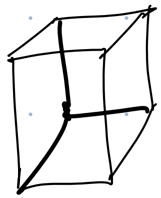
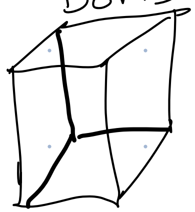
Problem: Quantum Bomb Detection (Elitzur-Vaidman)

Dud



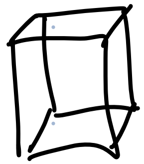
or

Bomb



Dud: photon

Dud



(passes through

Bomb : photon

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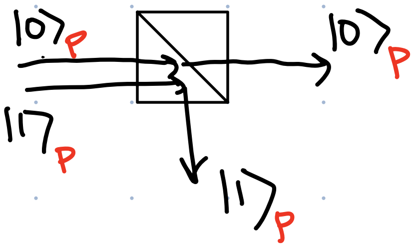
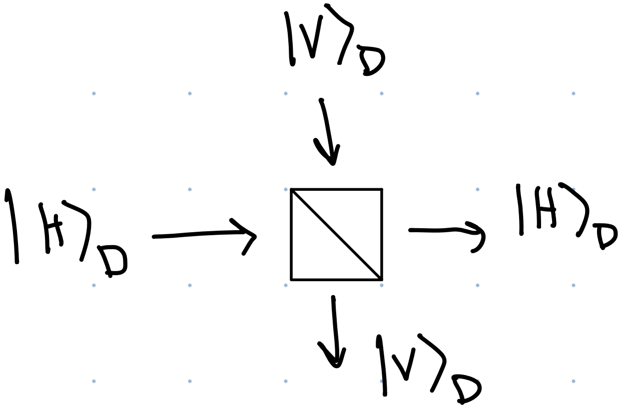
Want to give friend a rainbow bomb not a dud. How??

New Tool: Beamsplitter



- Transmist if photon $|0\rangle_P$
- Reflects if $|1\rangle_P$



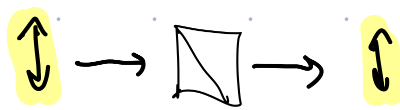
"P" for polarization

"D" for direction

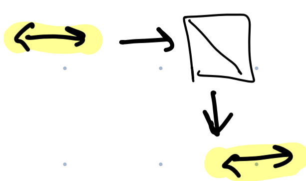
$\{|V\rangle_D, |H\rangle_D\}$ standard basis states

Behavior:

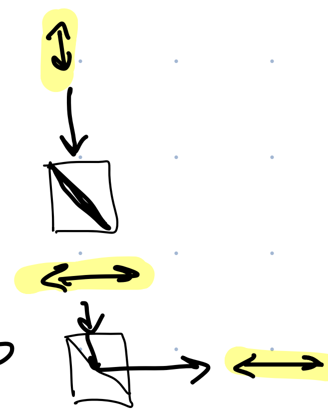
$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$



$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$



$|0\rangle_P|V\rangle_D \rightarrow \boxed{|0\rangle_P|V\rangle_D}$



$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$

A) $|0\rangle_P|H\rangle_D$ B) $|0\rangle_P|V\rangle_D$ C) $|1\rangle_P|H\rangle_D$ D) $|1\rangle_P|V\rangle_D$

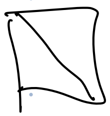
Quantum Gate

Operation that changes a quantum state without collapse.

We will describe gates using their effect on s.b. states

$|0\rangle, |1\rangle$

$|00\rangle, |01\rangle \ldots$



Beam-Splitter

$|0\rangle_P |H\rangle_D \to |0\rangle_P |H\rangle_D$

$|1\rangle_P |H\rangle_D \to |1\rangle_P |V\rangle_D$

$|0\rangle_P |V\rangle_D \to |0\rangle_P |V\rangle_D$

$|1\rangle_P |V\rangle_D \to |1\rangle_P |H\rangle_D$

Mirror

$|H\rangle_D \to |V\rangle_D$

$|V\rangle_D \to |H\rangle_D$

$\updownarrow \Rightarrow \leftrightarrow$

Wave plate

θ

Only affects polarization

45°

$|0\rangle_P \to |+\rangle_P$

$|1\rangle_P \to |-\rangle_P$

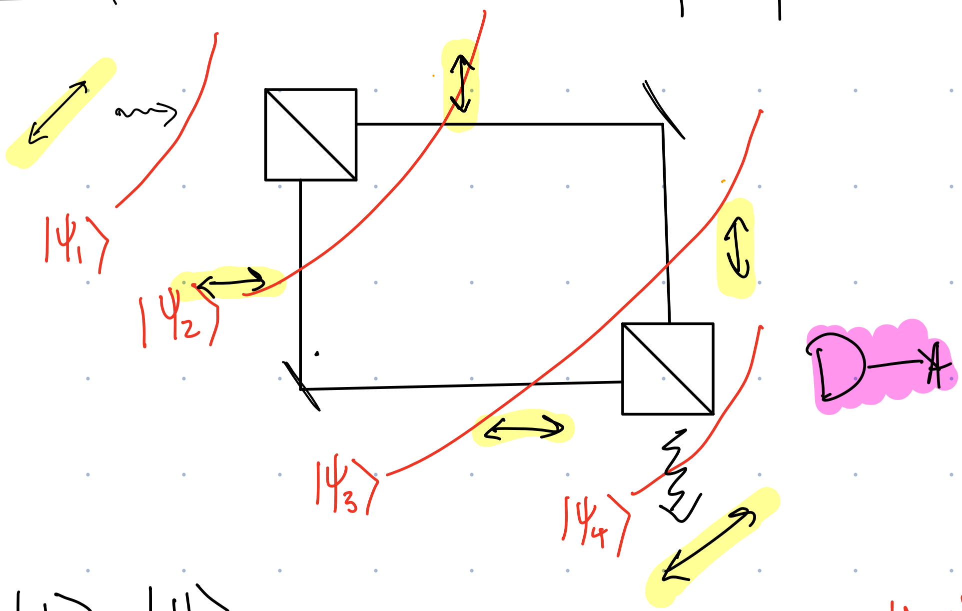
90°

$|0\rangle_P \to |1\rangle_P$

$|1\rangle_P \to |0\rangle_P$

Interferometer

"Superposition"



$\boxbslash$

$$
\begin{aligned}
|0\rangle_P|H\rangle_D &\to |0\rangle_P|H\rangle_D \\
|1\rangle_P|H\rangle_D &\to |1\rangle_P|V\rangle_D \\
|0\rangle_P|V\rangle_D &\to |0\rangle_P|V\rangle_D \\
|1\rangle_P|V\rangle_D &\to |1\rangle_P|H\rangle_D
\end{aligned}
$$

$\diagdown$

$$
\begin{aligned}
|H\rangle_D &\to |V\rangle_D \\
|V\rangle_D &\to |H\rangle_D
\end{aligned}
$$

$$|\psi_1\rangle = |+\rangle_P|H\rangle_D$$

Writing in standard basis

$$|\psi_2\rangle = \boxbslash\,|\psi_1\rangle = \boxbslash\,|+\rangle_P|H\rangle_D = \boxbslash\left(\tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{1}{\sqrt{2}}|1\rangle\right)|H\rangle_D =$$

$$= \boxbslash\left(\tfrac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \tfrac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D\right)$$ Distributing

$$= \tfrac{1}{\sqrt{2}}\,\boxbslash\,|0\rangle_P|H\rangle_D + \tfrac{1}{\sqrt{2}}\,\boxbslash\,|1\rangle_P|H\rangle_D$$ " commuting w/ constants

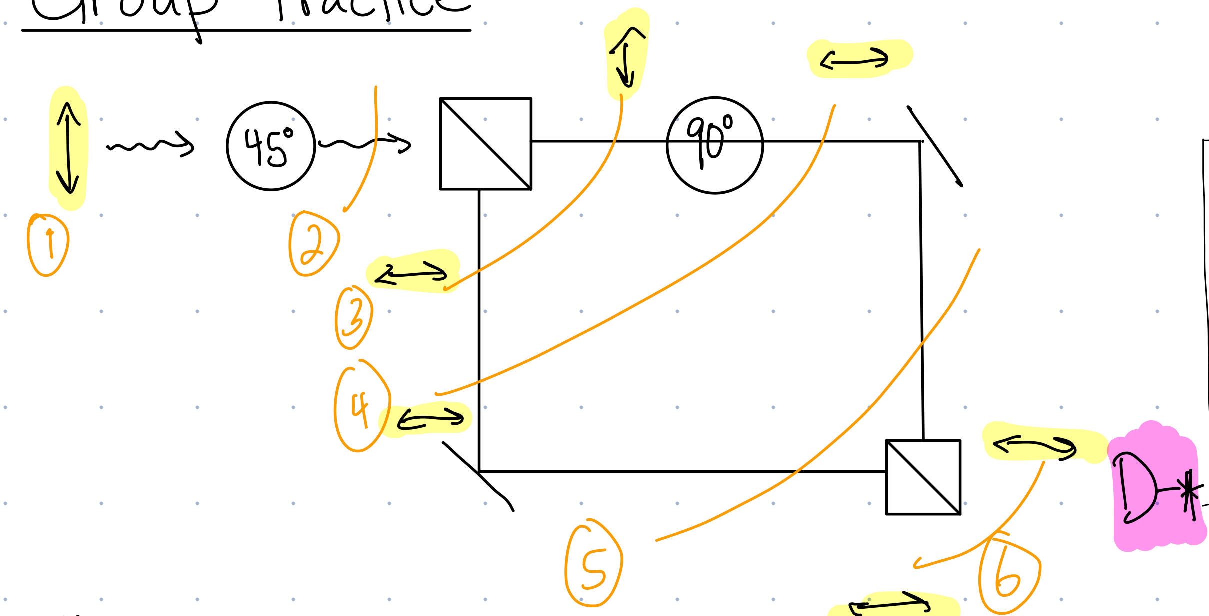
$$= \tfrac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \tfrac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D$$

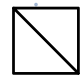
$$|\psi_3\rangle = \diagdown\left(\tfrac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \tfrac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D\right) = \tfrac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D + \tfrac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D$$

$$|\psi_4\rangle = \boxbslash\left(\tfrac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D + \tfrac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D\right) = \tfrac{1}{\sqrt{2}}\,\boxbslash\,|0\rangle_P|V\rangle_D + \tfrac{1}{\sqrt{2}}\,\boxbslash\,|1\rangle_P|H\rangle_D$$

$$= \tfrac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D + \tfrac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D = \left(\tfrac{1}{\sqrt{2}}|0\rangle_P + \tfrac{1}{\sqrt{2}}|1\rangle_P\right)|V\rangle_D = |+\rangle|V\rangle$$

Group Practice





$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$

$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$

$|0\rangle_P|V\rangle_D \rightarrow |0\rangle_P|V\rangle_D$

$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$

$|H\rangle_D \rightarrow |V\rangle_D$

$|V\rangle_D \rightarrow |H\rangle_D$

45°

$|0\rangle_P \rightarrow |+\rangle_P$

$|1\rangle_P \rightarrow |-\rangle_P$

90°

$|0\rangle_P \rightarrow |1\rangle_P$

$|1\rangle_P \rightarrow |0\rangle_P$

$$|\psi_1\rangle = |0\rangle_P|H\rangle_D$$

$$|\psi_2\rangle = |+\rangle_P|H\rangle_D$$

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D$$

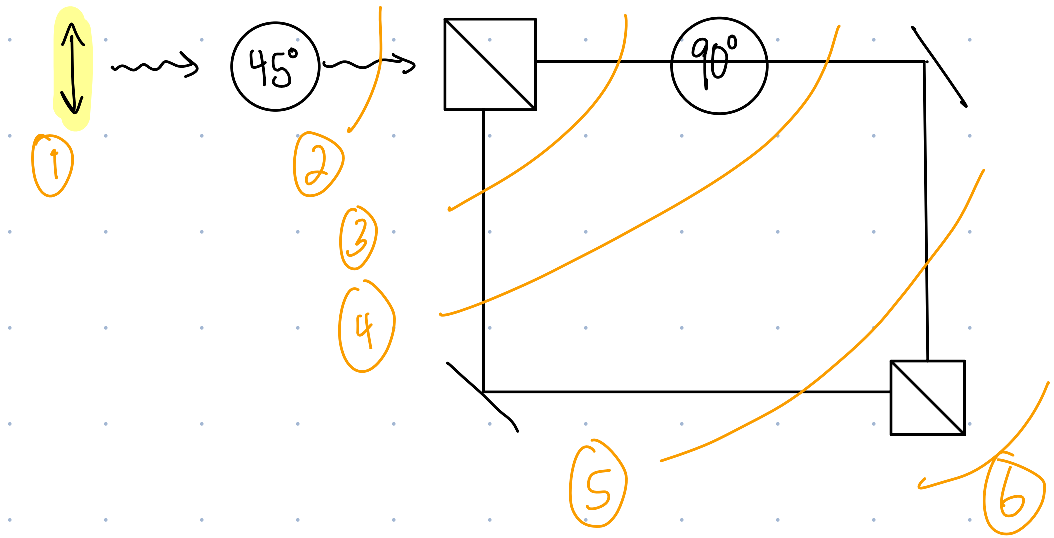
$$|\psi_4\rangle = \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D$$

$$|\psi_5\rangle = \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D$$

$$|6\rangle = \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D$$

$$= \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D = |1\rangle_P\left(\frac{1}{\sqrt{2}}|H\rangle_D + \frac{1}{\sqrt{2}}|V\rangle_D\right)$$

Group Practice



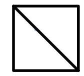
$|\psi_1\rangle =$

$|\psi_2\rangle =$

$|\psi_3\rangle =$

$|\psi_4\rangle =$

$|\psi_5\rangle =$



$$|0\rangle_P |H\rangle_D \rightarrow |0\rangle_P |H\rangle_D$$
$$|1\rangle_P |H\rangle_D \rightarrow |1\rangle_P |V\rangle_D$$
$$|0\rangle_P |V\rangle_D \rightarrow |0\rangle_P |V\rangle_D$$
$$|1\rangle_P |V\rangle_D \rightarrow |1\rangle_P |H\rangle_D$$

$$|H\rangle_D \rightarrow |V\rangle_D$$
$$|V\rangle_D \rightarrow |H\rangle_D$$

45°

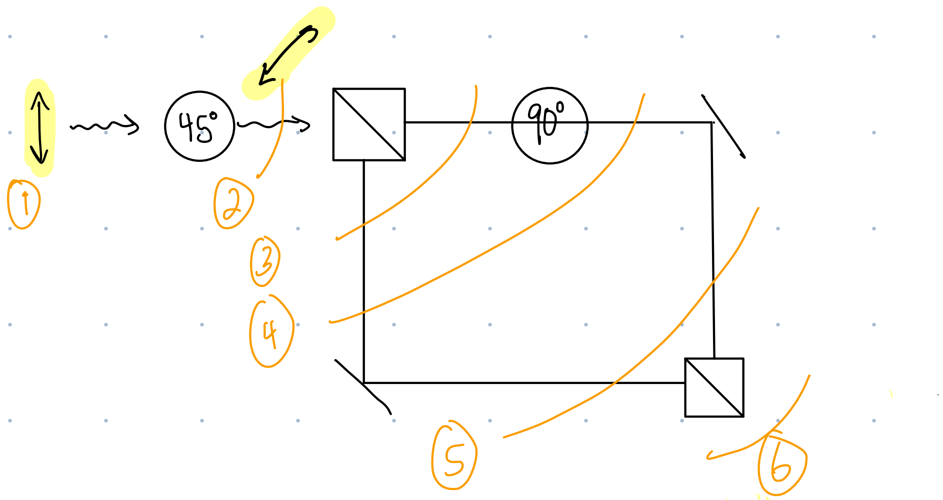
$$|0\rangle_P \rightarrow |+\rangle_P$$
$$|1\rangle_P \rightarrow |-\rangle_P$$

90°

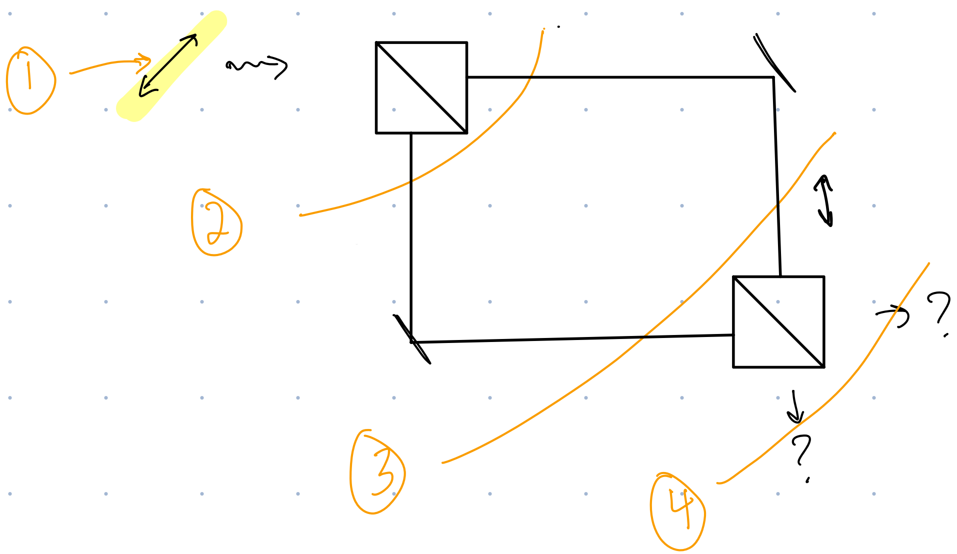
$$|0\rangle_P \rightarrow |1\rangle_P$$
$$|1\rangle_P \rightarrow |0\rangle_P$$

$|\psi_6\rangle =$

$=$



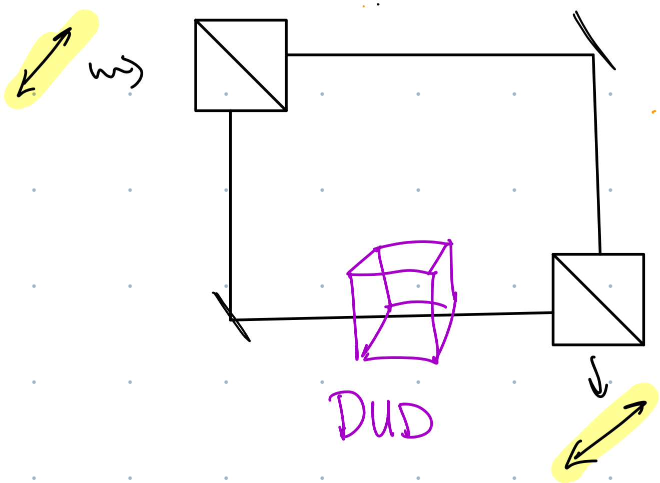
VS



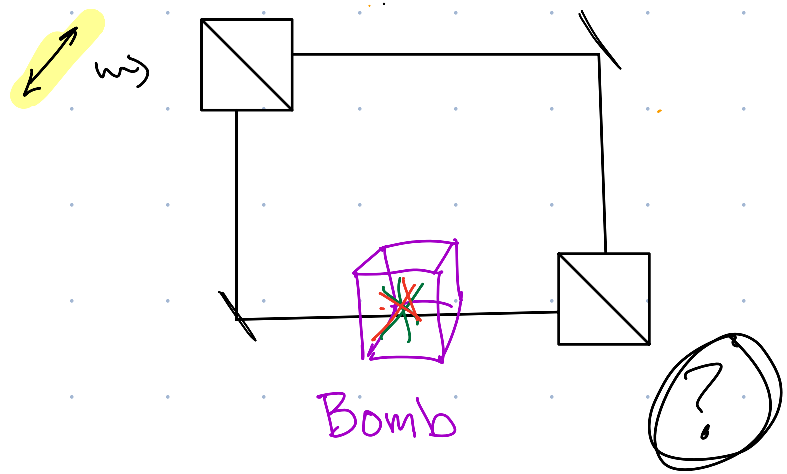
Big ideas:

- One photon takes 2 paths (superposition)
- Path + polarization are entangled
- Objects in arm affect the output photon state

Bomb Test



vs



Intuition:

- Will bomb always explode?
- Bomb makes a measurement:
  -  → photon in lowarm
  -  → photon in upper arm

Partial Q. Measurement (in standard basis)

- State to be measured $|\psi\rangle_{AB}$
- Want to measure qubit A with $M_A = \{|0\rangle, |1\rangle\}$

- To determine outcome:

REWRITE

- Write $|\psi\rangle_{AB}$ in standard basis:

$$|\psi\rangle_{AB} = a_{00}\overset{|0\rangle|0\rangle}{|00\rangle_{AB}} + a_{01}\overset{|0\rangle|1\rangle}{|01\rangle_{AB}} + a_{10}|10\rangle_{AB} + a_{11}|11\rangle_{AB}$$

- Factor/Group A kets:

$$|\psi\rangle_{AB} = |0\rangle_A\left(a_{00}|0\rangle_B + a_{01}|1\rangle_B\right) + |1\rangle_A\left(a_{10}|0\rangle_B + a_{11}|1\rangle_B\right)$$

- Calculate outcome probabilities/collapse state
  - Outcome $|0\rangle$
    - Probability: $P(|0\rangle_A) = |a_{00}|^2 + |a_{01}|^2$
    - Collapse: $\frac{1}{\sqrt{P(|0\rangle_A)}}|0\rangle_A\left(a_{00}|0\rangle_B + a_{01}|1\rangle_B\right) = \frac{a_{00}}{\sqrt{P(|0\rangle)}}|00\rangle_{AB} + \frac{a_{01}}{\sqrt{P(|0\rangle)}}|01\rangle_{AB}$

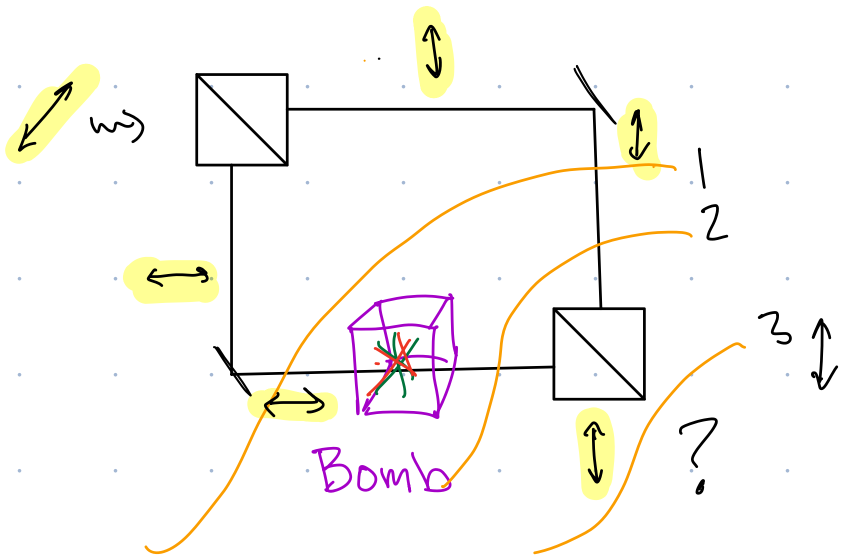
- Outcome $|1\rangle$
  - Probability : $P(|1\rangle_A) = |a_{10}|^2 + |a_{11}|^2$
  - Collapse: $\dfrac{a_{10}}{\sqrt{p(|1\rangle)}}|10\rangle_{AB} + \dfrac{a_{11}}{\sqrt{p(|1\rangle)}}|11\rangle_{AB}$

Back to the Bomb

Our two qubits are polarization + path (direction) ex: $\left(|+\rangle_P |H\rangle_D \right)$

Is the bomb measuring the polarization qubit or path qubit?

A: Polarization   B: Path (direction) $M_D = \{|H\rangle, |V\rangle\}$



$$|\psi_1\rangle = \frac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D + 0|0\rangle_P|H\rangle_D + 0|1\rangle_P|V\rangle_D$$

$$= \left(\frac{1}{\sqrt{2}}|0\rangle_P + 0|1\rangle_P\right)|V\rangle_D + \left(\frac{1}{\sqrt{2}}|1\rangle_P + 0|0\rangle_P\right)|H\rangle_D$$

- Outcome $|H\rangle_D$:

Prob: $\left|\frac{1}{\sqrt{2}}\right|^2 + |0|^2 = \frac{1}{2}$

Collapses: $\frac{1}{\sqrt{\frac{1}{2}}}\left(\frac{1}{\sqrt{2}}|1\rangle_P + 0|0\rangle_P\right)|H\rangle_D$

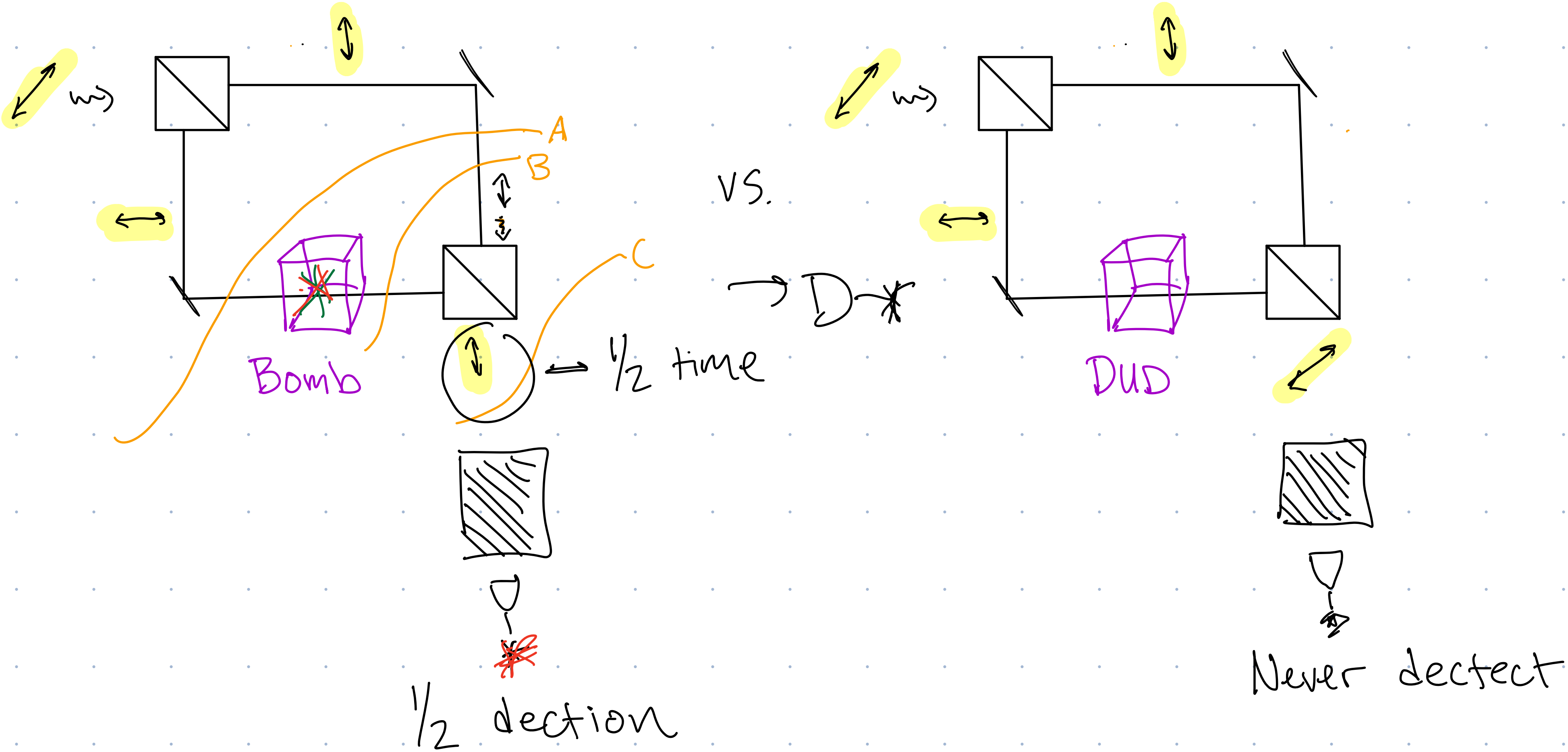
- Outcome $|V\rangle_D$:

Prob: $\left|\frac{1}{\sqrt{2}}\right|^2 + |0|^2 = \frac{1}{2}$

Collapse: $\frac{1}{\sqrt{\frac{1}{2}}}\left(\frac{1}{\sqrt{2}}|0\rangle_P + \cancel{0|1\rangle_P}\right)|V\rangle_D$

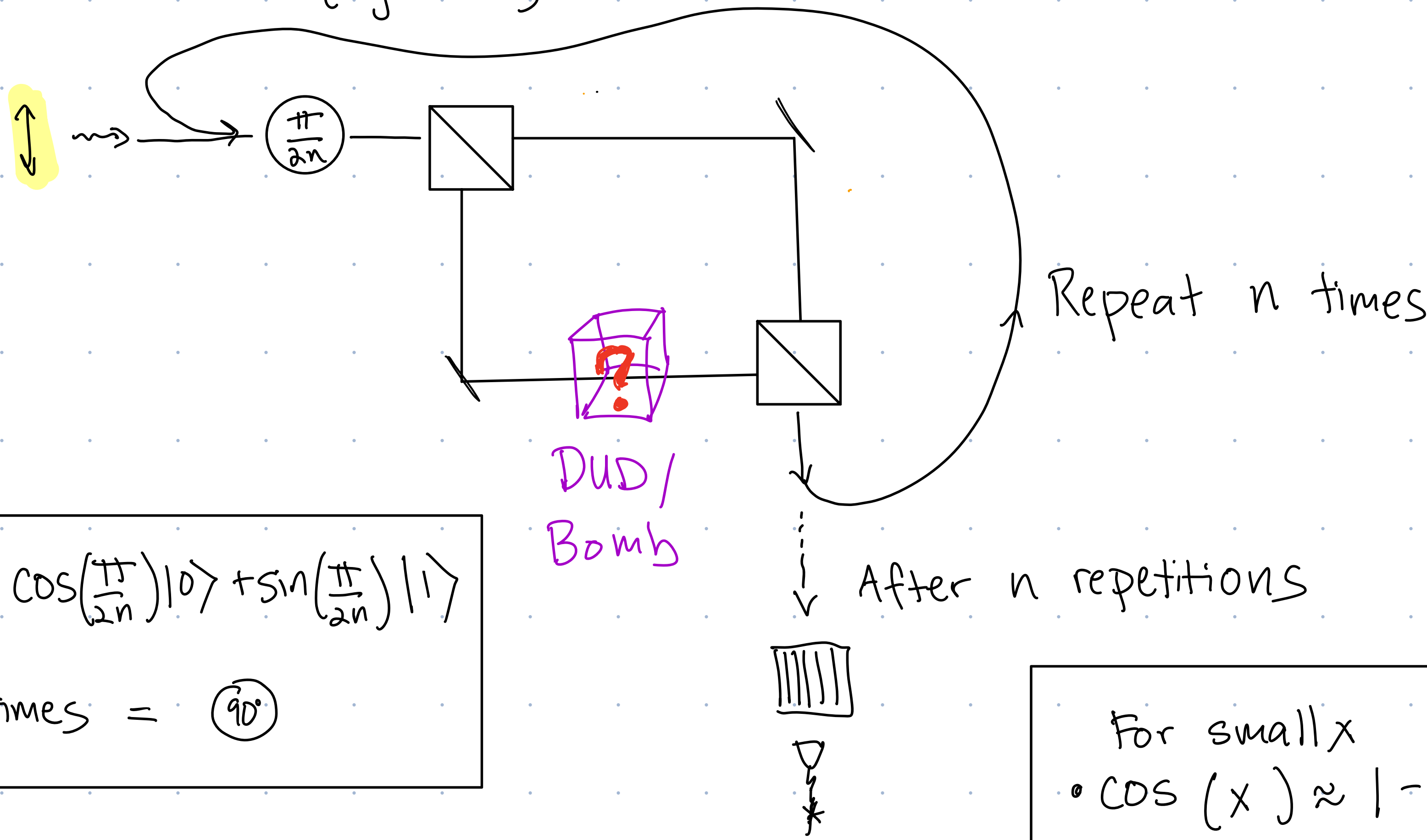
$= |0\rangle_P|V\rangle_D$

If no explosion



Group Work: Analyze the following set up.

Choose $n \gg 1$. (Eg. 1000)



$\frac{\pi}{2n}$ $|0\rangle \to \cos\left(\frac{\pi}{2n}\right)|0\rangle + \sin\left(\frac{\pi}{2n}\right)|1\rangle$

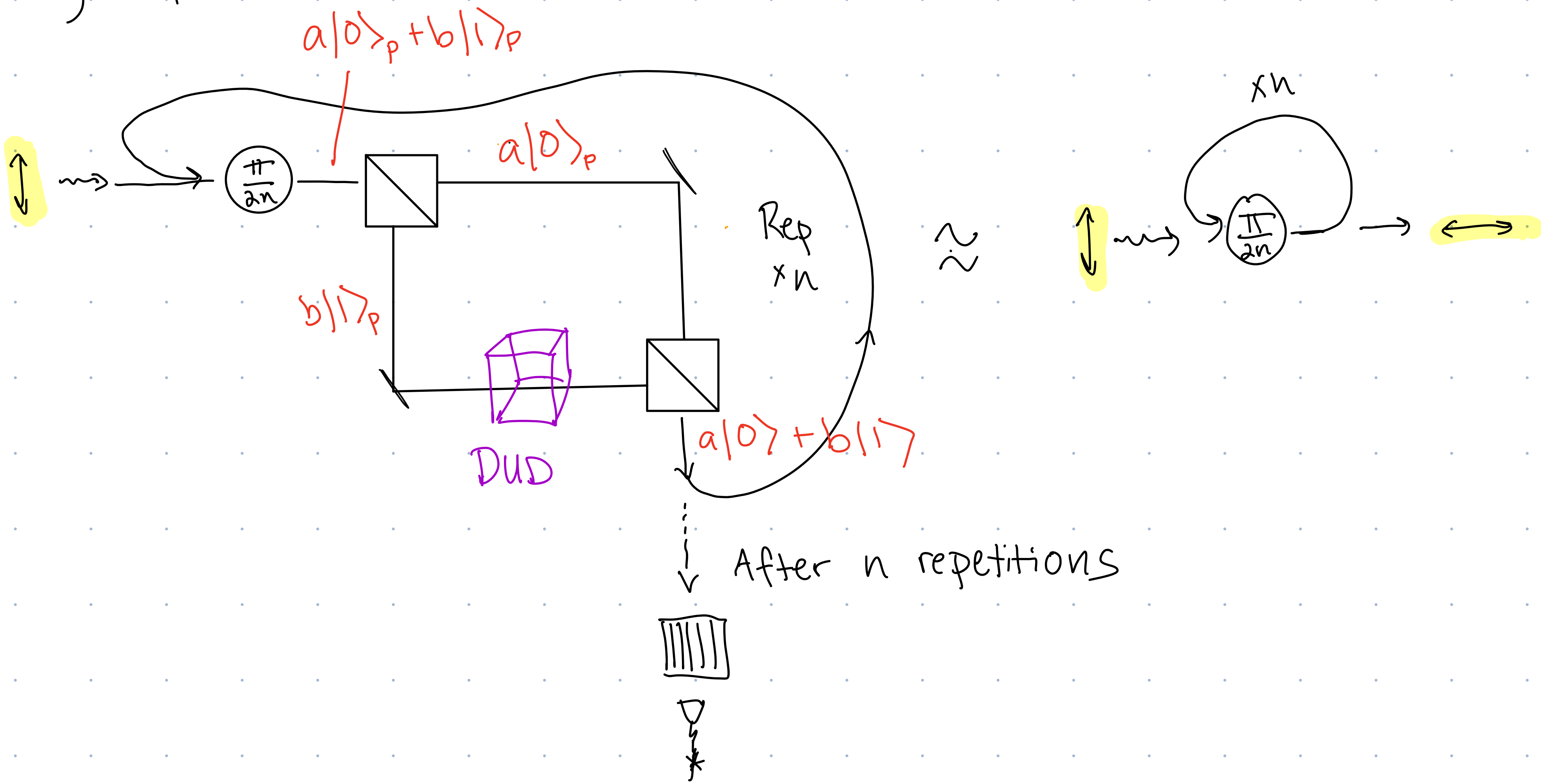
$\frac{\pi}{2n} \times n$ times $= 90°$

For small x

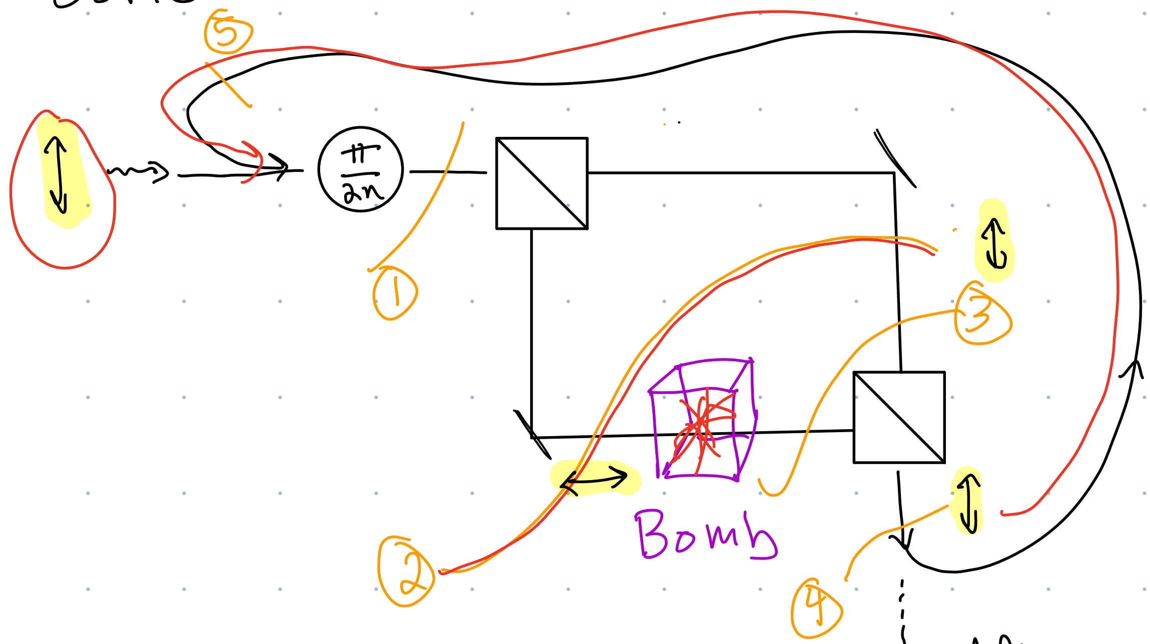
- $\cos(x) \approx 1 - \frac{x^2}{2}$
- $(1-x)^k \approx 1 - kx$

1) What happens if dud?
2) If bomb, probability of NO explosion
3) If bomb but no explosion, probability of detection?

1) If Dud:

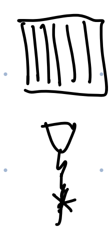


If Bomb:



Repeat n times

After n repetitions



$|\psi_1\rangle = \left(\cos\frac{\pi}{2n}|0\rangle + \sin\frac{\pi}{2n}|1\rangle\right)|H\rangle$

$|\psi_2\rangle = \boxed{\cos\frac{\pi}{2n}|0\rangle}|V\rangle_D + \sin\frac{\pi}{2n}|1\rangle|H\rangle_D$

$P = \left|\cos\frac{\pi}{2n}\right|^2$ $\qquad \cancel{\frac{1}{\cos\frac{\pi}{2n}}\cos\frac{\pi}{2n}}\boxed{|0\rangle|V\rangle_D}$

Prob of outcome $|V\rangle$ is $\cos^2\left(\frac{\pi}{2n}\right)$ state collapses to $|0\rangle_p|V\rangle_D$

Prob of no explosion:

$\left(\cos^2\frac{\pi}{2n}\right)^n \approx \left(1-\frac{\pi^2}{8n^2}\right)^{2n}$

$\approx \left(1-\frac{2n\pi^2}{8n^2}\right)$

$$= \left(1 - \frac{\pi^2}{4n}\right)$$

$$\approx 1$$