqrt{\frac{2}{10}}|10\rangle - i\sqrt{\frac{4}{10}}|11\rangle$$

Measure 2nd qubit with $M = \{|0\rangle, |1\rangle\}$

$$\left(\sqrt{\frac{2}{10}}|1\rangle\right)|0\rangle + \left(\sqrt{\frac{4}{10}}|0\rangle - i\sqrt{\frac{4}{10}}|1\rangle\right)|1\rangle$$

Prob of $|0\rangle$: $\left|\sqrt{\frac{2}{10}}\right|^2 = \frac{2}{10} = \frac{1}{5}$

State collapses to

$$\frac{1}{\sqrt{\frac{2}{10}}}\sqrt{\frac{2}{10}}|1\rangle|0\rangle = |1\rangle|0\rangle$$
$$= |10\rangle$$

Exercise for Practice:

Prob:

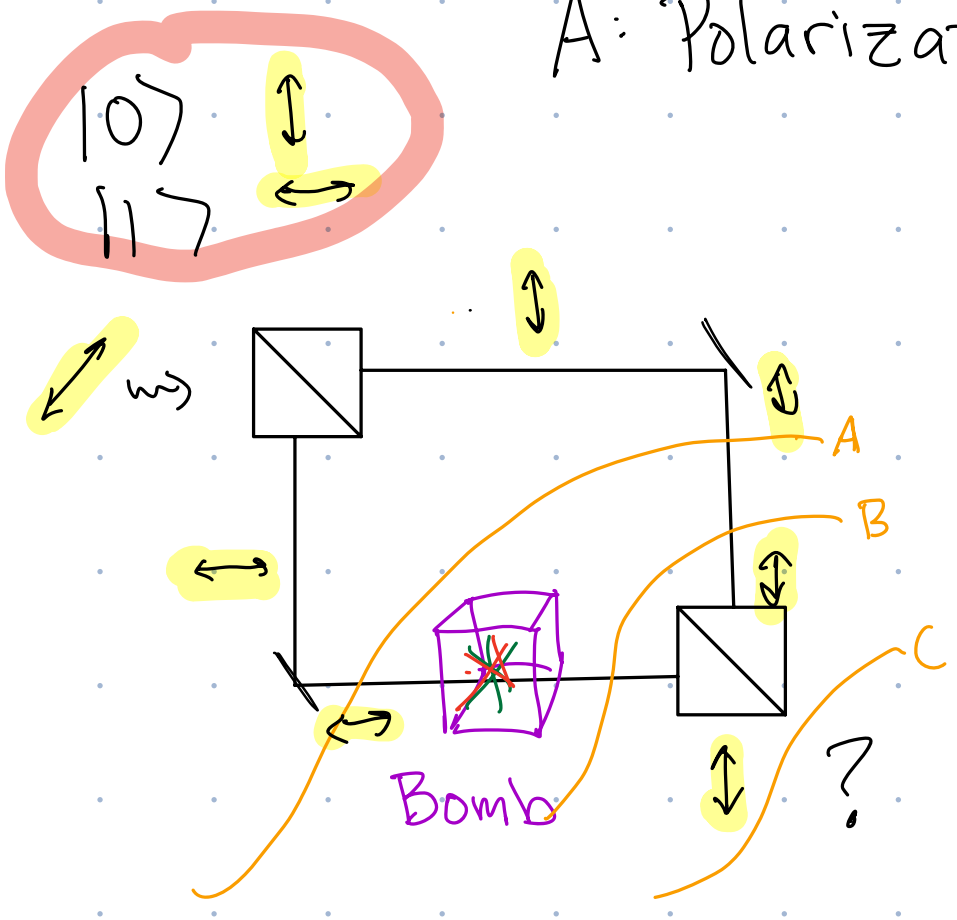
Collapse

Back to the Bomb

Our two qubits are polarization + path (direction) $(|+\rangle_P |H\rangle_D)$

Is the bomb measuring the polarization qubit or path qubit?

A: Polarization (P) B: Path (D)



$$|\psi_A\rangle = \frac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D$$

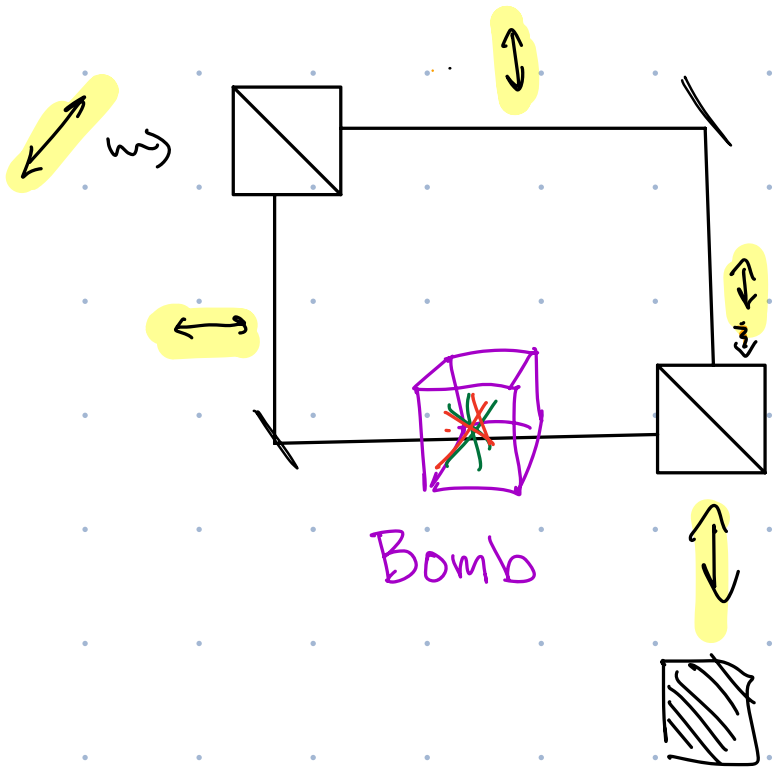
$$\left(\frac{1}{\sqrt{2}}|0\rangle_P\right)|V\rangle_D + \left(\frac{1}{\sqrt{2}}|1\rangle_P\right)|H\rangle_D$$

$$P(\text{explosion}) = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$$

$$P(\text{no explosion}) = \frac{1}{2}$$

Collapse: $\frac{1}{\sqrt{\frac{1}{2}}}\frac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D = |0\rangle_P|V\rangle_D$

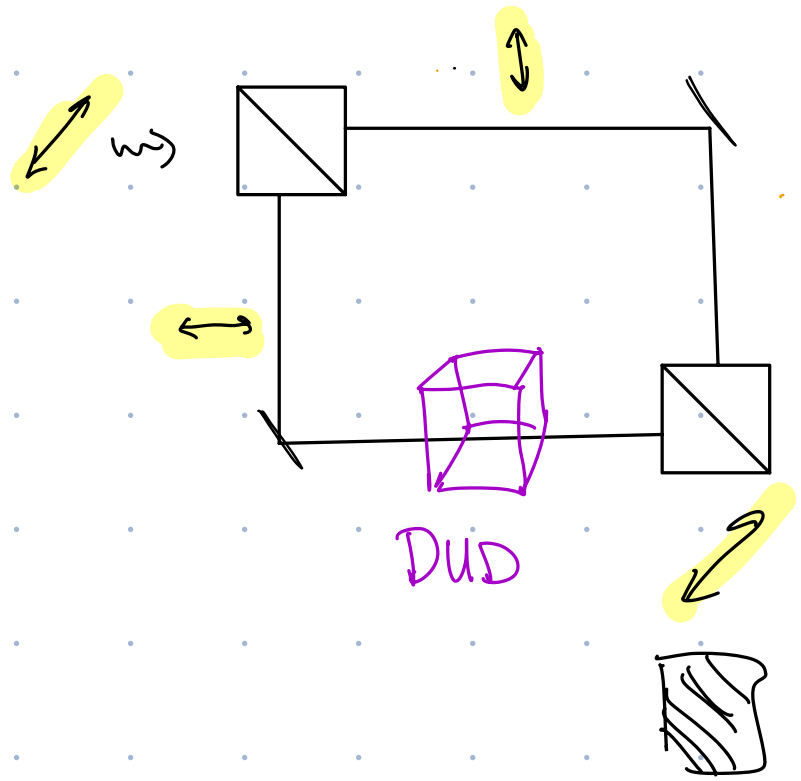
If no explosion



50%
Detection



VS.



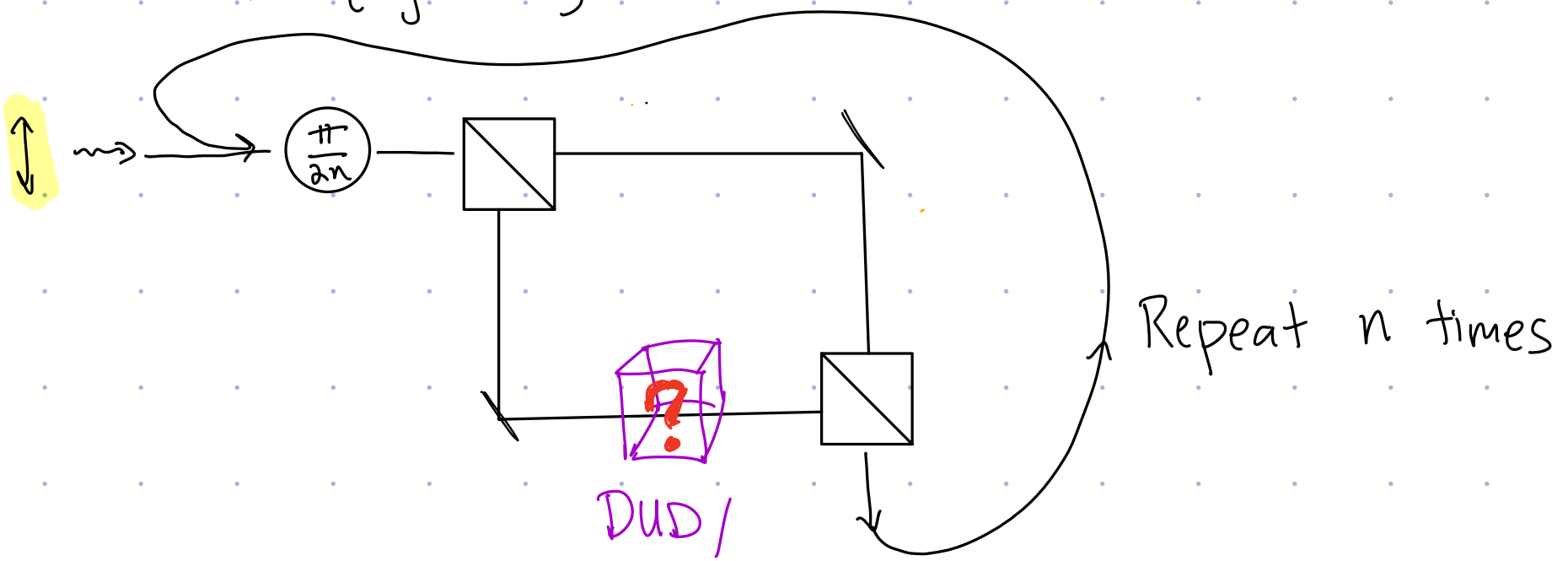


Never get a detection

Can we do better?

Group Work: Analyze the following set up.

Choose $n \gg 1$. (Eg. 1000)



$\frac{\pi}{2n}|0\rangle \rightarrow \cos\left(\frac{\pi}{2n}\right)|0\rangle + \sin\left(\frac{\pi}{2n}\right)|1\rangle$

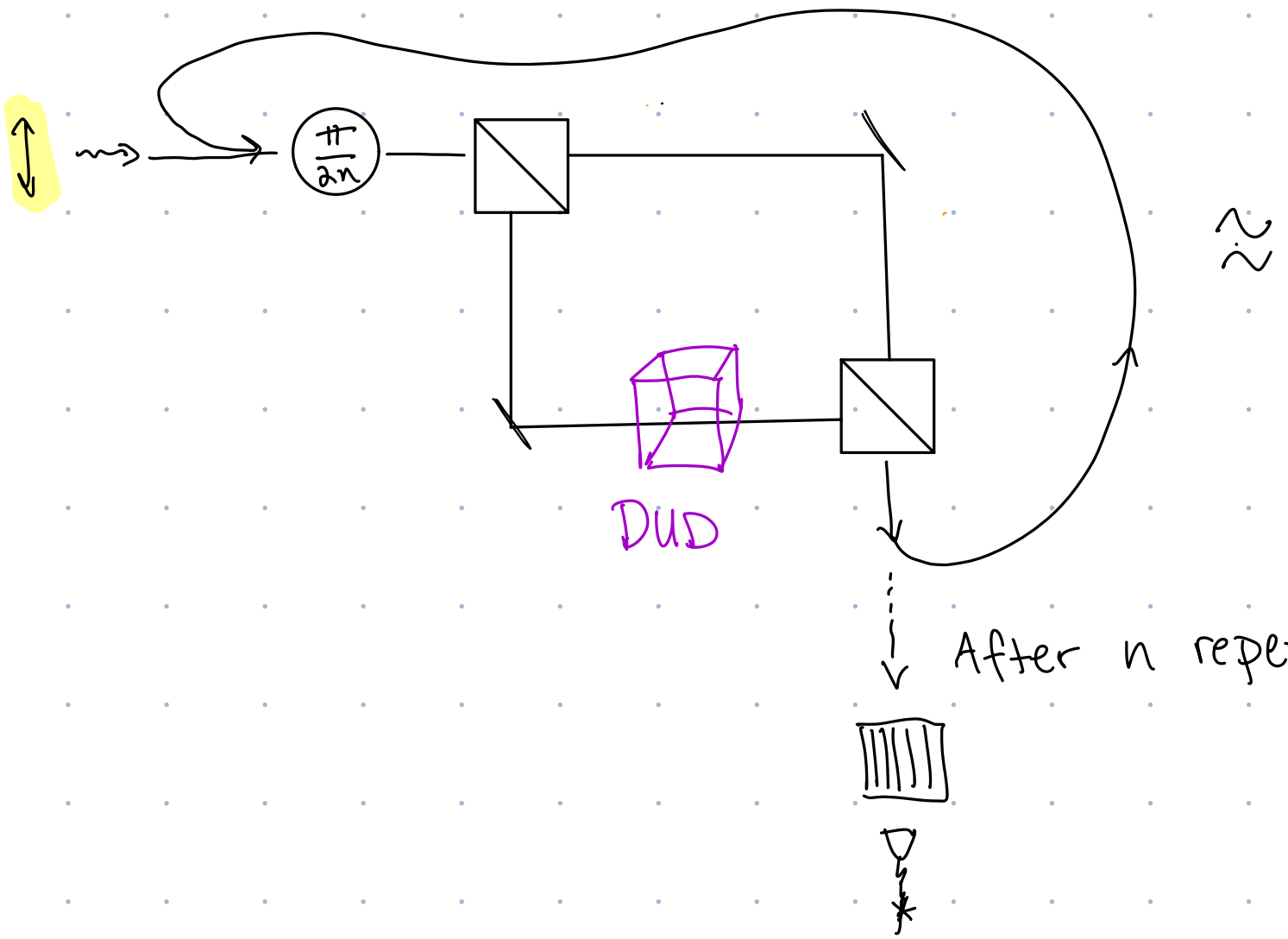
$\frac{\pi}{2n}$ x n times = 90°

For small x

- $\cos(x) \approx 1 - \frac{x^2}{2}$
- $(1-x)^k \approx 1 - kx$

1) What happens if dud?
2) If bomb, probability of NO explosion
3) If bomb but no explosion, probability of detection?

1) If Dud:



≈

After n repetitions

If Bomb:

