Quantum Gates

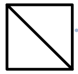
Learning Goals

- Describe quantum gates using ket action
- Describe sufficient + necessary properties of gates
- Apply ket formalism to analyze novel gate applications

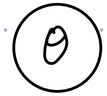
Announcements

Exit Tickets

Previous examples of gates (also called operations/unitaries)

 Beamsplitter

$$|0\rangle_P |H\rangle_D \rightarrow |0\rangle_P |H\rangle_D$$
$$|1\rangle_P |H\rangle_D \rightarrow |1\rangle_P |V\rangle_D$$
$$|0\rangle_P |V\rangle_D \rightarrow |0\rangle_P |V\rangle_D$$
$$|1\rangle_P |V\rangle_D \rightarrow |1\rangle_P |H\rangle_D$$

 Wave plate

$$|0\rangle \rightarrow \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$$
$$|1\rangle \rightarrow -\sin\theta\,|0\rangle + \cos\theta\,|1\rangle$$

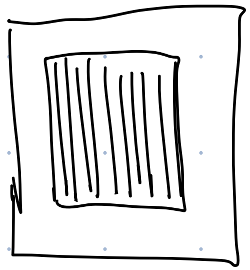
Properties of Gates (Necessary + Sufficient)

iff

Are these gates? Why/Why not?

$|0\rangle \to |0\rangle + |1\rangle$

$|1\rangle \to |0\rangle - |1\rangle$



$|0\rangle \to$

$|1\rangle \to$

$|00\rangle \to$

$|01\rangle \to$

$|10\rangle \to$

$|11\rangle \to$

Famous Gates:

Paulis

I
$|0\rangle \rightarrow$
$|1\rangle \rightarrow$

X
$|0\rangle \rightarrow$
$|1\rangle \rightarrow$

Y
$|0\rangle \rightarrow$
$|1\rangle \rightarrow$

Z
$|0\rangle \rightarrow$
$|1\rangle \rightarrow$

Hadamard

H
$|0\rangle \rightarrow$
$|1\rangle \rightarrow$

CNOT

$|00\rangle \rightarrow$
$|01\rangle \rightarrow$
$|10\rangle \rightarrow$
$|11\rangle \rightarrow$

Apply left to right

Matrix Note

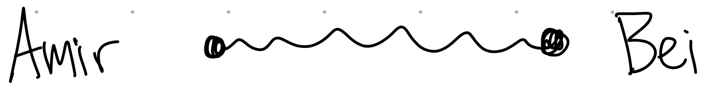
Hadamard

H $\boxed{\begin{array}{l}|0\rangle \to |+\rangle \\ |1\rangle \to |-\rangle\end{array}}$

What is $H|+\rangle$?

Single Qubit Gates on 2-qubit States

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$



We write:

$$|\psi\rangle_{AB} \Rightarrow ?$$

$$U_{AB}|\psi\rangle_{AB} =$$

Group Problem

Suppose Amir + Bei share the 2-qubit state $|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.
What Paulis should they each apply to create the state $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$?

Write this effective 2 qubit gate as a transformation of standard basis states:

$|00\rangle \rightarrow$

$|01\rangle \rightarrow$

$|10\rangle \rightarrow$

$|11\rangle \rightarrow$