

Error Correction

Learning Goals

- Describe errors in quantum/classical systems
- Describe repetition codes + why they work
- Analyze how a repetition code corrects errors

If Q. Computers are so great, why haven't we built one?

- Classical Computers

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Solution: Repetition Code

- Sources of Quantum Errors
 - control lasers not perfect
(shape, frequency, focus, intensity)
 - imperfect vacuum
 - Stray magnetic, electric fields
 - Heat (anomalous heating)

Why won't repetition code work for quantum?

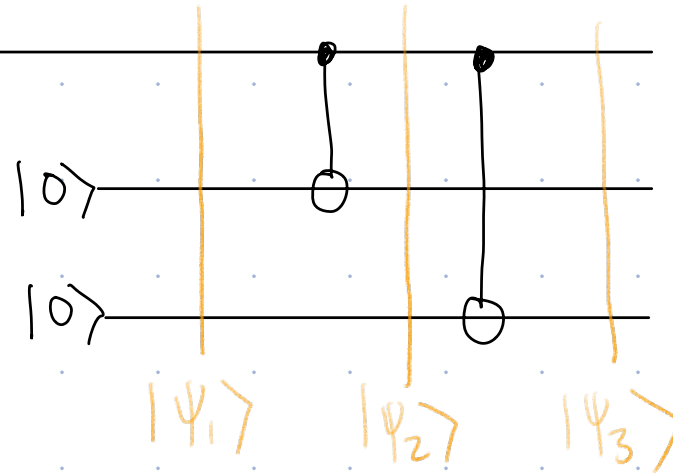
- A) MAJ not reversible
- B) Doesn't correct phase errors (Z)
- C) No fanout because can't copy quant
- D) There are uncountably infinitely many errors to deal with

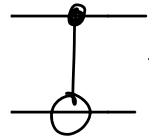
Shor's Repetition Code

Encoding:

$$|\psi\rangle = a|0\rangle + b|1\rangle$$

↑
original state
to protect from
errors. Don't
know a, b



A  = CNOT
B If $A = |1\rangle$,
 $\rightarrow X B$

$$|\psi_1\rangle = (a|0\rangle + b|1\rangle) |0\rangle |0\rangle = a|000\rangle + b|100\rangle$$

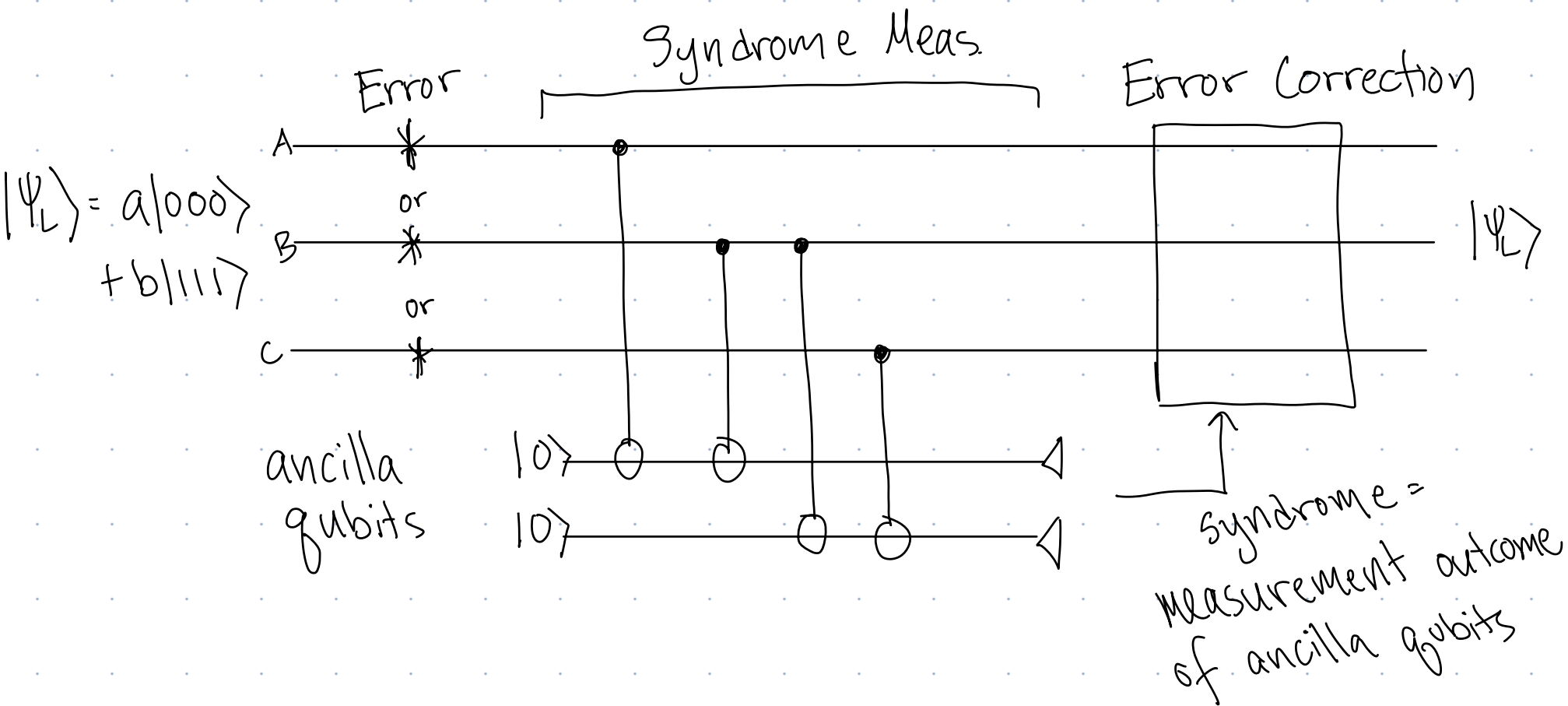
$$|\psi_2\rangle =$$

$$=$$

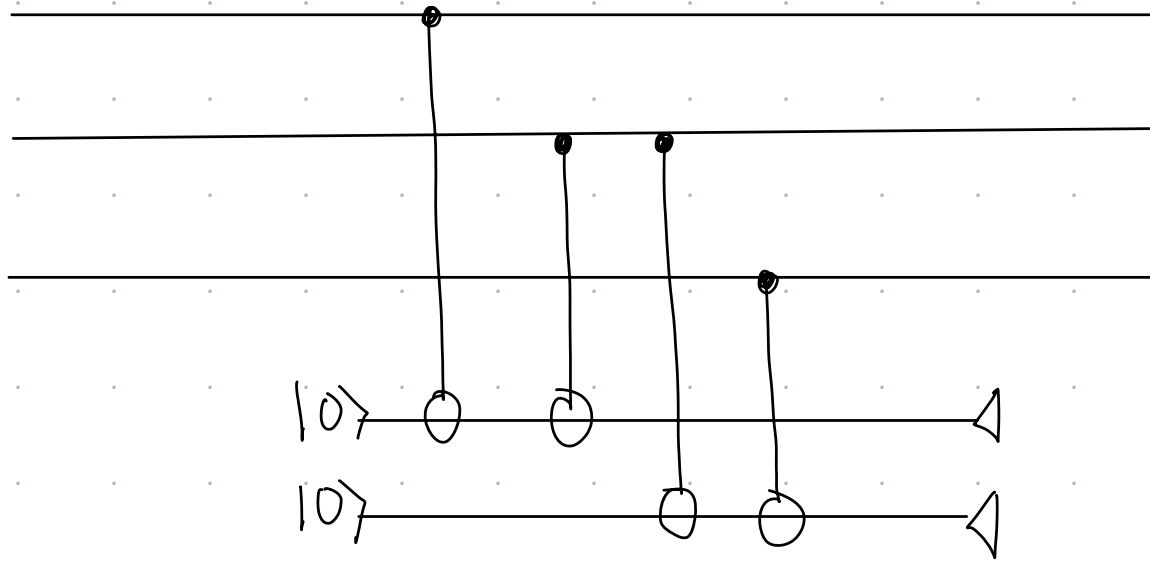
$$|\psi_3\rangle =$$

$$=$$

Error Correction Circuit



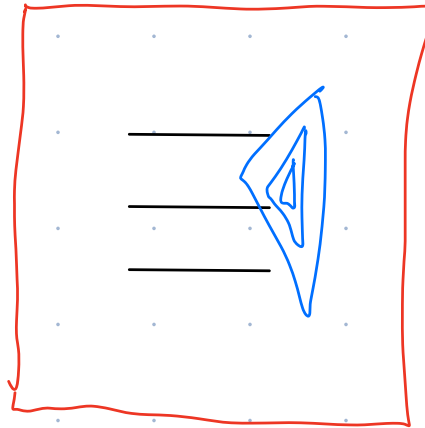
Syndrome Meas.



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Projective Measurement

1.



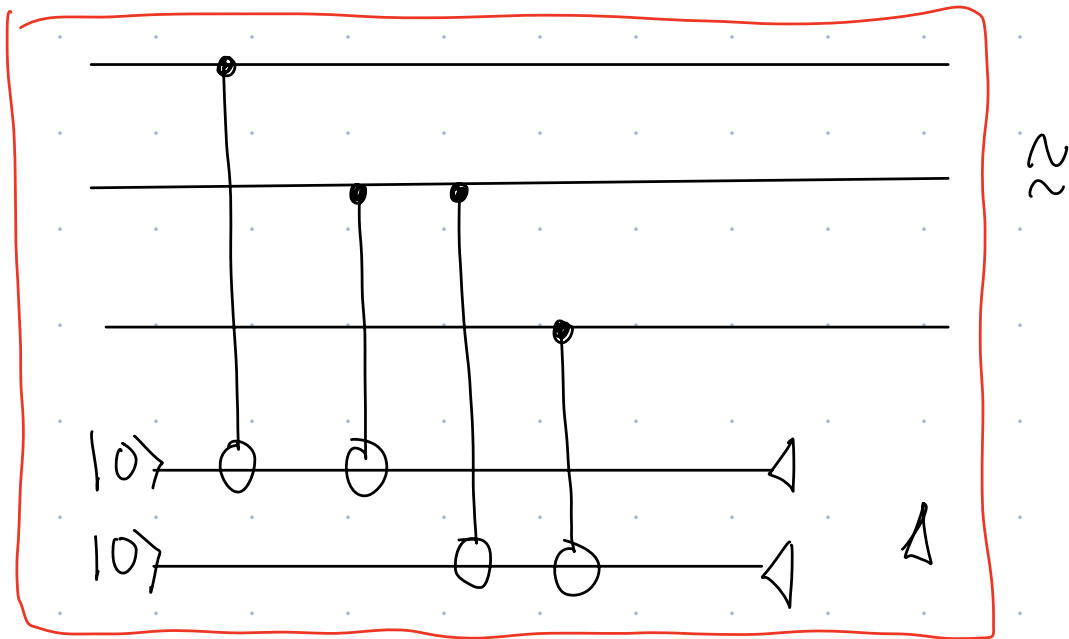
2.

3.

Projective Measurement:

4. Probability of outcome P_i

After outcome i , state collapses to



If measure $|\psi\rangle = \frac{1}{\sqrt{6}}|000\rangle + \sqrt{\frac{2}{6}}|001\rangle + \sqrt{\frac{3}{6}}|110\rangle$ with M ,
which outcomes are possible?

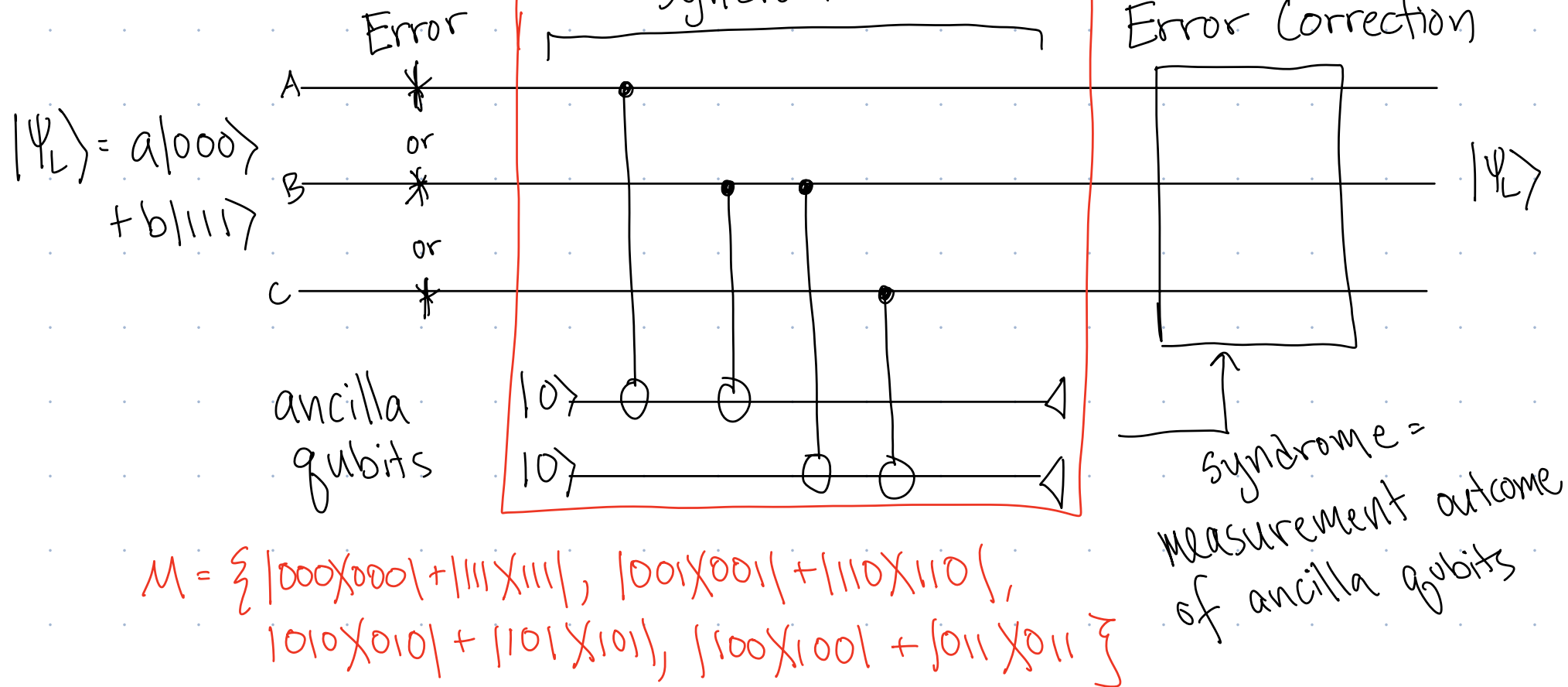
- A) P_0
- B) P_1
- C) P_2
- D) P_3

If measure $|\psi\rangle = \frac{1}{\sqrt{6}}|000\rangle + \sqrt{\frac{2}{6}}|001\rangle + \sqrt{\frac{3}{6}}|110\rangle$ with M

and get outcome $P_i = |001\rangle\langle 001| + |110\rangle\langle 110|$ what is

the resultant state? $|\psi\rangle \rightarrow \frac{P_i|\psi\rangle}{\sqrt{P(i)}} \quad P_r(i) = \langle\psi|P_i|\psi\rangle$

Group Exercise



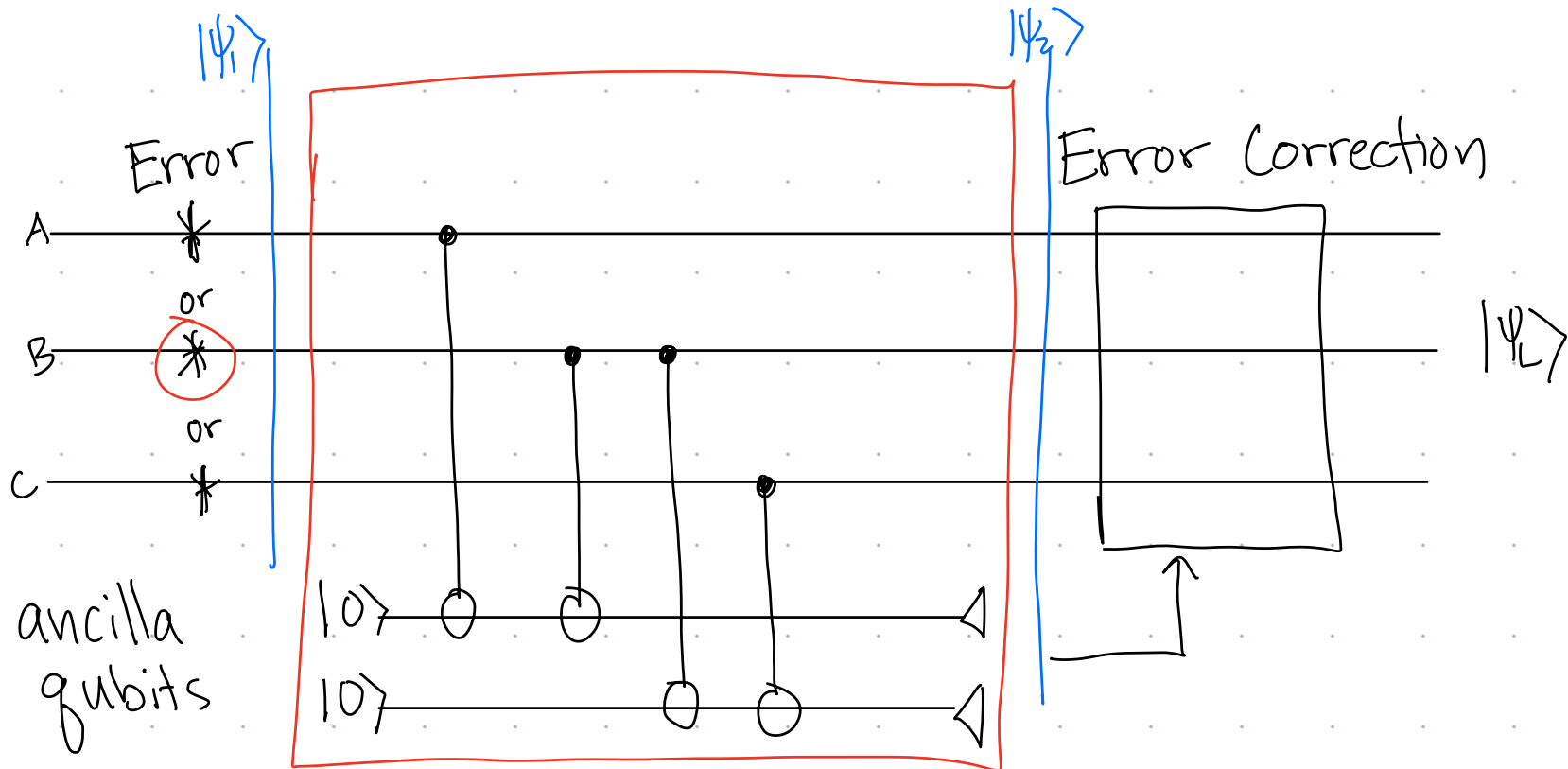
*: $R_x(\theta) = \cos\theta I + i\sin\theta X$

ex: $R_x(\theta)|1\rangle = \cos\theta I|1\rangle + i\sin\theta X|1\rangle = \cos\theta|1\rangle + i\sin\theta|0\rangle$

1. What are possible outcomes + what does state collapse to?
2. What error correction procedure should be used in each case?

Solution

$$|\psi_L\rangle = a|000\rangle + b|111\rangle$$



$$M = \{ |000\rangle\langle 000| + |111\rangle\langle 111|, |001\rangle\langle 001| + |110\rangle\langle 110|, \\ |101\rangle\langle 101| + |101\rangle\langle 101|, |100\rangle\langle 100| + |011\rangle\langle 011| \}$$

$$|\psi_1\rangle =$$

=

• Outcome

• Outcome

Error on A qubit:

Error on B qubit

Error on C qubit

Big Idea with Error Correction

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Need to Correct More Errors? Bigger Code!

$$|0_L\rangle = (|000\rangle_{123} + |111\rangle_{123}) (|000\rangle_{456} + |111\rangle_{456}) (|000\rangle_{789} + |111\rangle_{789})$$

$$|1_L\rangle = (|000\rangle - |111\rangle) (|000\rangle - |111\rangle) (|000\rangle - |111\rangle)$$

} 9-qubit
Shor Code

Projector onto

$$P_0 =$$

$$P_{x1} =$$

$$P_{z1} =$$

$$P_{y1} =$$

Correct With

2
b
,

Need to Correct More Errors? Bigger Code!

$$|0_L\rangle = (|000\rangle_{123} + |111\rangle_{123}) (|000\rangle_{456} + |111\rangle_{456}) (|000\rangle_{789} + |111\rangle_{789})$$

$$|1_L\rangle = (|000\rangle - |111\rangle) (|000\rangle - |111\rangle) (|000\rangle - |111\rangle)$$

} 9-qubit
Shor Code



Corrects any
single qubit
error

Which errors can be corrected by Shor's code?

$$|0_L\rangle = \left(|000\rangle_{123} + |111\rangle_{123} \right) \left(|000\rangle_{456} + |111\rangle_{456} \right) \left(|000\rangle_{789} + |111\rangle_{789} \right)$$

$$|1_L\rangle = \left(|000\rangle - |111\rangle \right) \left(|000\rangle - |111\rangle \right) \left(|000\rangle - |111\rangle \right)$$

A) $Z_1 Z_2$

E) $X_1 X_2$

B) $Z_4 Z_8$

F) $X_1 X_4$

C) $Z_1 Z_2 Z_3$

G) $Y_1 Z_4$

D) $X_1 Z_2$