Learning Goals

- Describe how quantum algorithms gain an advantage over probabilistic algorithms.
- Analyze circuits using the path integral formalism

PS 7

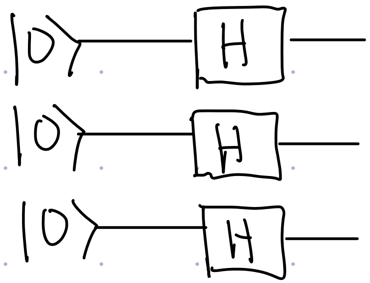
- Lot of assignments
- Office Hours

Exit Tickets

- Systems get destroyed when measured?
- Universal decomposition, inverse, classical alg, conditions for universality
- Approximations + errors
- Physical error rates

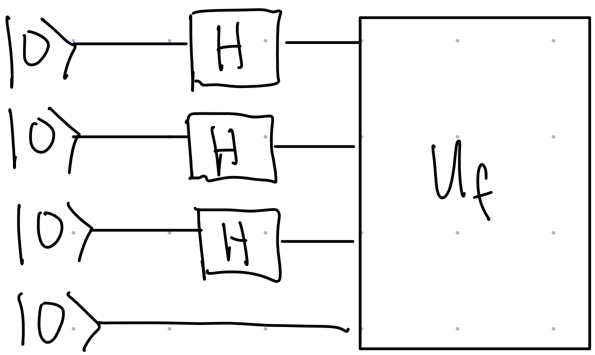
Quantum Secret Sauce?

SuperPosition?



$$\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$$
$$\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$$
$$\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$$

$$\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$$
$$= \frac{1}{2\sqrt{2}}\left(|000\rangle + |001\rangle + |010\rangle + |010\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle\right)$$



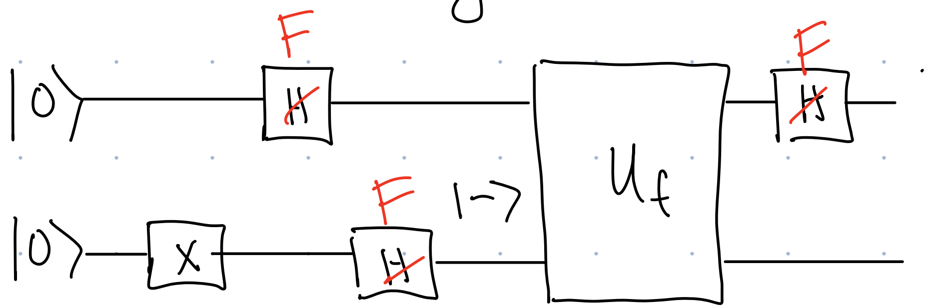
$$\Rightarrow \frac{1}{2\sqrt{2}}\big(|000\rangle|f(000)\rangle + |001\rangle|f(001)\rangle$$
$$+ |010\rangle|f(010)\rangle + \dots$$

But actually, this exponential scaling is not special

ex: 3 coin flips $\Rightarrow$ 8 possible outcomes $\rightarrow$ exponential

Quantum Computing	vs	Probabilistic Computing
$\sum_{i \in \{0,1\}^n} a_i \lvert i \rangle$	← State →	$\sum_{i \in \{0,1\}} a_i \lvert i \rangle$ (probability)
s.t. $\sum_i \lvert a_i \rvert^2 = 1$		$\sum_{i \in \{0,1\}^n} a_i = 1 \quad a_i \geq 0$
Probability of outcome i is $\lvert a_i \rvert^2$	← Measure →	Probability of outcome i is a_i
Unitary (preserve normalization, reversible)	← Gate →	Left Stochastic gates (preserves normalization, positivity)

Deutsch's Alg (Probabilistic Version)



X:

$$|0\rangle \to |1\rangle$$
$$|1\rangle \to |0\rangle$$

Unitary ✓

Left Stoch. ✓

U_f:

$$|0\rangle|0\rangle \to |0\rangle|f(0)\rangle$$
$$|0\rangle|1\rangle \to |0\rangle|\overline{f(0)}\rangle$$
$$|1\rangle|0\rangle \to |1\rangle|f(1)\rangle$$
$$|1\rangle|1\rangle \to |1\rangle|\overline{f(1)}\rangle$$

Unitary ✓

Left Stoch. ✓

H:

$$|0\rangle \to \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$
$$|1\rangle \to \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

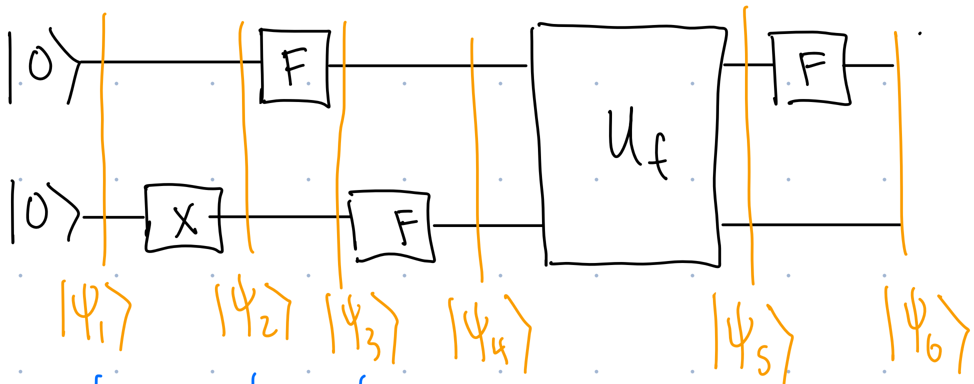
Unitary ✓

Left Stoch. ✗

Instead: F:

$$|0\rangle \to \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle$$
$$|1\rangle \to \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle$$

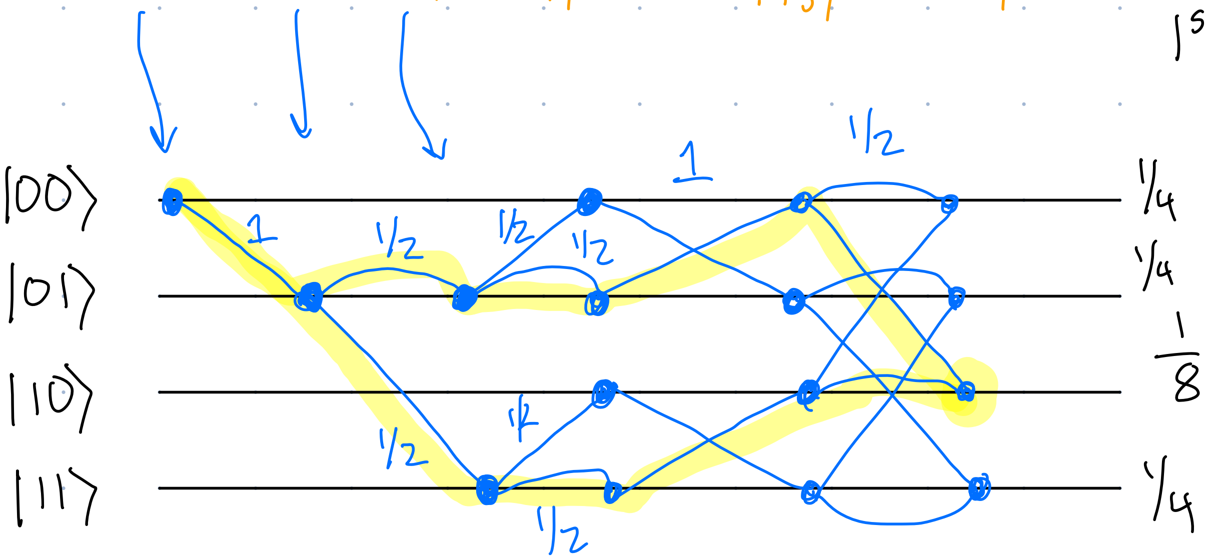
Path Integral Analysis (Probabilistic)



For exercise, set

$$f(0) = f(1) = 1$$

What is probability of 1^{st} bit being 1?

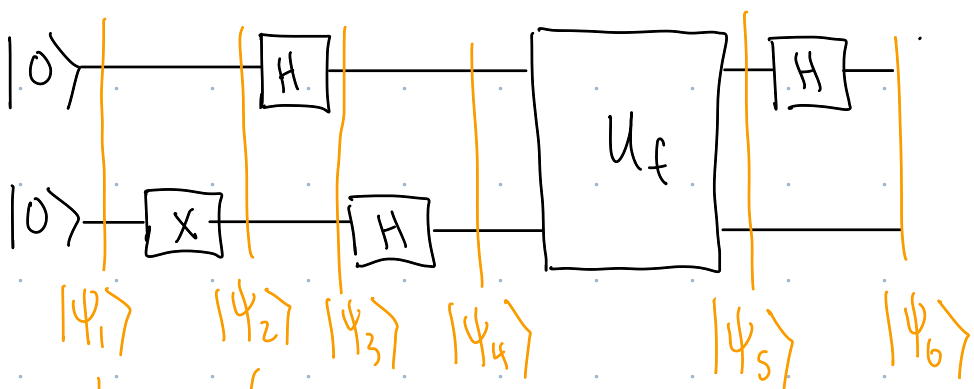


$\frac{1}{8}$ per path × 2 path = 1/4

To analyze

- Multiply probabilities on a path to get prob of path
- Add probabilities of all paths terminating at state to get probability of that outcome

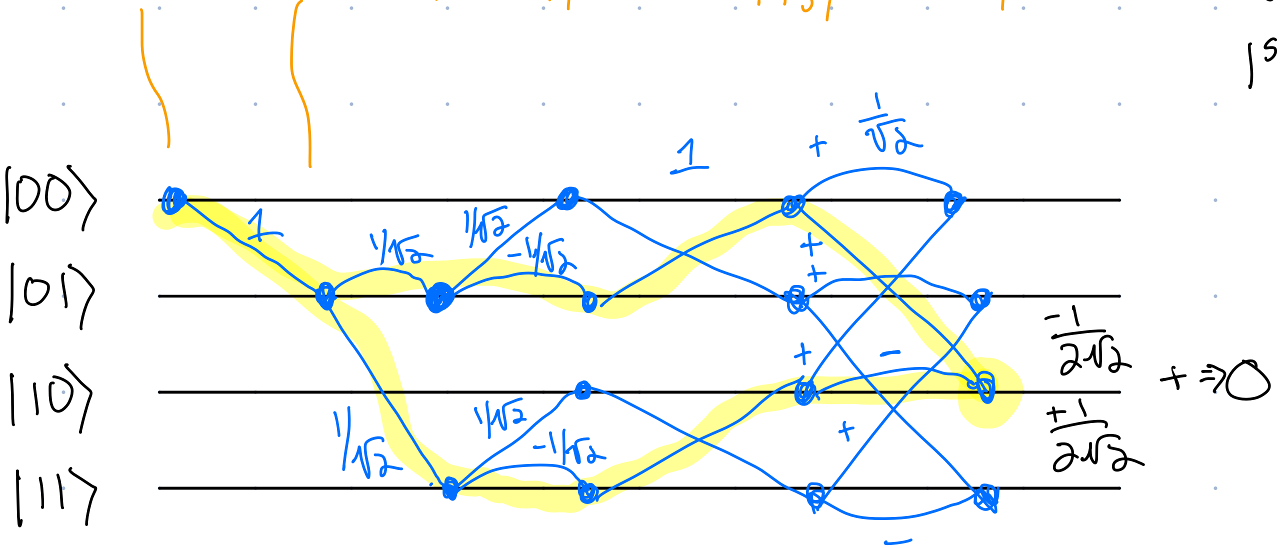
Path Integral Analysis (Quantum)



For exercise, set

$f(0) = f(1) = 1$

What is probability of 1^{st} qubit being 1?



$|0|^2 = 0$

To analyze

- Multiply ~~probabilities~~ amplitudes on a path to get ~~prob~~ amplitude of path
- Add ~~probabilities~~ amplitudes of all paths terminating at state ~~to get probability of that outcome~~ then take abs. value squared, ⇒ probability

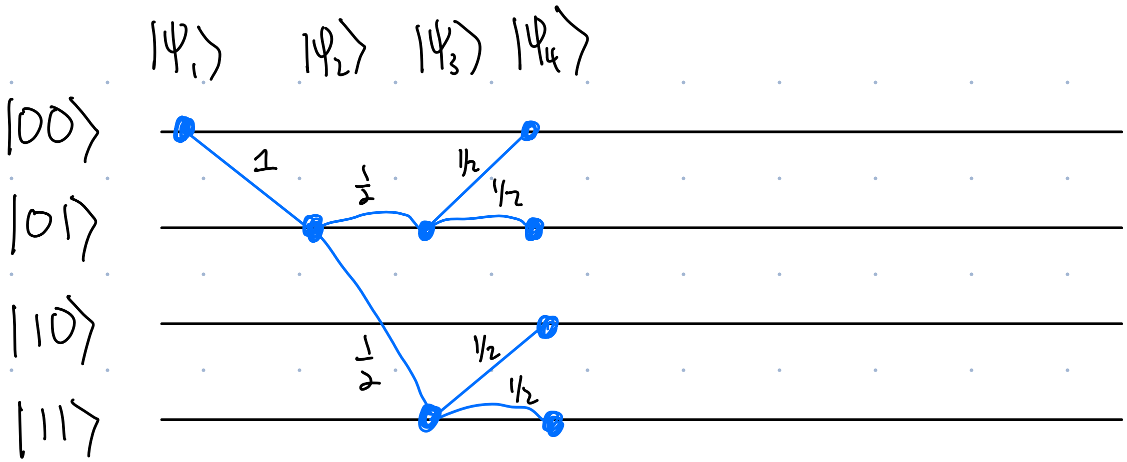
Quantum Secret Sauce (Algorithms)

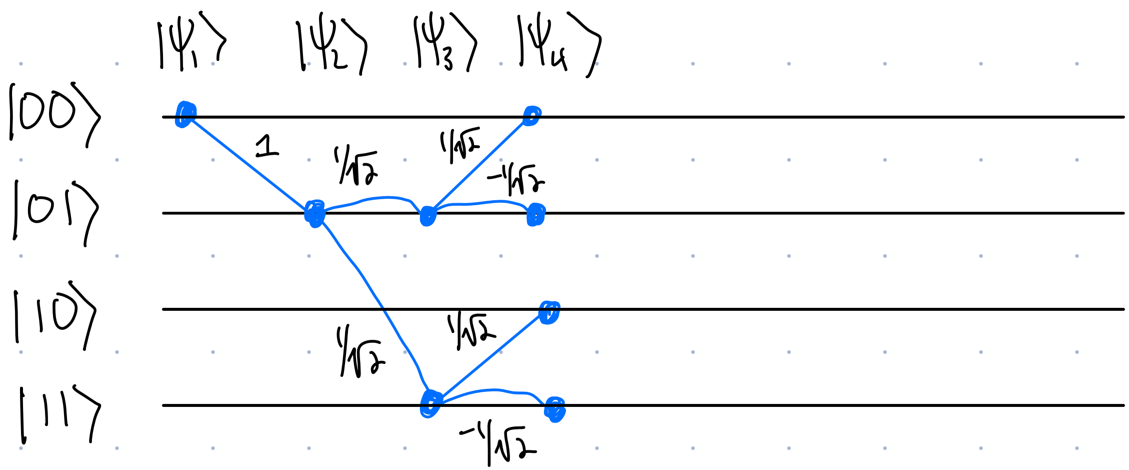
Superposition + Interference

↓

pos + neg phases on different paths

concel out undesirable outcomes


|ψ1⟩ |ψ2⟩ |ψ3⟩ |ψ4⟩
|00⟩
|01⟩
|10⟩
|11⟩
1
1/2
1/2
1/2
1/2
1/2
1/2


|ψ1⟩ |ψ2⟩ |ψ3⟩ |ψ4⟩
|00⟩
|01⟩
|10⟩
|11⟩
1
1/√2
1/√2
−1/√2
1/√2
1/√2
−1/√2