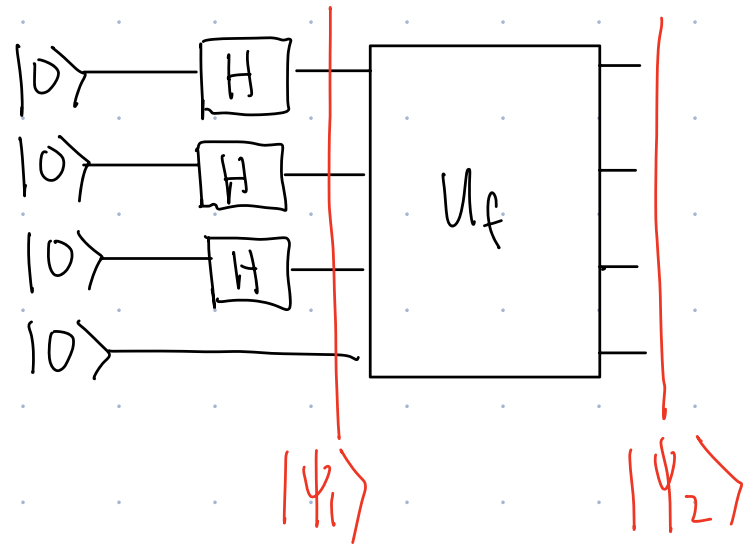


Learning Goals

- Describe how quantum algorithms gain an advantage over probabilistic algorithms.
- Analyze circuits using the path integral formalism

Quantum Secret Sauce?



$$|\psi_1\rangle =$$

$$|\psi_2\rangle =$$

=

$|x\rangle, |b\rangle \rightarrow$ standard basis states

$$U_f |x\rangle |b\rangle = |x\rangle |b \oplus f(x)\rangle$$

But actually, this exponential scaling is not special

ex:

Quantum Computing vs Probabilistic Computing

$$\sum_{i \in \{0,1\}^n} a_i |i\rangle$$

← State →

$$\text{s.t. } \sum_i |a_i|^2 = 1, a_i \in \mathbb{C}$$

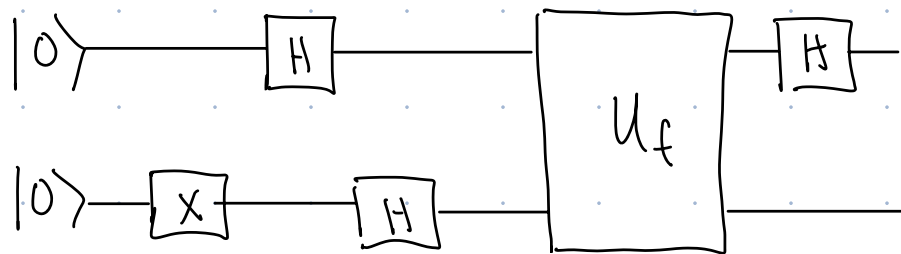
Probability of
outcome i is $|a_i|^2$

← Measure →

← Gate →

(preserves normalization,
reversibility)

Deutsch's Alg (Probabilistic Version)



$$X: \begin{array}{l} |0\rangle \rightarrow |1\rangle \\ |1\rangle \rightarrow |0\rangle \end{array}$$

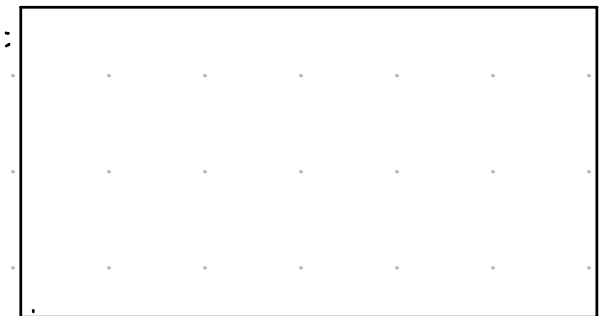
Unitary
Left Stoch.

$$U_f: \begin{array}{l} |0\rangle|0\rangle \rightarrow |0\rangle|f(0)\rangle \\ |0\rangle|1\rangle \rightarrow |0\rangle|1 \oplus f(0)\rangle \\ |1\rangle|0\rangle \rightarrow |1\rangle|f(1)\rangle \\ |1\rangle|1\rangle \rightarrow |1\rangle|1 \oplus f(1)\rangle \end{array}$$

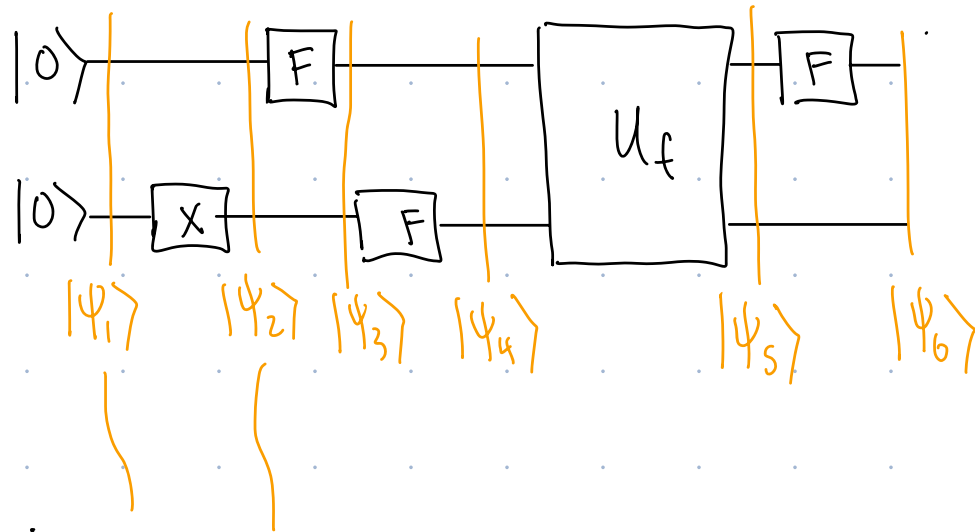
Unitary
Left Stoch.

$$H: \begin{array}{l} |0\rangle \rightarrow \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\ |1\rangle \rightarrow \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \end{array}$$

Unitary
Left Stoch.



Path Integral Analysis (Probabilistic)



Assume
 $f(0) = f(1) = 1$

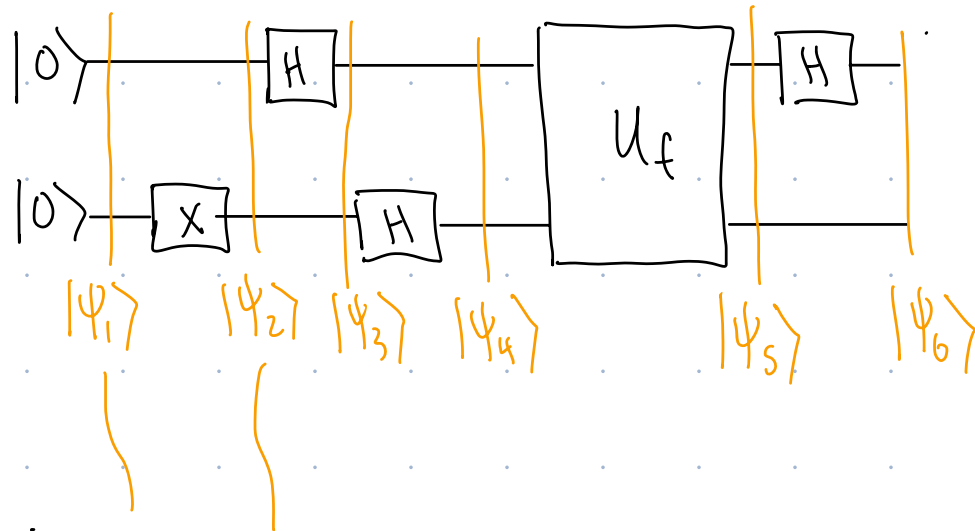
What is probability of
1st bit being 1?

$|00\rangle$ _____
 $|01\rangle$ _____
 $|10\rangle$ _____
 $|11\rangle$ _____

To analyze

- Multiply probabilities on a path to get prob of path
- Add probabilities of all paths terminating at state to get probability of that outcome

Path Integral Analysis (Quantum)



For exercise, set

$$f(0) = f(1) = 1$$

What is probability of
1st qubit being 1?

$|00\rangle$ _____
 $|01\rangle$ _____
 $|10\rangle$ _____
 $|11\rangle$ _____

To analyze

- Multiply probabilities on a path to get prob of path
- Add probabilities of all paths terminating at state to get probability of that outcome

Quantum Secret Sauce (Algorithms)