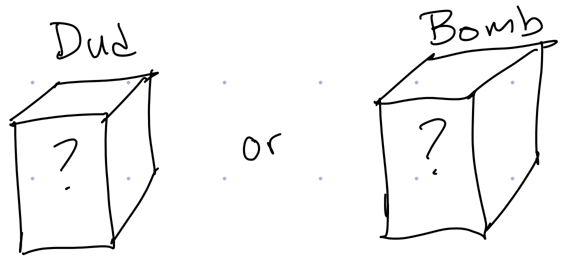
BOMB DETECTION

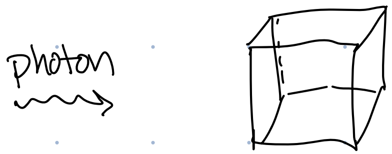
Learning Goals

- Describe quantum operations
- Describe partial q. measurements
- Use rules of q. ops + partial meas. to analyze interferometer scenarios

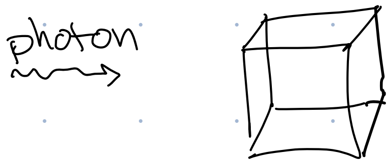
Problem: Quantum Bomb Detection (Elitzur-Vaidman)



Dud:

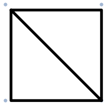


Bomb:

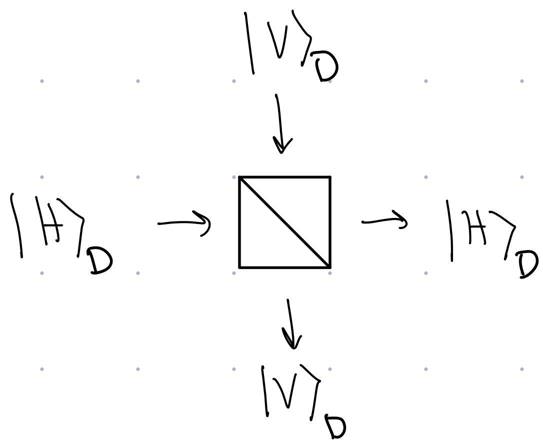


Want to give friend a rainbow bomb not a dud. How??

New Tool: Beamsplitter



- Transmits $|0\rangle_P$ (vertically polarized light)
- Reflects $|1\rangle_P$ (horiz. " ")



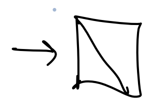
"P" for polarization

"D" for direction

$\{|V\rangle_D, |H\rangle_D\}$ standard basis states

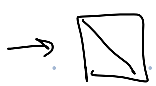
Behavior:

$|0\rangle_P |H\rangle_D \rightarrow$

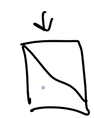


$|0\rangle_P |V\rangle_D \rightarrow$

$|1\rangle_P |H\rangle_D \rightarrow$



$|1\rangle_P |V\rangle_D \rightarrow$

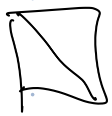


A) $|0\rangle_P |H\rangle_D$ B) $|0\rangle_P |V\rangle_D$ C) $|1\rangle_P |H\rangle_D$ D) $|1\rangle_P |V\rangle_D$

Quantum Gate

Operation that changes q. state without collapse (no info extracted)

Can describe gate via action on standard basis states



Beam-Splitter

$|0\rangle_P |H\rangle_D \rightarrow$

$|1\rangle_P |H\rangle_D \rightarrow$

$|0\rangle_P |V\rangle_D \rightarrow$

$|1\rangle_P |V\rangle_D \rightarrow$

Mirror 

$|H\rangle_D \rightarrow$

$|V\rangle_D \rightarrow$

Wave plate

θ

Only affects polarization

45°

$|0\rangle_P \rightarrow$

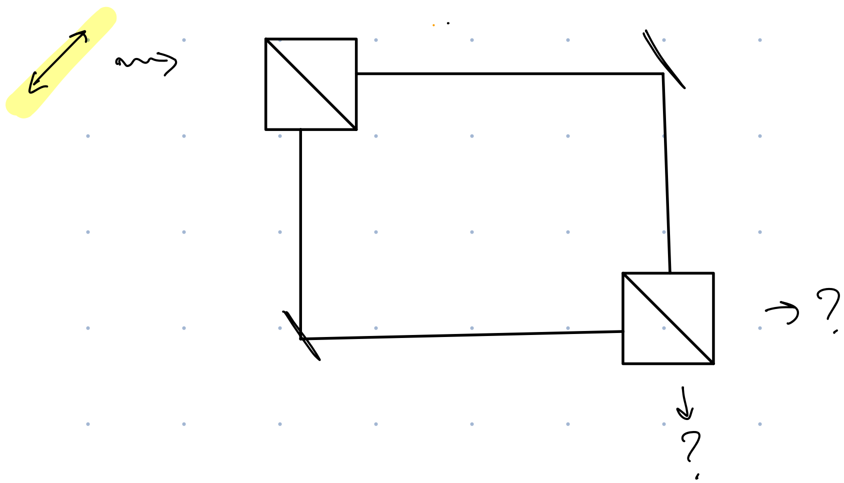
$|1\rangle_P \rightarrow$

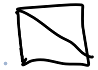
90°

$|0\rangle_P \rightarrow$

$|1\rangle_P \rightarrow$

Interferometer





$$|0\rangle_P |H\rangle_D \longrightarrow |0\rangle_P |H\rangle_D$$
$$|1\rangle_P |H\rangle_D \longrightarrow |1\rangle_P |V\rangle_D$$
$$|0\rangle_P |V\rangle_D \longrightarrow |0\rangle_P |V\rangle_D$$
$$|1\rangle_P |V\rangle_D \longrightarrow |1\rangle_P |H\rangle_D$$

$$|H\rangle_D \longrightarrow |V\rangle_D$$
$$|V\rangle_D \longrightarrow |H\rangle_D$$

$|\psi_1\rangle =$

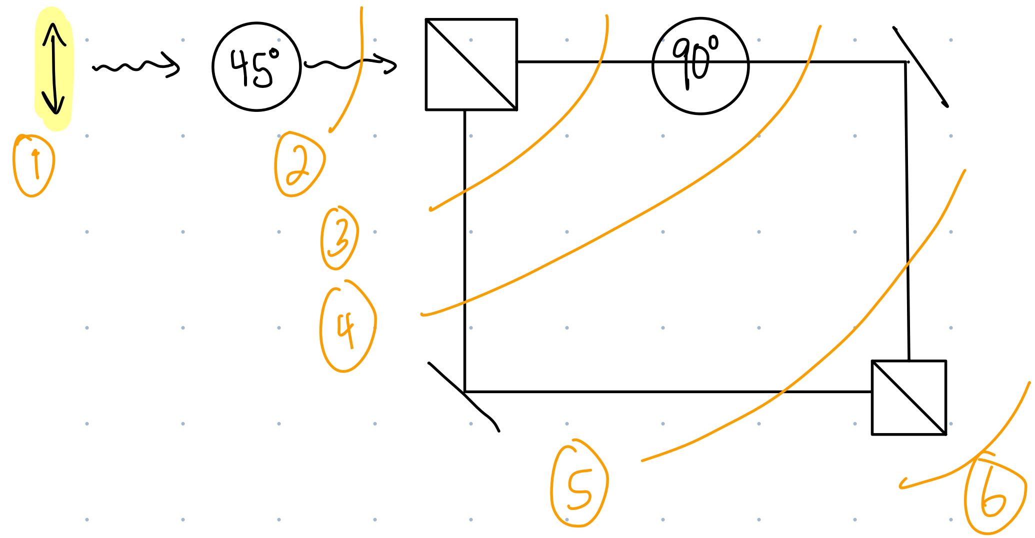
$|\psi_2\rangle =$

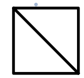
$|\psi_3\rangle =$

$|\psi_4\rangle =$

Group Practice

QI4





$$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$$
$$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$$
$$|0\rangle_P|V\rangle_D \rightarrow |0\rangle_P|V\rangle_D$$
$$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$$

$$|H\rangle_D \rightarrow |V\rangle_D$$
$$|V\rangle_D \rightarrow |H\rangle_D$$

45°

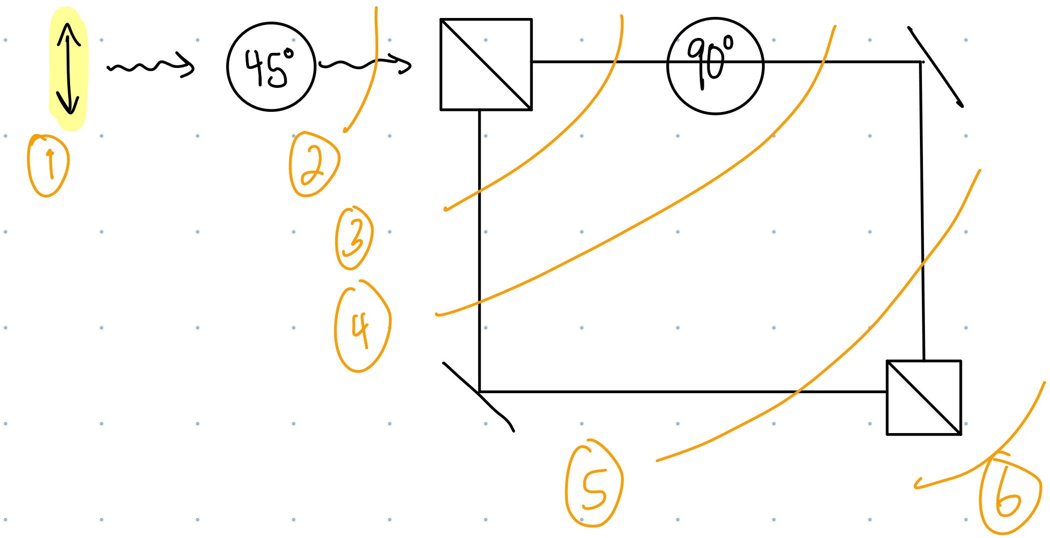
$$|0\rangle_P \rightarrow |+\rangle_P$$
$$|1\rangle_P \rightarrow |-\rangle_P$$

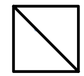
90°

$$|0\rangle_P \rightarrow |1\rangle_P$$
$$|1\rangle_P \rightarrow |0\rangle_P$$

Group Practice

QI4





$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$

$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$

$|0\rangle_P|V\rangle_D \rightarrow |0\rangle_P|V\rangle_D$

$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$

$|H\rangle_D \rightarrow |V\rangle_D$

$|V\rangle_D \rightarrow |H\rangle_D$

45°

$|0\rangle_P \rightarrow |+\rangle_P$

$|1\rangle_P \rightarrow |-\rangle_P$

90°

$|0\rangle_P \rightarrow |1\rangle_P$

$|1\rangle_P \rightarrow |0\rangle_P$

$|\psi_1\rangle =$

$|\psi_2\rangle =$

$|\psi_3\rangle =$

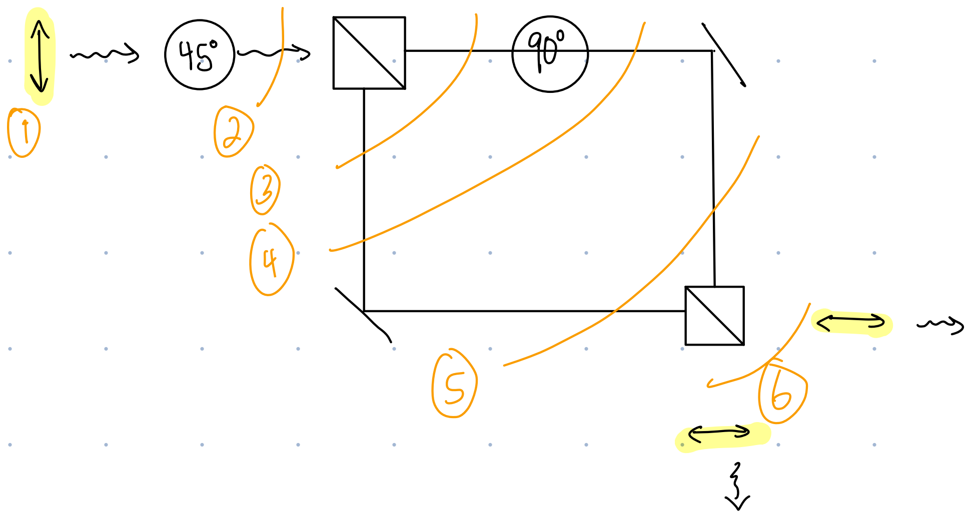
$|\psi_4\rangle =$

$=$

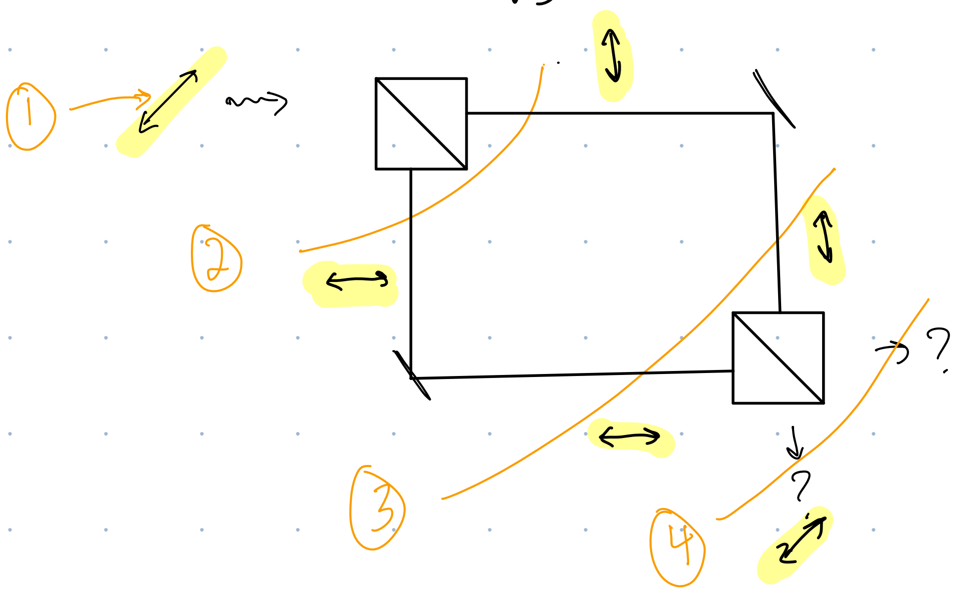
$|\psi_5\rangle =$

$|\psi_6\rangle =$

$=$



VS



Big ideas:

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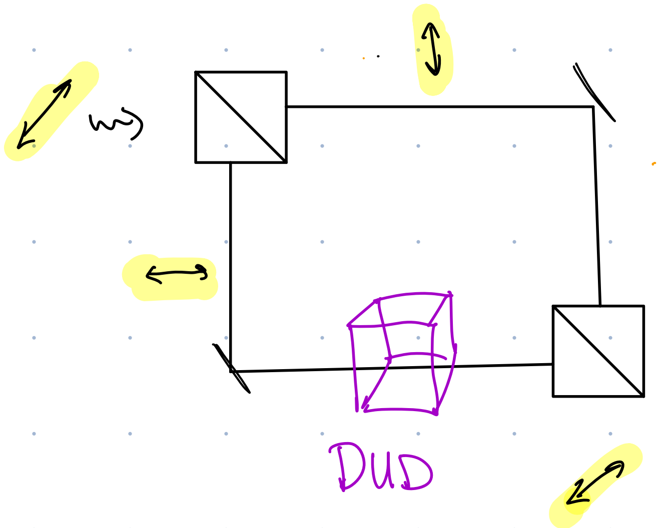
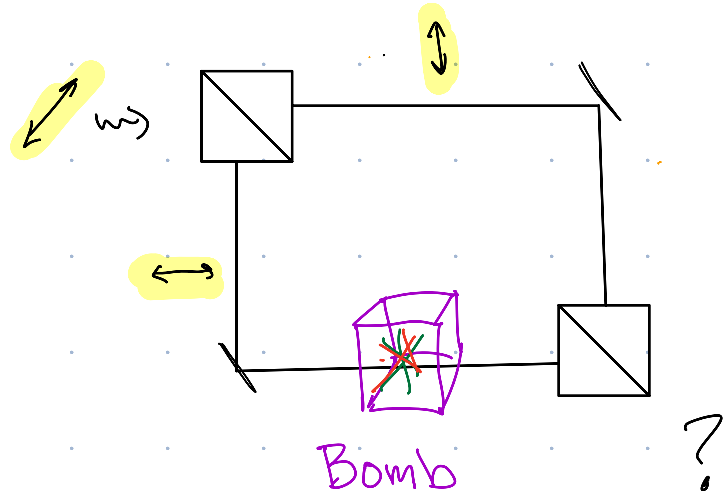
-
-
-

Bomb Test



vs



Same as empty interferometer

Intuition:

-
-
-

Partial Q. Measurement (in standard basis)

- State to be measured: $|\psi\rangle_{AB}$ Needs 2 parts! (A & B)
- Want to measure qubit A in $M_A = \{|0\rangle, |1\rangle\}$ No measurement of B. • Asking if A is in state $|0\rangle$ or $|1\rangle$ • Not asking B anything
- To determine outcome:
  - Write $|\psi\rangle_{AB}$ in standard basis:

$$|\psi\rangle_{AB} = a_{00}|00\rangle_{AB} + a_{01}|01\rangle_{AB} + a_{10}|10\rangle_{AB} + a_{11}|11\rangle_{AB}$$

  - Rewrite: Factor/group A kets: (numbers/scalars commute!)

$$|\psi\rangle_{AB} = |0\rangle_A\left(a_{00}|0\rangle_B + a_{01}|1\rangle\right) + |1\rangle\left(a_{10}|0\rangle + a_{11}|1\rangle\right)$$

  - Calculate outcome probabilities/collapse state
    - Outcome $|0\rangle$
      - Probability: $|a_{00}|^2 + |a_{01}|^2 = p(|0\rangle_A)$
      - Collapse state: $\frac{1}{\sqrt{p(|0\rangle_A)}}|0\rangle\left(a_{00}|0\rangle + a_{01}|1\rangle\right) = \frac{a_{00}}{\sqrt{p(|0\rangle_A)}}|00\rangle + \frac{a_{01}}{\sqrt{p(|0\rangle_A)}}|01\rangle$

Needed for state to be properly normalized

- Outcome $|1\rangle$
  - Probability: $|a_{10}|^2 + |a_{11}|^2 = p(|1\rangle_A)$
  - Collapse state: $\frac{1}{\sqrt{p(|1\rangle_A)}} |1\rangle\left(a_{10}|0\rangle + a_{11}|1\rangle\right) = \frac{a_{10}}{\sqrt{p(|1\rangle_A)}}|10\rangle + \frac{a_{11}}{\sqrt{p(|1\rangle_A)}}|11\rangle$

* Partial measurement destroys entanglement

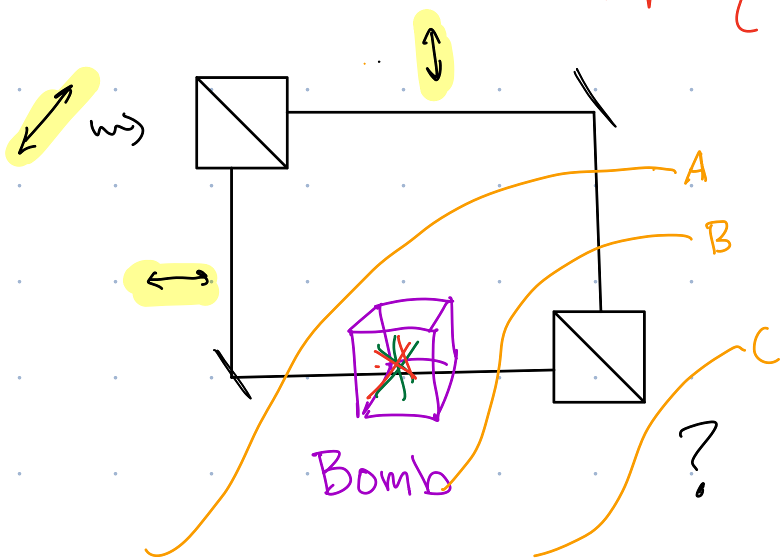
Back to the Bomb

Our two qubits are polarization + path (direction) $\left(|+\rangle_P |H\rangle_D\right)$

Is the bomb measuring the polarization qubit or path qubit?

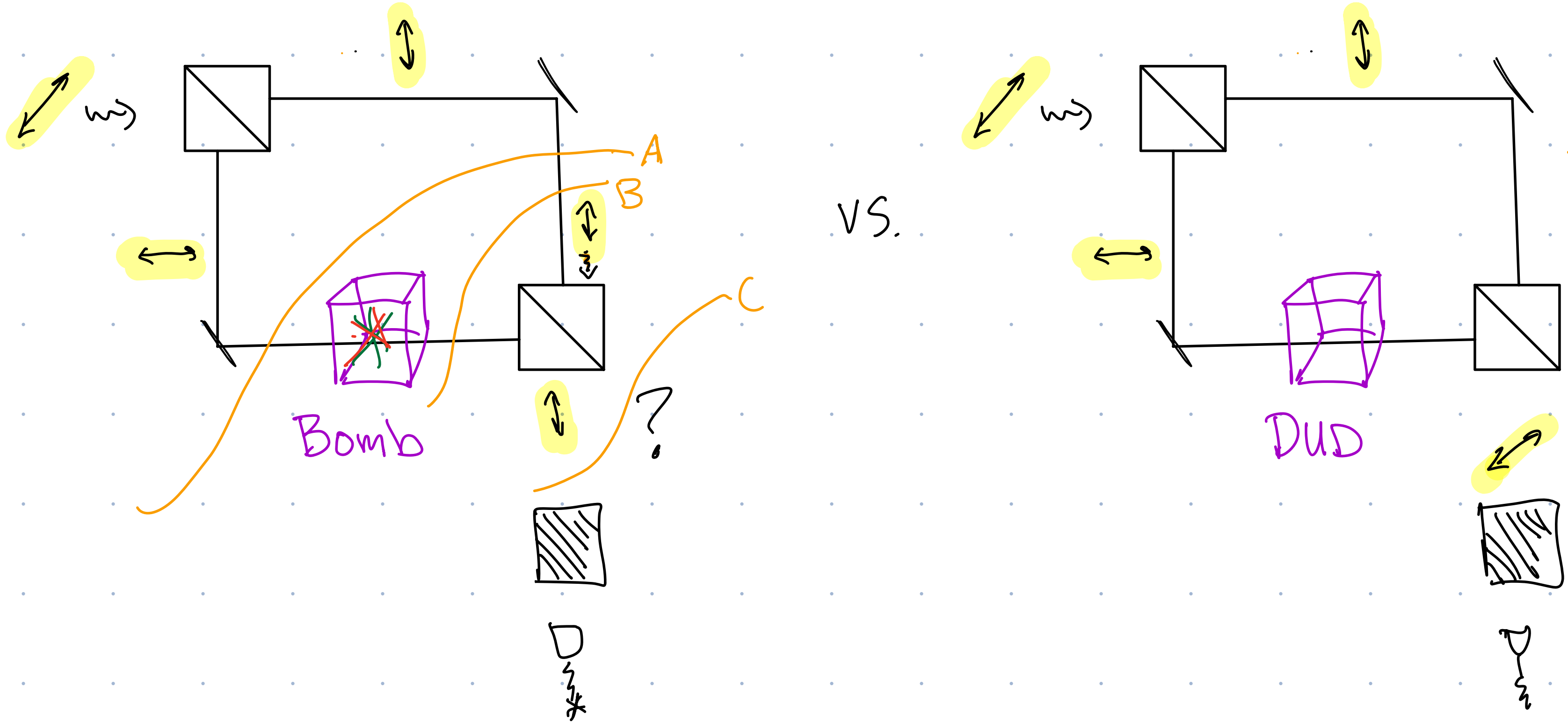
A: Polarization B: Path (direction)

$M = \{ |H\rangle_D, |V\rangle_D \}$



$$|\psi_A\rangle = \frac{1}{\sqrt{2}} |0\rangle_P |V\rangle_D + \frac{1}{\sqrt{2}} |1\rangle_P |H\rangle_D$$

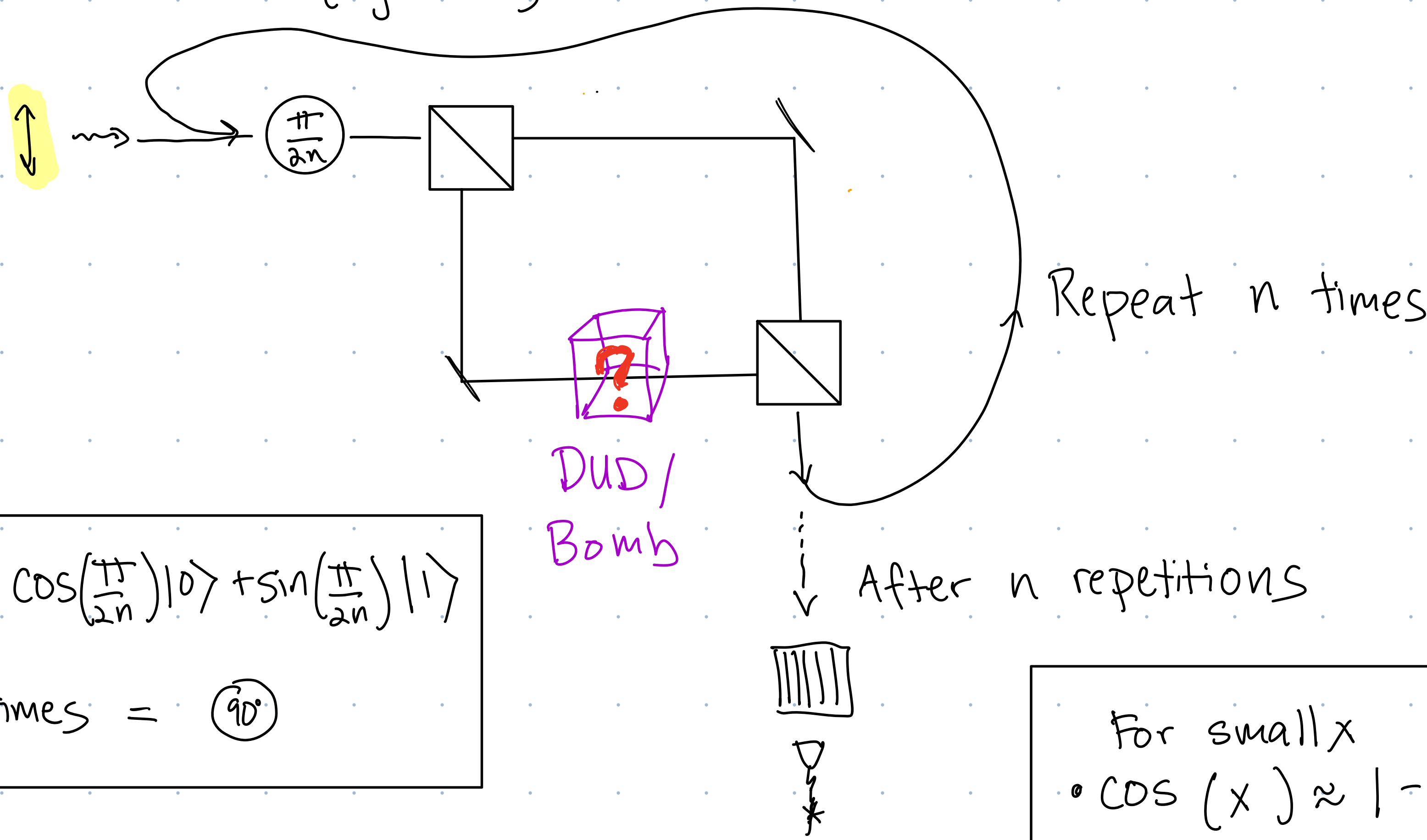
If no explosion



Can we do better?

Group Work: Analyze the following set up.

Choose $n \gg 1$. (Eg. 1000)



$\frac{\pi}{2n}\; |0\rangle \to \cos\left(\frac{\pi}{2n}\right)|0\rangle + \sin\left(\frac{\pi}{2n}\right)|1\rangle$

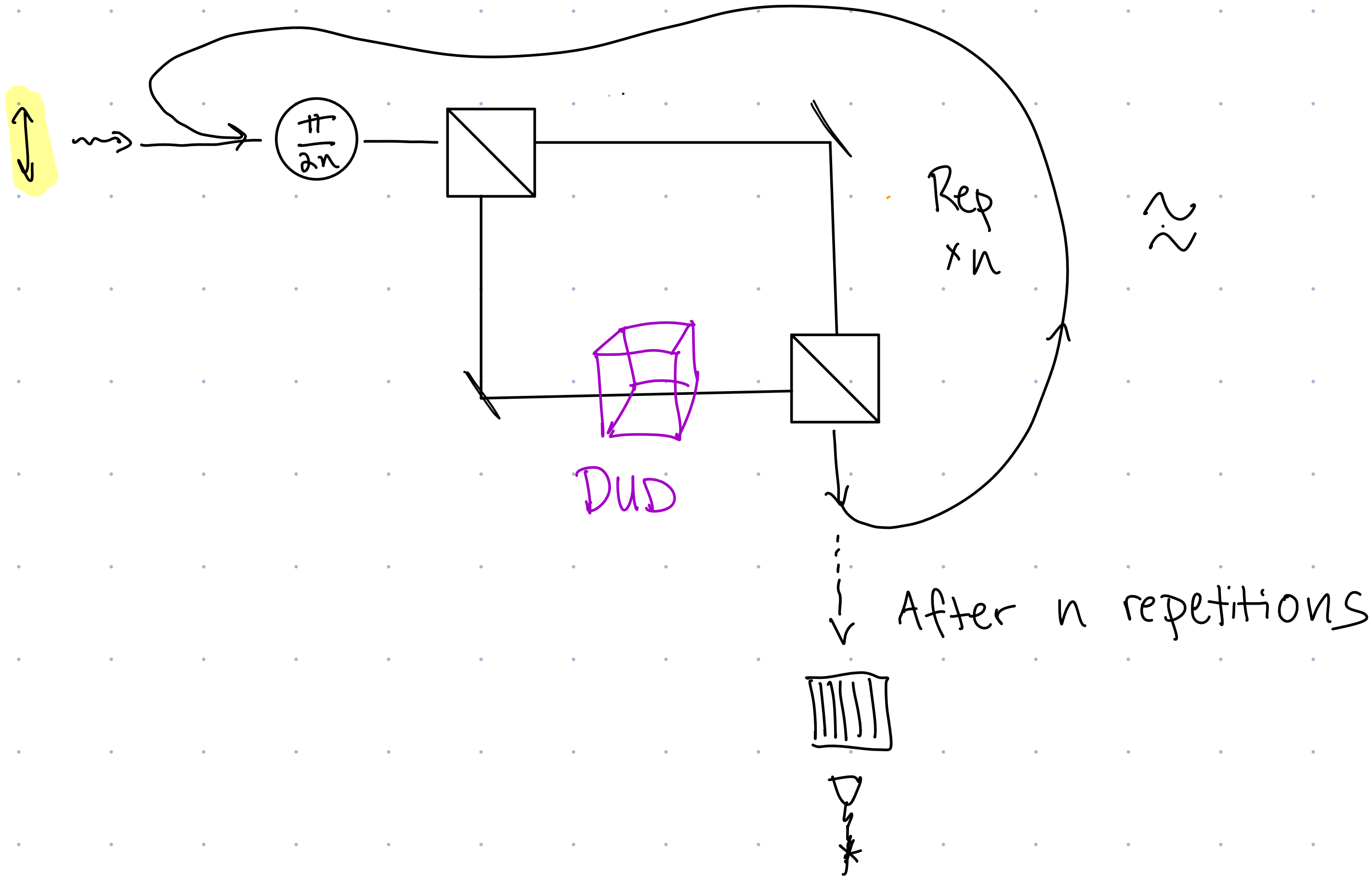
$\frac{\pi}{2n} \times n \text{ times} = 90°$

For small x

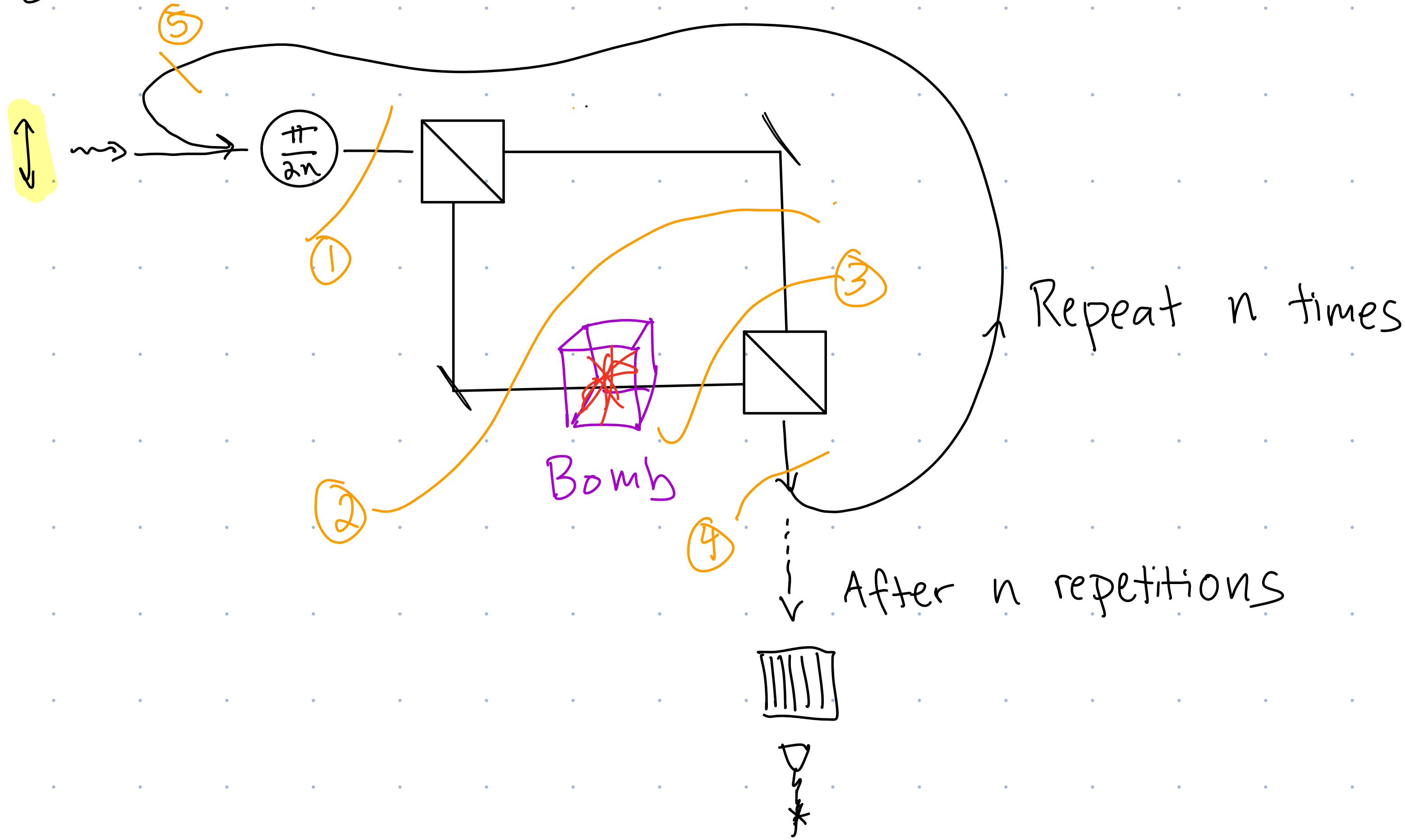
- $\cos(x) \approx 1 - \frac{x^2}{2}$
- $(1-x)^k \approx 1 - kx$

1) What happens if dud?
2) If bomb, probability of NO explosion
3) If bomb but no explosion, probability of detection?

1) If Dud:



If Bomb:



$|\psi_1\rangle =$

$|\psi_2\rangle =$