Learning Goals

- Describe quantum gates using ket action
- Describe sufficient + necessary properties of gates
- Apply ket formalism to analyze novel gate applications

Announcements

Exit Tickets

Previous examples of gates (also called operations/unitaries)

⧅ Beamsplitter

$$\boxed{\begin{aligned} |0\rangle_P |H\rangle_D &\rightarrow |0\rangle_P |H\rangle_D \\ |1\rangle_P |H\rangle_D &\rightarrow |1\rangle_P |V\rangle_D \\ |0\rangle_P |V\rangle_D &\rightarrow |0\rangle_P |V\rangle_D \\ |1\rangle_P |V\rangle_D &\rightarrow |1\rangle_P |H\rangle_D \end{aligned}}$$

θ Wave plate

$$|0\rangle \rightarrow \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$$

$$|1\rangle \rightarrow -\sin\theta\,|0\rangle + \cos\theta\,|1\rangle$$

Take q states to q states:

$$⧅\left(\frac{1}{\sqrt{5}}|0\rangle|H\rangle + \sqrt{\frac{2}{5}}\,i\,|1\rangle|H\rangle + \sqrt{\frac{3}{5}}\,|1\rangle|V\rangle\right)$$

$$\theta\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)$$

$$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta\,|0\rangle + \frac{1}{\sqrt{2}}\sin\theta\,|1\rangle$$

Reversible $|\psi\rangle$ ⧅ ⧅ $|\psi\rangle$

θ $-\theta$

Properties of Gates (Necessary + Sufficient)

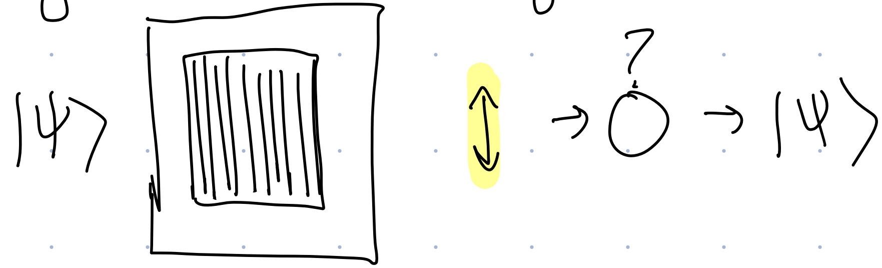
- Take quantum states to quantum states
- Reversible

iff

Are these gates? Why/Why not?

$$\boxed{\begin{aligned} |0\rangle &\to |0\rangle + |1\rangle \\ |1\rangle &\to |0\rangle - |1\rangle \end{aligned}}$$

X $q \to q$



X reversible

Gate is a unitary operation

- a unitary is a multidimensional genaralization of reflections and rotations
- Must take an orthonormal set to an orthonormal set

$$\left.\begin{aligned} |0\rangle &\to |\psi_1\rangle \\ |1\rangle &\to |\psi_2\rangle \end{aligned}\right\} \text{Valid if } \{|\psi_1\rangle, |\psi_2\rangle\} \text{ orthonormal}$$

$$\left.\begin{aligned} |00\rangle &\to |\psi_1\rangle \\ |01\rangle &\to |\psi_2\rangle \\ |10\rangle &\to |\psi_3\rangle \\ |11\rangle &\to |\psi_4\rangle \end{aligned}\right\} \text{Valid if } \langle\psi_i|\psi_j\rangle = \delta_{ij} \text{ (orthonormal)}$$

Famous Gates:

Paulis

I

$|0\rangle \rightarrow$

$|1\rangle \rightarrow$

X

$|0\rangle \rightarrow$

$|1\rangle \rightarrow$

Y

$|0\rangle \rightarrow$

$|1\rangle \rightarrow$

Z

$|0\rangle \rightarrow$

$|1\rangle \rightarrow$

Hadamard

H

$|0\rangle \rightarrow$

$|1\rangle \rightarrow$

CNOT

$|00\rangle \rightarrow$

$|01\rangle \rightarrow$

$|10\rangle \rightarrow$

$|11\rangle \rightarrow$

controlled (based) on 1st qubit, apply NOT to 2nd

Apply left to right

$HY|0\rangle =$

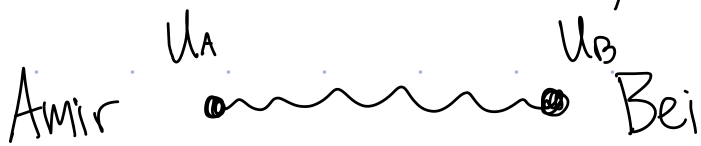
Hadamard

H $\boxed{\begin{array}{l} |0\rangle \to |+\rangle \\ |1\rangle \to |-\rangle \end{array}}$

What is $H|+\rangle$?

Single Qubit Gates on 2-qubit States

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$



$$U_{AB}|\psi\rangle_{AB} =$$

Group Problem

Suppose Amir + Bei share the 2-qubit state $|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.
What Paulis should they each apply to create the state $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$?

Write this effective 2 qubit gate as a transformation of standard basis states:

$|00\rangle \rightarrow$

$|01\rangle \rightarrow$

$|10\rangle \rightarrow$

$|11\rangle \rightarrow$