BOMB DETECTION

COME TO OH!

Learning Goals

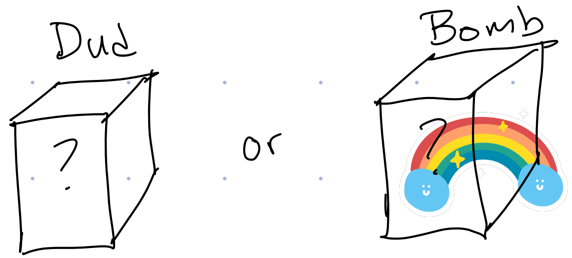
- Describe quantum operations
- Describe partial q. measurements
- Use rules of q. ops + partial meas. to analyze interferometer scenarios

Exit Tickets

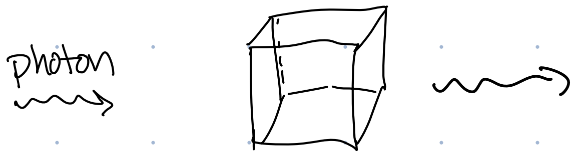
- Have not yet done bomb test!
- "I don't understand the intuition despite being able to solve for outcomes."
- Partial measurement destroys entanglement... what about if both qubits are measured, is entanglement destroyed?

Review: interferometer, partial measurement

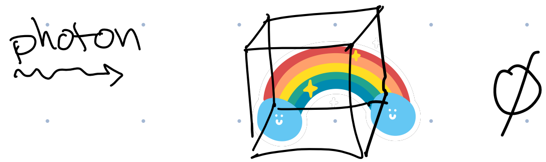
Problem: Quantum Bomb Detection (Elitzur-Vaidman)



Dud:

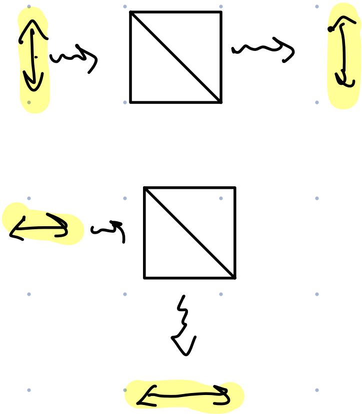


Bomb:

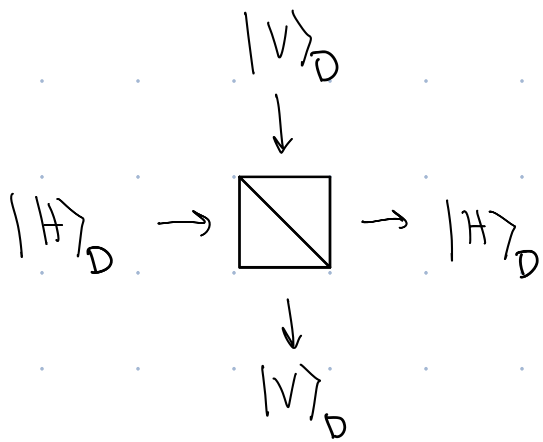


Want to give friend a rainbow bomb not a dud. How??

New Tool: Beamsplitter



- Transmits $|0\rangle_P$ (vertically polarized light)
- Reflects $|1\rangle_P$ (horiz. " ")



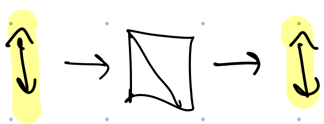
"P" for polarization

"D" for direction

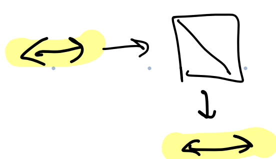
$\{|V\rangle_D, |H\rangle_D\}$ standard basis states

Behavior:

$|0\rangle_P|H\rangle_D \to |0\rangle_P|H\rangle_D$

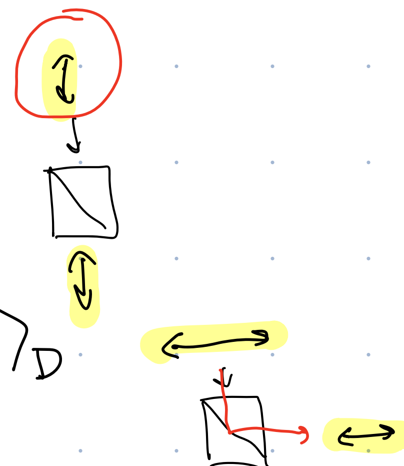


$|1\rangle_P|H\rangle_D \to |1\rangle_P|V\rangle_D$



$|0\rangle_P|V\rangle_D \to |0\rangle_P|V\rangle_D$

$|1\rangle_P|V\rangle_D \to |1\rangle_P|H\rangle_D$

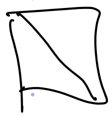


A) $|0\rangle_P|H\rangle_D$ B) $|0\rangle_P|V\rangle_D$ C) $|1\rangle_P|H\rangle_D$ D) $|1\rangle_P|V\rangle_D$

Quantum Gate

Operation that changes q. state without collapse (no info extracted)

Can describe gate via action on standard basis states



Beam-Splitter

$$\boxed{\begin{aligned} |0\rangle_P|H\rangle_D &\to |0\rangle_P|H\rangle_D \\ |1\rangle_P|H\rangle_D &\to |1\rangle_P|V\rangle_D \\ |0\rangle_P|V\rangle_D &\to |0\rangle_P|V\rangle_D \\ |1\rangle_P|V\rangle_D &\to |1\rangle_P|H\rangle_D \end{aligned}}$$

Mirror 

$$\boxed{\begin{aligned} |H\rangle_D &\to |V\rangle_D \\ |V\rangle_D &\to |H\rangle_D \end{aligned}}$$

Wave plate

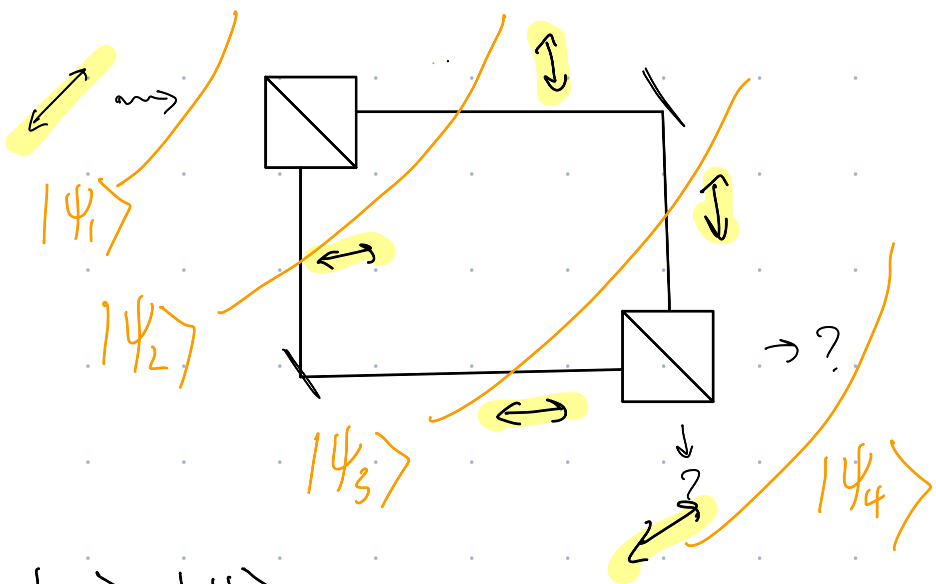
θ

Only affects polarization

$45°$ $$\boxed{\begin{aligned} |0\rangle_P &\to |+\rangle_D \\ |1\rangle_P &\to |-\rangle_P \end{aligned}}$$

$90°$ $$\boxed{\begin{aligned} |0\rangle_P &\to |1\rangle_P \\ |1\rangle_P &\to |0\rangle_P \end{aligned}}$$

Interferometer



$\boxed{\diagdown}$

$$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$$
$$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$$
$$|0\rangle_P|V\rangle_D \rightarrow |0\rangle_P|V\rangle_D$$
$$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$$

$\diagdown$

$$|H\rangle_D \rightarrow |V\rangle_D$$
$$|V\rangle_D \rightarrow |H\rangle_D$$

$$|\psi_1\rangle = |+\rangle_P|H\rangle_D$$

$$|\psi_1\rangle = \left(\frac{1}{\sqrt{2}}|0\rangle_P + \frac{1}{\sqrt{2}}|1\rangle_P\right)|H\rangle_D = \frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D$$

$$|\psi_2\rangle = \boxed{\diagdown}\left(\frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D\right) = \frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D$$

$$|\psi_3\rangle = \diagdown\left(\frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D\right) = \frac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D$$

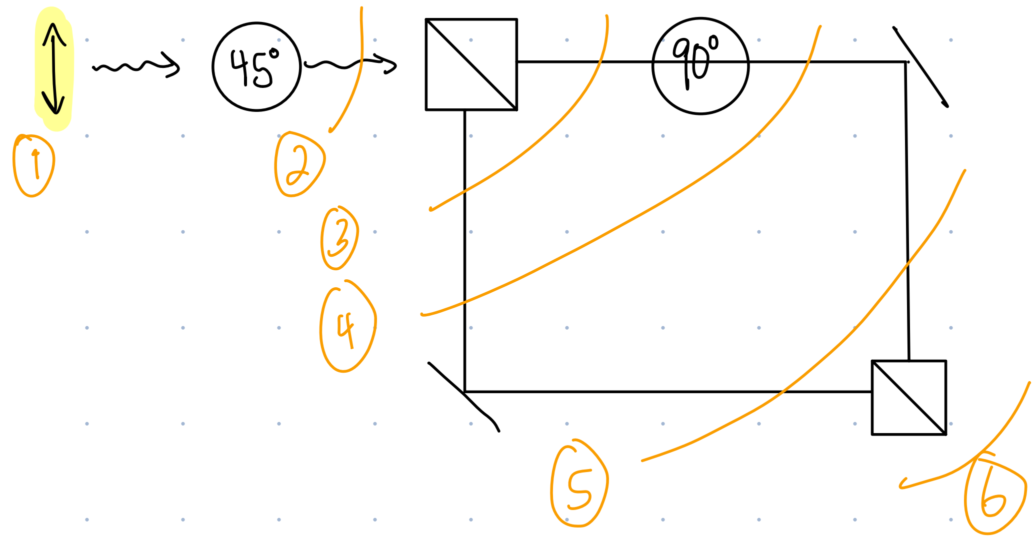
$$|\psi_4\rangle = \boxed{\diagdown}\left(\frac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D\right) = \frac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D$$

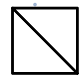
$$= \left(\frac{1}{\sqrt{2}}|0\rangle_P + \frac{1}{\sqrt{2}}|1\rangle_P\right)|V\rangle_D = |+\rangle_P|V\rangle_D$$

Group Practice

Family weekend fun? Group Style?

QI4





$$|0\rangle_P |H\rangle_D \rightarrow |0\rangle_P |H\rangle_D$$
$$|1\rangle_P |H\rangle_D \rightarrow |1\rangle_P |V\rangle_D$$
$$|0\rangle_P |V\rangle_D \rightarrow |0\rangle_P |V\rangle_D$$
$$|1\rangle_P |V\rangle_D \rightarrow |1\rangle_P |H\rangle_D$$

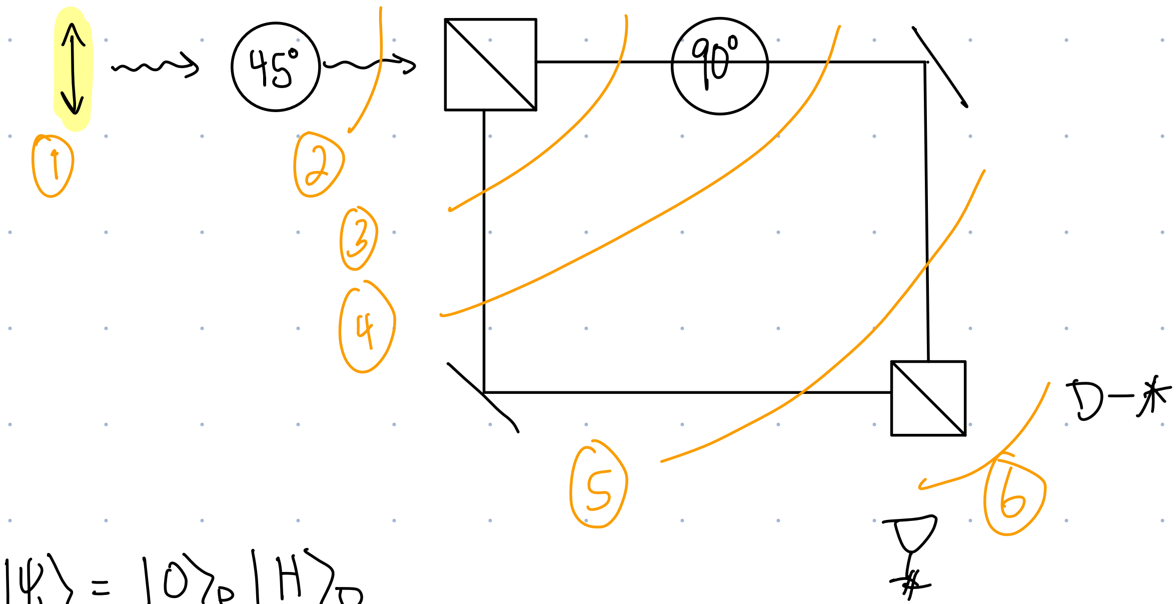
$$|H\rangle_D \rightarrow |V\rangle_D$$
$$|V\rangle_D \rightarrow |H\rangle_D$$

45°

$$|0\rangle_P \rightarrow |+\rangle_P$$
$$|1\rangle_P \rightarrow |-\rangle_P$$

$$|0\rangle_P \rightarrow |1\rangle_P$$
$$|1\rangle_P \rightarrow |0\rangle_P$$

Group Practice



$|0\rangle_P |H\rangle_D \rightarrow |0\rangle_P |H\rangle_D$

$|1\rangle_P |H\rangle_D \rightarrow |1\rangle_P |V\rangle_D$

$|0\rangle_P |V\rangle_D \rightarrow |0\rangle_P |V\rangle_D$

$|1\rangle_P |V\rangle_D \rightarrow |1\rangle_P |H\rangle_D$

$|H\rangle_D \rightarrow |V\rangle_D$

$|V\rangle_D \rightarrow |H\rangle_D$

45°

$|0\rangle_P \rightarrow |+\rangle_P$

$|1\rangle_P \rightarrow |-\rangle_P$

90°

$|0\rangle_P \rightarrow |1\rangle_P$

$|1\rangle_P \rightarrow |0\rangle_P$

$$|\psi_1\rangle = |0\rangle_P |H\rangle_D$$

$$|\psi_2\rangle = |+\rangle_P |H\rangle_D = \frac{1}{\sqrt{2}}|0\rangle_P |H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P |H\rangle_D$$

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}|0\rangle_P |H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P |V\rangle_D$$

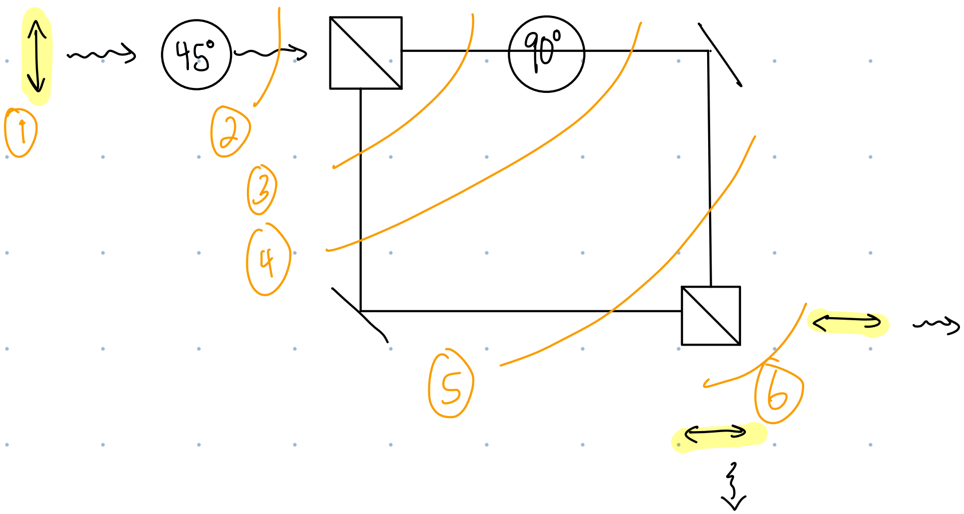
$$|\psi_4\rangle = \frac{1}{\sqrt{2}}|1\rangle_P |H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P |V\rangle_D$$

$$|\psi_5\rangle = \frac{1}{\sqrt{2}}|1\rangle_P |V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P |H\rangle_D$$

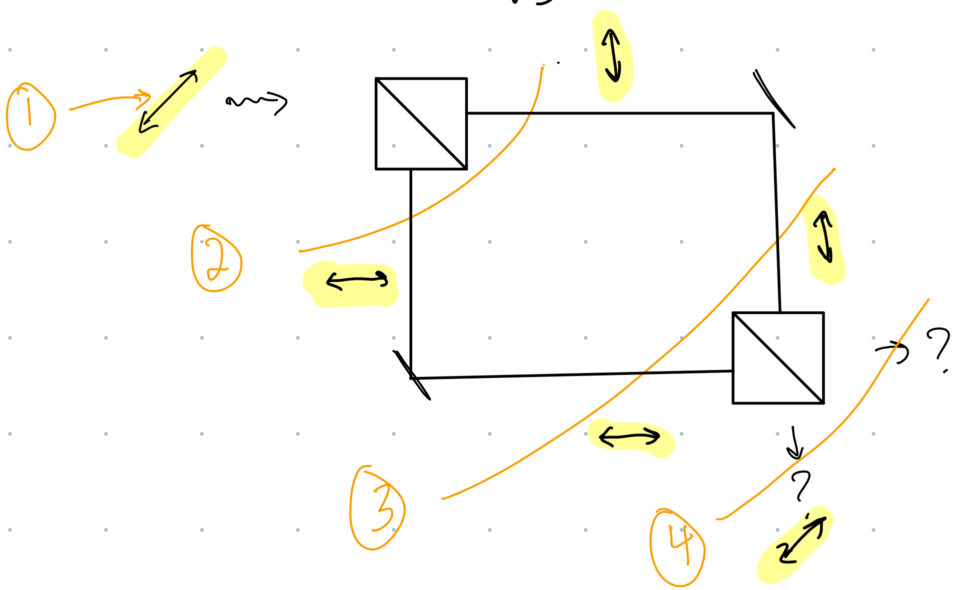
$$|\psi_6\rangle = \frac{1}{\sqrt{2}}|1\rangle_P |H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P |V\rangle_D = |1\rangle_P \left(\frac{1}{\sqrt{2}}|H\rangle_D + \frac{1}{\sqrt{2}}|V\rangle_D\right)$$

$|\psi_6\rangle =$

$=$



VS



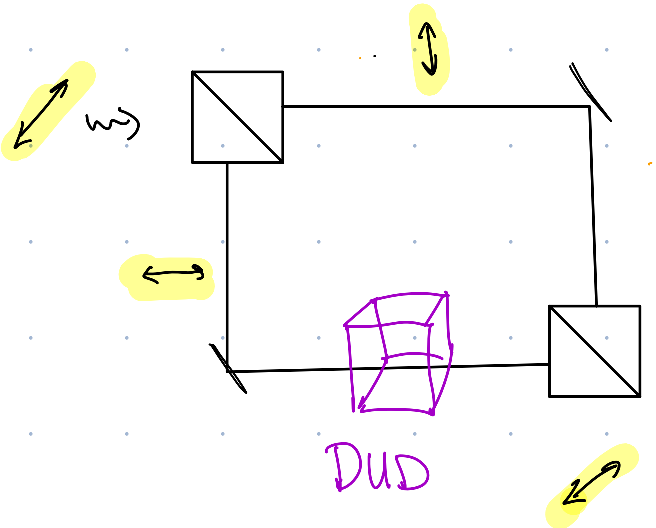
Big ideas:

- One photon takes 2 paths
- Direction & polarization are entangled
- Objects in arm affects output
-

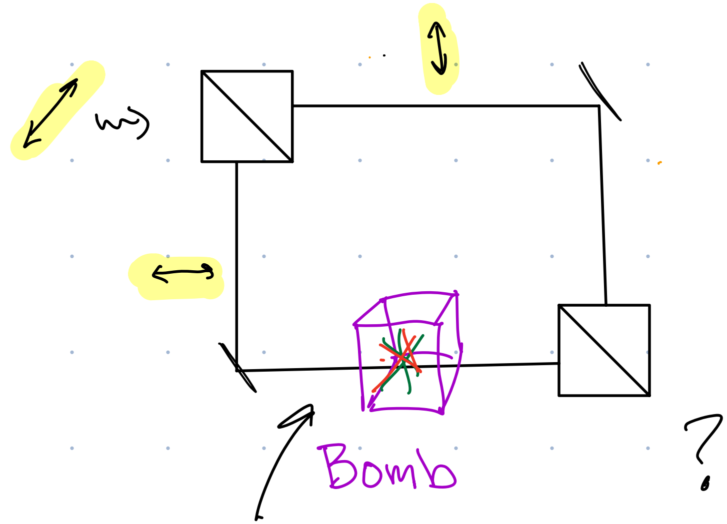
$\Downarrow$

Put potential bomb in arm

Bomb Test



vs



Same as empty interferometer

Partial Measurement

Partial Q. Measurement (in standard basis)

- State to be measured: $|\psi\rangle_{AB}$ Needs 2 parts! (A & B)
- Want to measure qubit A in $M_A = \{|0\rangle, |1\rangle\}$ No measurement of B. • Asking if A is in state $|0\rangle$ or $|1\rangle$ • Not asking B anything
- To determine outcome:
  - Write $|\psi\rangle_{AB}$ in standard basis:

$$|\psi\rangle_{AB} = \boxed{a_{00}|00\rangle_{AB} + a_{01}|01\rangle_{AB}} + a_{10}|10\rangle_{AB} + a_{11}|11\rangle_{AB}$$

  - Rewrite: Factor/group A kets: (numbers/scalars commute!)

$$|\psi\rangle_{AB} = |0\rangle_A (a_{00}|0\rangle_B + a_{01}|1\rangle_B) + |1\rangle (a_{10}|0\rangle_B + a_{11}|1\rangle_B)$$

  - Calculate outcome probabilities / collapse state
    - Outcome $|0\rangle$
      - Probability: $|a_{00}|^2 + |a_{01}|^2 = p(|0\rangle_A)$
      - Collapse state: $\frac{1}{\sqrt{p(|0\rangle_A)}} \boxed{|0\rangle(a_{00}|0\rangle + a_{01}|1\rangle)} =$

$$\frac{a_{00}}{\sqrt{p_0}}|00\rangle + \frac{a_{01}}{\sqrt{p_0}}|01\rangle$$

- Outcome $|1\rangle$
  - Probability: $|a_{10}|^2 + |a_{11}|^2 = p(|1\rangle_A)$
  - Collapse state: $\frac{1}{\sqrt{p(|1\rangle_A)}} |1\rangle \left(a_{10}|0\rangle + a_{11}|1\rangle\right) = \frac{a_{10}}{\sqrt{p(|1\rangle_A)}}|10\rangle + \frac{a_{11}}{\sqrt{p(|1\rangle_A)}}|11\rangle$

* Partial measurement destroys entanglement

$$= \frac{a_{10}}{\sqrt{p(|1\rangle_A)}}|10\rangle + \frac{a_{11}}{\sqrt{p(|1\rangle_A)}}|11\rangle$$

$$= |1\rangle \left(\frac{a_{10}}{\sqrt{p_1}}|0\rangle + \frac{a_{11}}{\sqrt{p_1}}|1\rangle \right)$$

$$\frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right) = |+\rangle|+\rangle$$

Example:

$$\frac{1}{\sqrt{10}}|00\rangle + \sqrt{\frac{4}{10}}|01\rangle + \frac{1}{\sqrt{10}}|10\rangle - i\sqrt{\frac{4}{10}}|11\rangle$$

$=$

Measure 2^{nd} qubit with $M = \{|0\rangle, |1\rangle\}$

$$\left(\frac{1}{\sqrt{10}}|0\rangle + \frac{1}{\sqrt{10}}|1\rangle\right)|0\rangle + \left(\sqrt{\frac{4}{10}}|0\rangle - i\sqrt{\frac{4}{10}}|1\rangle\right)|1\rangle$$

Prob of $|0\rangle$: $\left|\frac{1}{\sqrt{10}}\right|^2 + \left|\frac{1}{\sqrt{10}}\right|^2 = \frac{1}{10} + \frac{1}{10} = \frac{1}{5}$

State collapses to

$1/\sqrt{2}$ $1/\sqrt{2}$

$$\frac{1}{\sqrt{\frac{1}{5}}}\left(\frac{1}{\sqrt{10}}|0\rangle + \frac{1}{\sqrt{10}}|1\rangle\right)|0\rangle = \left(\frac{1}{\sqrt{\frac{10}{5}}}|0\rangle + \frac{1}{\sqrt{\frac{10}{5}}}|1\rangle\right)|0\rangle$$

Extra practice: Prob $(|1\rangle)$, collapses

$\frac{4}{5}$ $|\circlearrowleft\rangle|1\rangle$ $\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle\right)|1\rangle$

$$A\left(\frac{1}{\sqrt{10}}|0\rangle+\frac{1}{\sqrt{10}}|1\rangle\right)|0\rangle$$

$$\frac{A}{\sqrt{10}}|00\rangle+\frac{A}{\sqrt{10}}|10\rangle$$

$$\left|\frac{A}{\sqrt{10}}\right|^2+\left|\frac{A}{\sqrt{10}}\right|^2=1$$

$$\frac{2A^2}{10}=1 \Rightarrow A^2=5$$

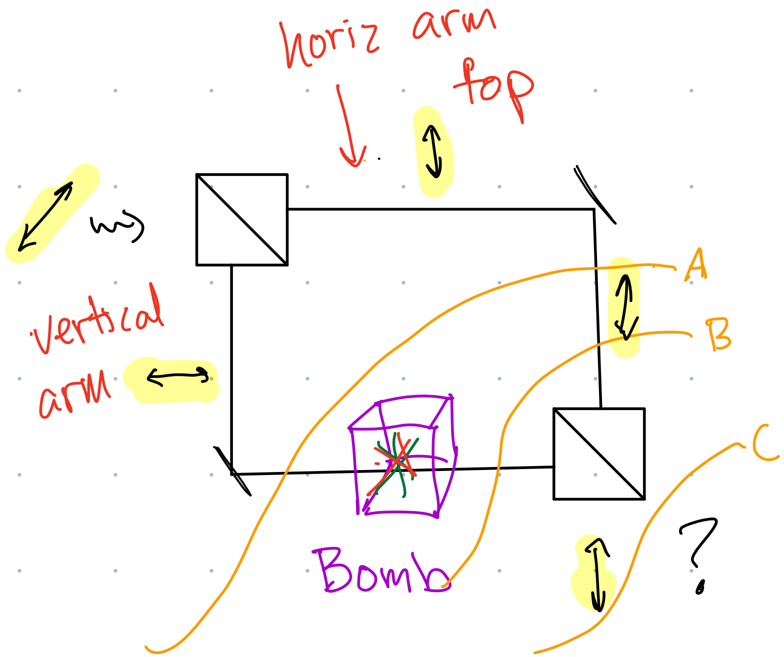
$$A=\sqrt{5}$$

Back to the Bomb

Our two qubits are polarization + path (direction)

Is the bomb measuring the polarization qubit or path qubit?

1/ A: Polarization   2/ B: Path (direction)



Prob Explosion? Output if no explosion?

$$|\psi_A\rangle = \frac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D \cancel{+0|0\rangle_P|H\rangle_D + 0|1\rangle_P|V\rangle_D}$$

$$M = \{|H\rangle_D, |V\rangle_D\}$$

Outcome $|H\rangle$

Prob: $\left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$

Collapse $\sqrt{2}\left(\frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D\right)$

$= |1\rangle_P|H\rangle_D$

Outcome $|V\rangle$

Prob: $\left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$

Collapse to $\frac{1}{\sqrt{\frac{1}{2}}}\frac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D$

$= |0\rangle_P|V\rangle_D$

After beamsplitter: $|0\rangle_P|V\rangle_D$

Beam Splitter Demo!

Phases

$\underbrace{\quad}$

$1, -1, i, -i$

$e^{i\omega}$ $\omega \in \mathbb{R}$

$\boxed{\text{Global Phase}}$ vs. Relative Phase / Local Phase?

$$-i\left(\frac{i}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)$$

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle = |\circlearrowleft\rangle$$

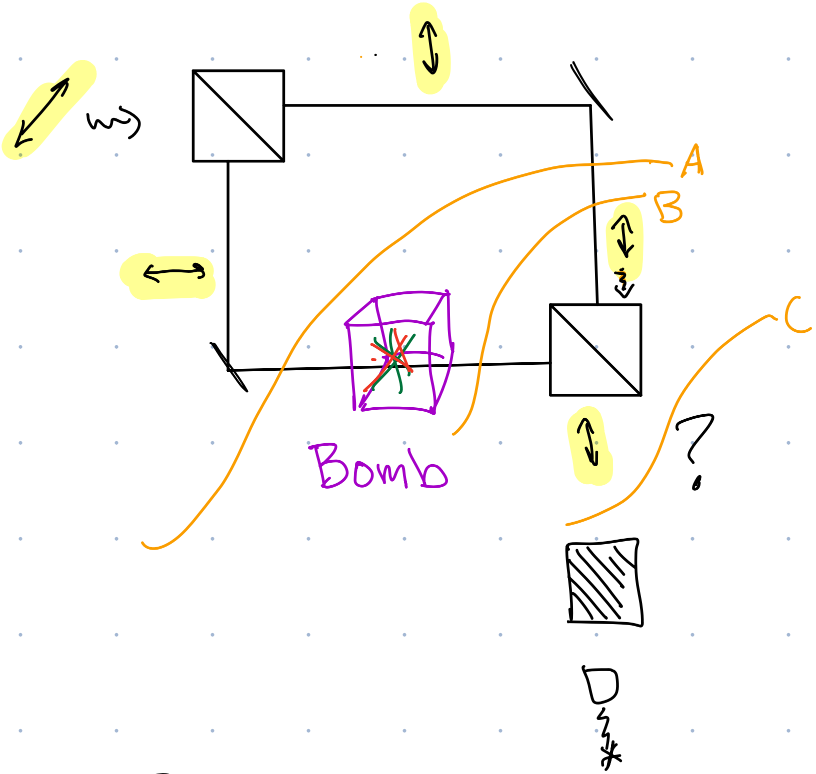
$$-|1\rangle = |1\rangle$$

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \neq \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

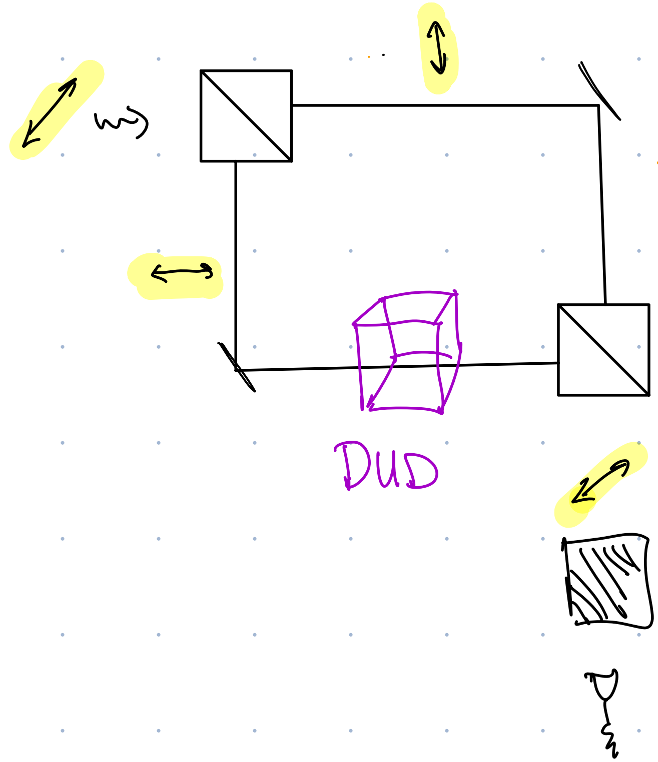
$$\| $$

$$e^{i\pi/8}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) = \frac{e^{i\pi/8}}{\sqrt{2}}|0\rangle + \frac{e^{i\pi/8}}{\sqrt{2}}|1\rangle$$

If no explosion



vs.



Explode: 1/2

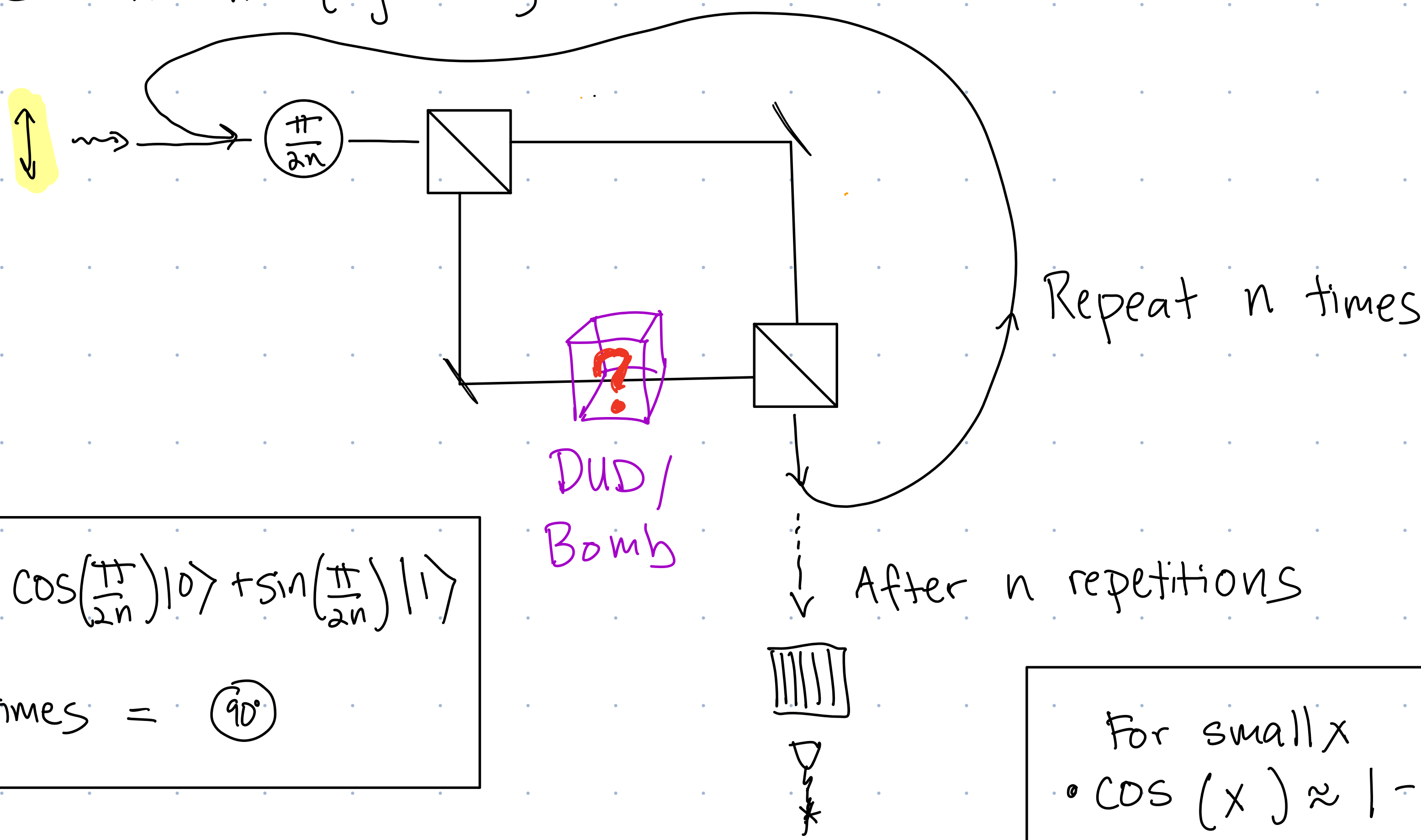
No explosion 1/2 → Block by filter 1/2

→ Not blocked 1/2

Can we do better?

Group Work: Analyze the following set up.

Choose $n \gg 1$. (Eg. 1000)



$\frac{\pi}{2n}\ |0\rangle \rightarrow \cos\left(\frac{\pi}{2n}\right)|0\rangle + \sin\left(\frac{\pi}{2n}\right)|1\rangle$

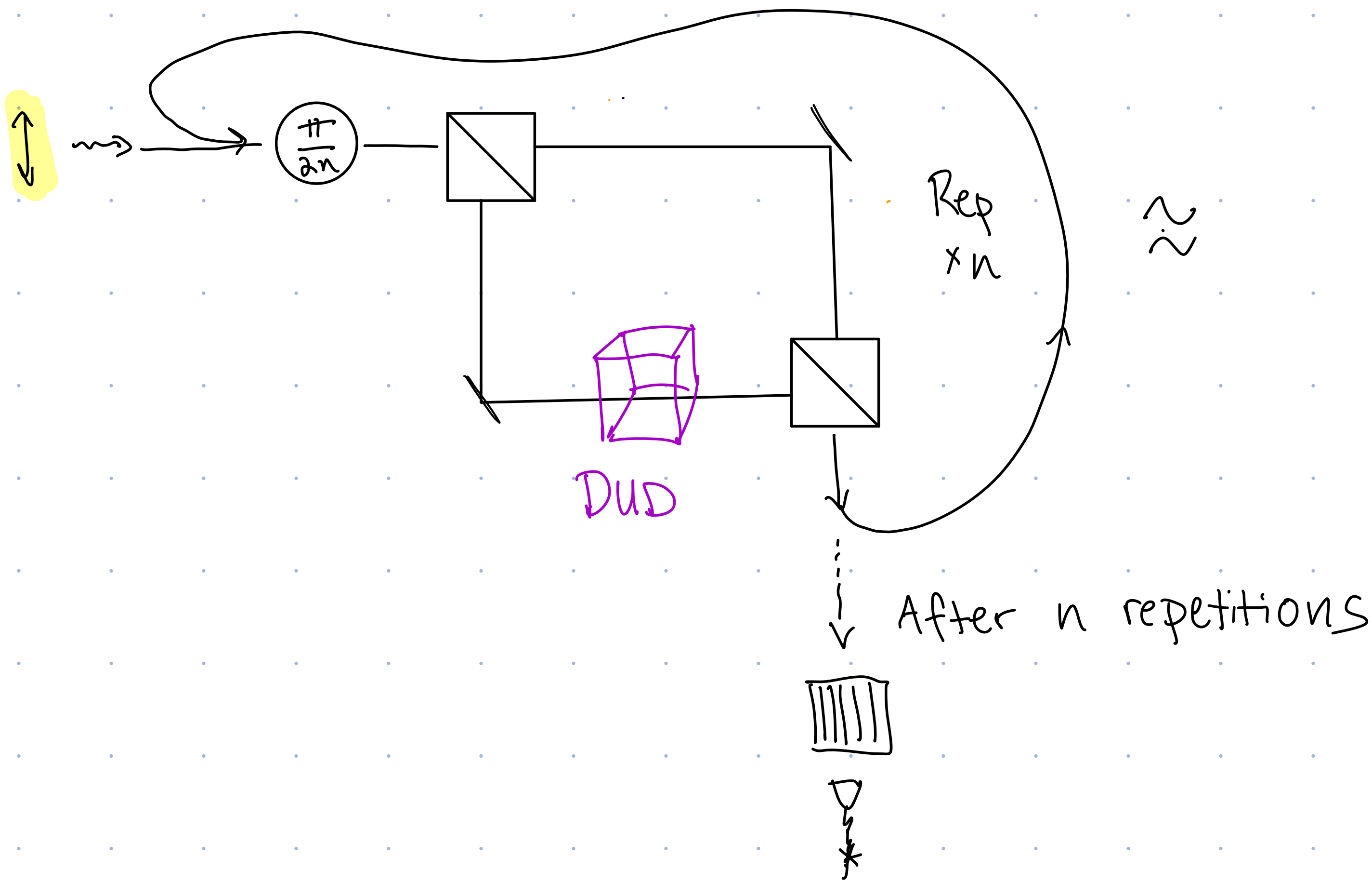
$\frac{\pi}{2n}$ x n times = $90°$

For small x

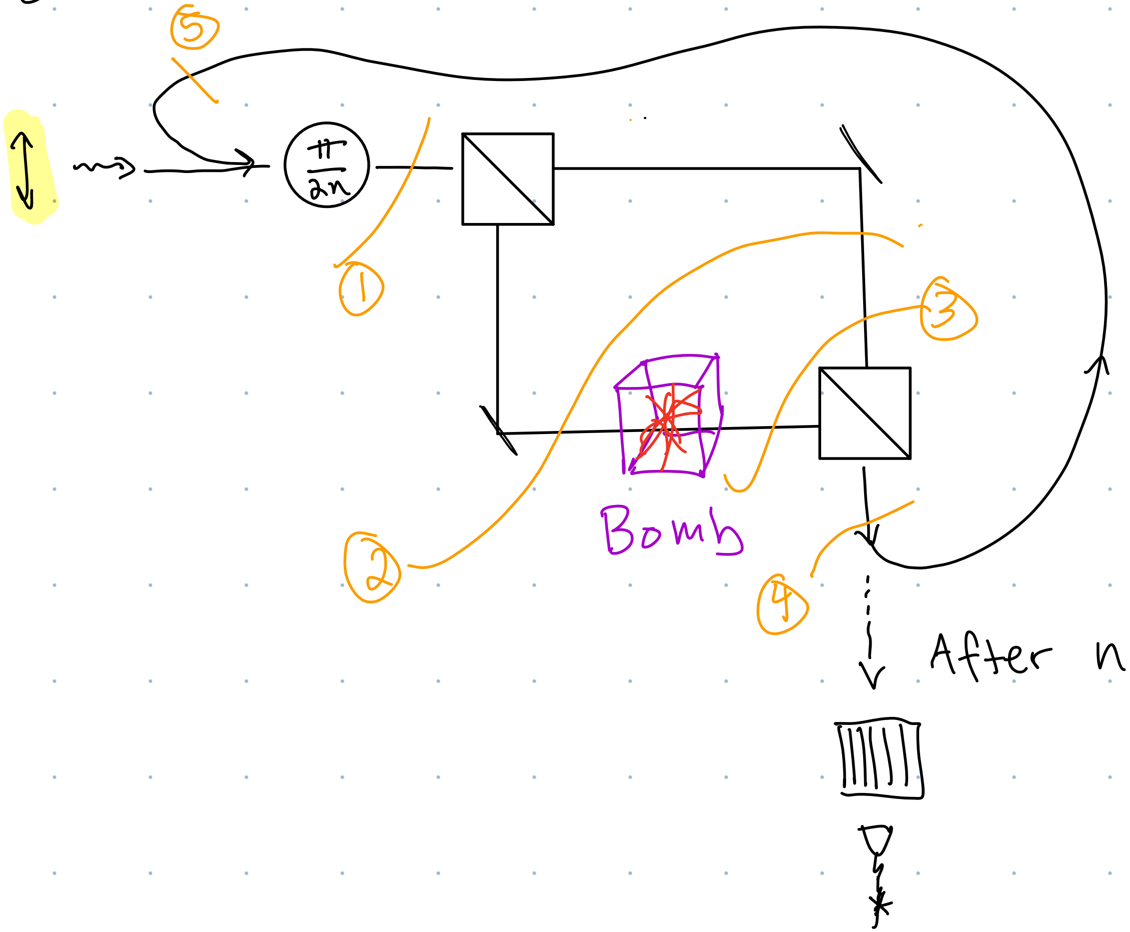
- $\cos(x) \approx 1 - \frac{x^2}{2}$
- $(1-x)^k \approx 1 - kx$

1) What happens if dud?
2) If bomb, probability of NO explosion
3) If bomb but no explosion, probability of detection?

1) If Dud:



If Bomb:



Repeat n times

After n repetitions

$|\psi_1\rangle =$

$|\psi_2\rangle =$