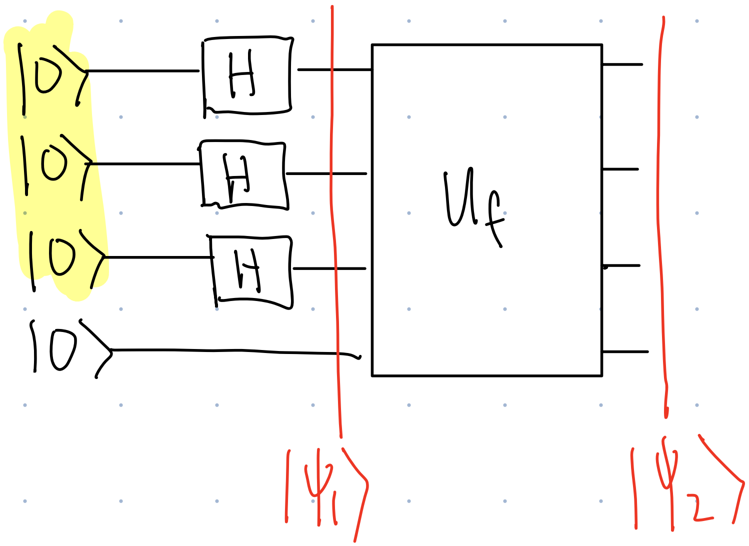
Learning Goals

- Describe how quantum algorithms gain an advantage over probabilistic algorithms.
- Analyze circuits using the path integral formalism

Quantum Secret Sauce?

$|x\rangle, |b\rangle \rightarrow$ standard basis states



$$U_f \underset{\substack{\uparrow \\ \text{3-qubit} \\ \text{s.b. state}}}{|x\rangle} |b\rangle = |x\rangle |b \oplus f(x)\rangle$$

$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \frac{1}{\sqrt{2}}(\ ) \frac{1}{\sqrt{2}}(\ )$

$$|\psi_1\rangle = (H \otimes H \otimes H \otimes I)|0000\rangle = |+\rangle|+\rangle|+\rangle|0\rangle$$

$$= \frac{1}{2\sqrt{2}}\left(|000\rangle|0\rangle + |001\rangle|0\rangle + |010\rangle|0\rangle + |011\rangle|0\rangle + \dots\right)$$

$$= \frac{1}{2\sqrt{2}} \sum_{x \in \{0,1\}^3} |x\rangle|0\rangle$$

exponential growth: 3 H's $\Rightarrow$ 2^3 s.b. states

$$|\psi_2\rangle = U_f \frac{1}{2\sqrt{2}} \sum_{x \in \{0,1\}^3} |x\rangle|0\rangle = \frac{1}{2\sqrt{2}} \sum_{x \in \{0,1\}^3} U_f|x\rangle|0\rangle$$

$$= \frac{1}{2\sqrt{2}} \sum_{x \in \{0,1\}^3} |x\rangle|f(x)\rangle$$

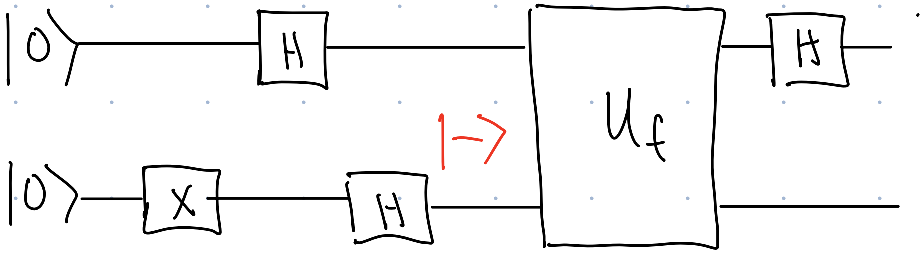
With 1 query, get info about all outputs of function.

But actually, this exponential scaling is not special

ex: 3 coin flips $\Rightarrow$ 8 possible outcomes $\Rightarrow f \rightarrow$ equal chance of every output of f

Quantum Computing	vs	Probabilistic Computing
$\sum_{i \in \{0,1\}^n} a_i \lvert i\rangle$	$\leftarrow$ State $\rightarrow$	$\sum_{i \in \{0,1\}^n} a_i \lvert i\rangle$ (probability)
s.t. $\sum_i \lvert a_i\rvert^2 = 1$, $a_i \in \mathbb{C}$		$1 \geq a_i \geq 0$ $\quad \sum a_i = 1$
Probability of outcome i is $\lvert a_i\rvert^2$	$\leftarrow$ Measure $\rightarrow$	Prob of outcome i a_i
Unitary (preserves normalization, reversibility)	$\leftarrow$ Gate $\rightarrow$	Left stochastic (preserve normalization, positivity)

Deutsch's Alg (Probabilistic Version)



X:

$$|0\rangle \to |1\rangle$$
$$|1\rangle \to |0\rangle$$

Unitary ✓

Left Stoch. ✓

U_f:

$$|0\rangle|0\rangle \to |0\rangle|f(0)\rangle$$
$$|0\rangle|1\rangle \to |0\rangle|1 \oplus f(0)\rangle$$
$$|1\rangle|0\rangle \to |1\rangle|f(1)\rangle$$
$$|1\rangle|1\rangle \to |1\rangle|1 \oplus f(1)\rangle$$

Unitary ✓

Left Stoch. ✓

H:

$$|0\rangle \to \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$
$$|1\rangle \to \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

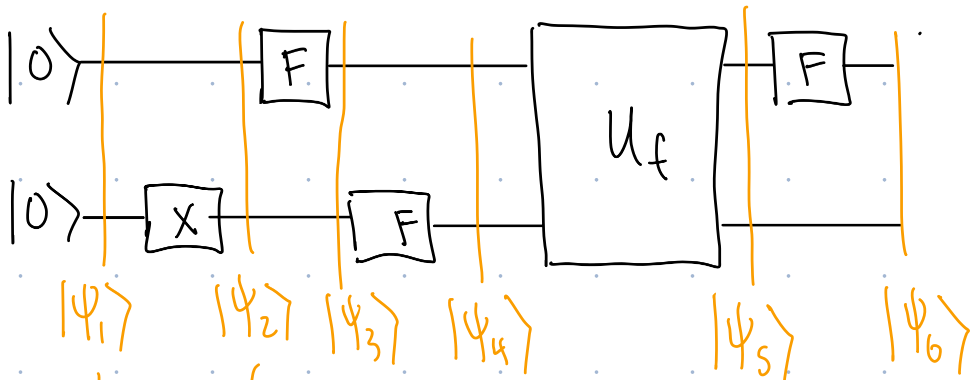
Unitary ✓

Left Stoch. ✗

F:

$$|0\rangle \to \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle$$
$$|1\rangle \to \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle$$

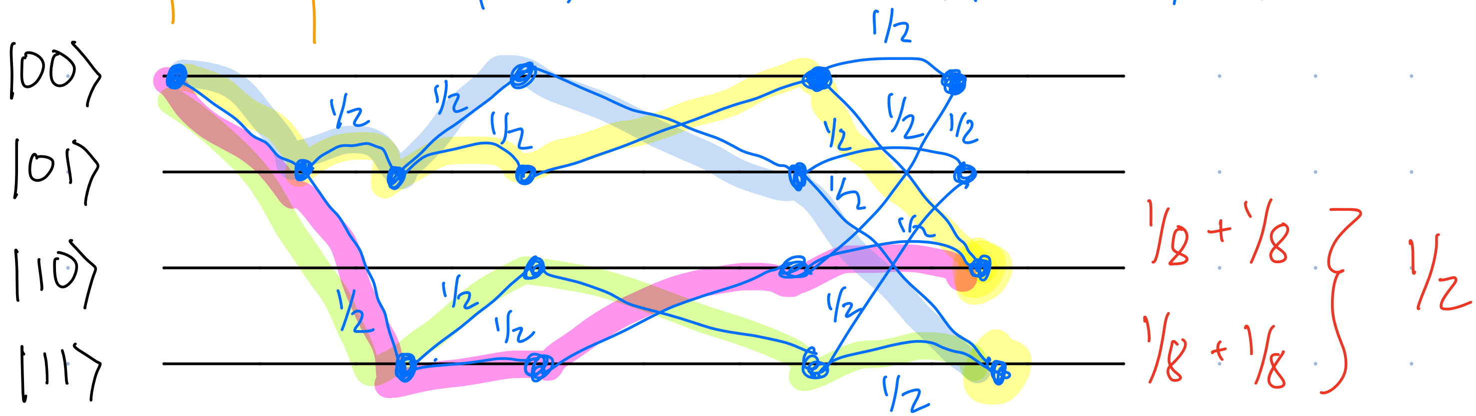
Path Integral Analysis (Probabilistic)



Assume
$f(0) = f(1) = 1$

What is probability of 1^{st} bit being 1?

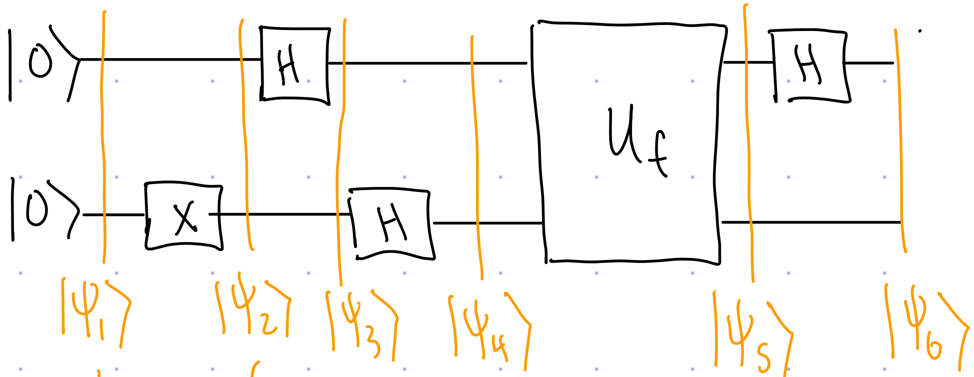
$U_f|00\rangle = |0\rangle|1\rangle$ $\quad$ $U_f|01\rangle = |0\rangle|0\rangle$



To analyze

- Multiply probabilities on a path to get prob of path
- Add probabilities of all paths terminating at state to get probability of that outcome

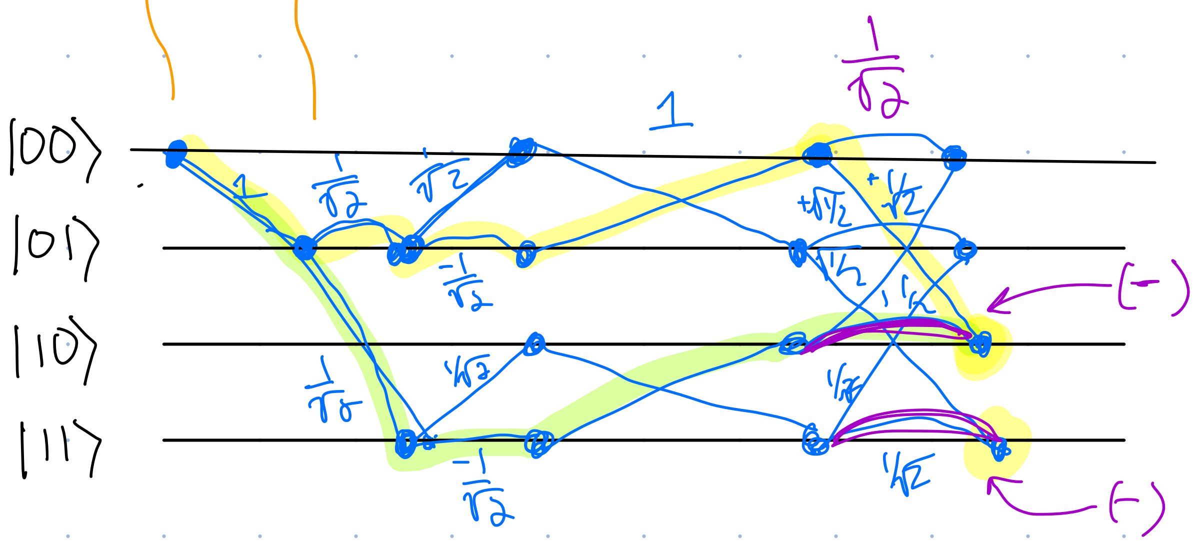
Path Integral Analysis (Quantum)



For exercise, set

$f(0) = f(1) = 1$

What is probability of 1^{st} qubit being 1?



$\frac{-1}{2\sqrt{2}} + \frac{-1}{2\sqrt{2}} = \left|\frac{-1}{\sqrt{2}}\right|^2 = \frac{1}{2}$

$\frac{-1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = 0$ 0 prob of outcome $|10\rangle$

To analyze

- Multiply ~~probabilities~~ amplitudes on a path to get ~~prob~~ amplitude of path
- Add ~~probabilities~~ amplitudes of all paths terminating at state and take abs. value sq. to get probability of that outcome

Quantum Secret Sauce (Algorithms)