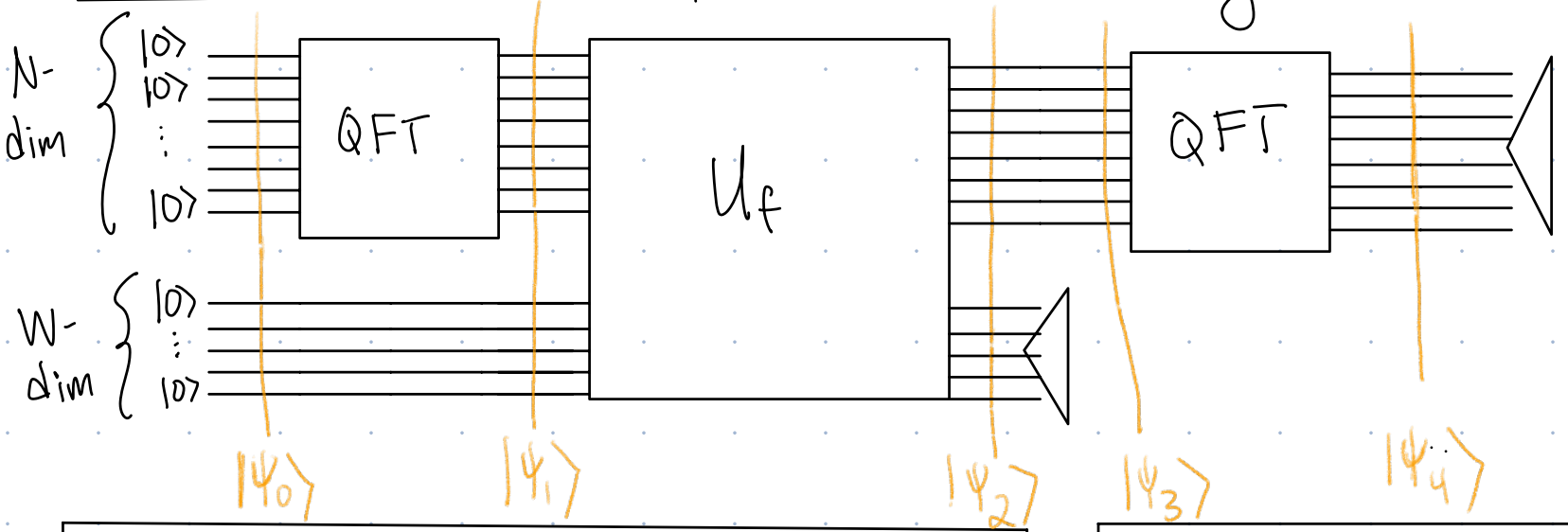
Quantum Circuit for Period Finding

GROUP WORK Version



QFT: Quantum Forier Transform

$$|x\rangle \longrightarrow \frac{1}{\sqrt{N}} \sum_{y=0}^{N-1} e^{2\pi i xy/N} |y\rangle$$

↑ N-dim standard basis state

U_f:

$$|x\rangle|y\rangle = |x\rangle|f(x)+y \bmod W\rangle$$

↑ N-dim standard basis state

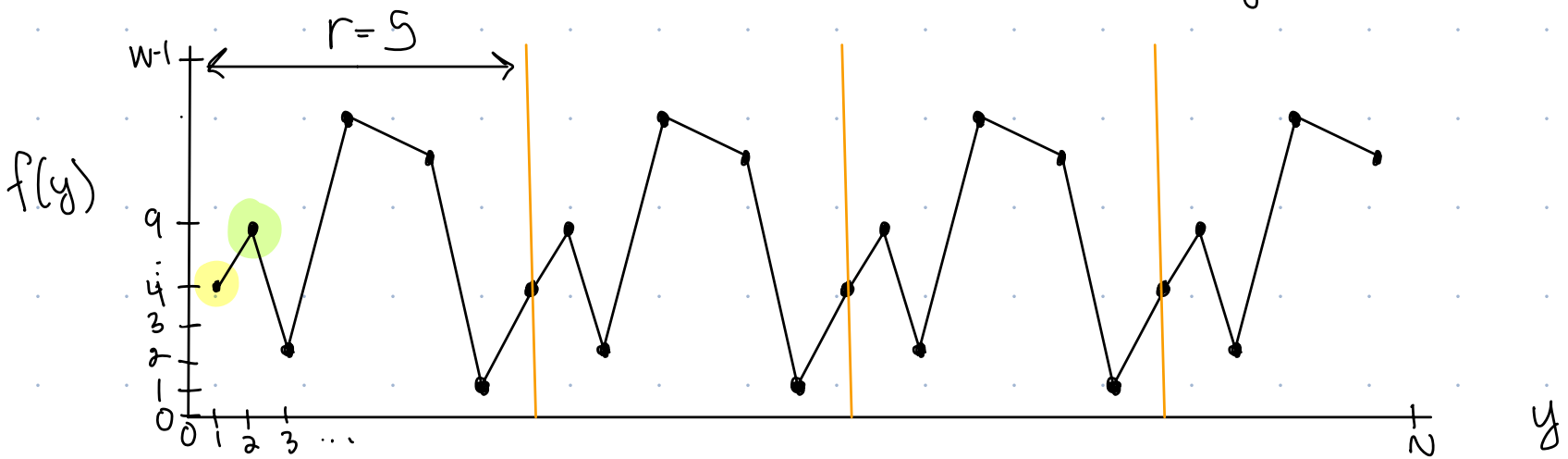
↑ W-dim standard basis state

$|\psi_0\rangle =$

$|\psi_1\rangle =$

$|\psi_2\rangle =$

For the following example, what does state $|\psi_3\rangle$ collapse to if outcome is $|4\rangle$? If outcome is $|9\rangle$? What is pattern if get outcome $|f(b^*)\rangle$

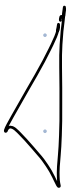


(outcome $f(b^*)$) $|\psi_3\rangle = \square \sum_{m=0}^{\square} |b^* \square\rangle$

Geometric Series Formula:

$$\sum_{m=0}^{t-1} a^m = \begin{cases} t & \text{if } a = 1 \\ \dfrac{1-a^t}{1-a} & \text{if } a \neq 1 \end{cases}$$

Reminder: $U\left(\sum_i a_i |i\rangle\right) = \sum_i a_i U|i\rangle$

 = Standard basis measurement

- Measure entire state:

$$|\psi\rangle = \sum_{x=0}^{N-1} a_x |x\rangle \Rightarrow \text{Prob of outcome } |x\rangle \text{ is } |a_x|^2$$

- Partial measurement (B system)

$$|\psi\rangle_{AB} = \sum_{x=0}^{N-1}\sum_{y=0}^{W-1} a_{xy}|x\rangle|y\rangle \Rightarrow \text{Prob of outcome } |y\rangle \text{ is}$$

$$P(|y\rangle) = \sum_{x=0}^{N-1} |a_{xy}|^2$$

$$\Rightarrow \text{Collapse to } \frac{1}{\sqrt{P(|y\rangle)}} \sum_{x=0}^{N-1} a_{xy}|x\rangle|\cancel{y}\rangle$$

Destroyed

only terms with that specific $|y\rangle$