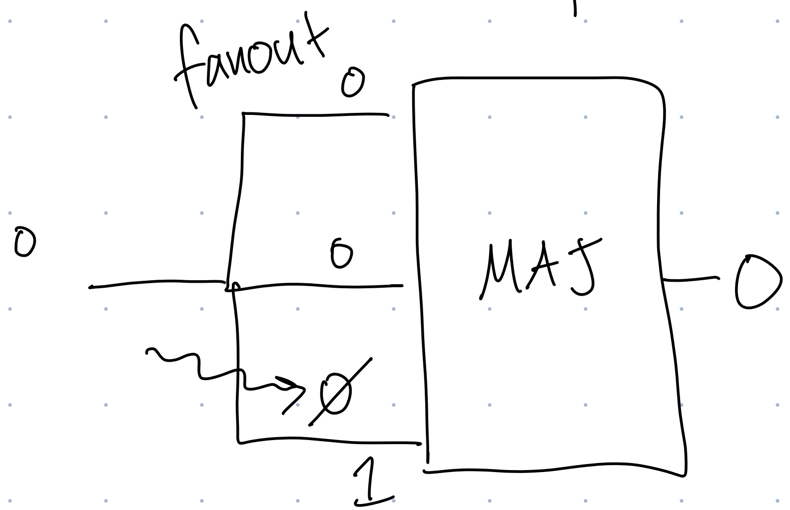
If Q. Computers are so great, why haven't we built one?

Errors - unwanted gates/measurements

- Classical Computers
  - cosmic rays "soft errors"
    - ↳ 1 bit flip / 4 GB per day

Solution: <u>Repetition Code</u>

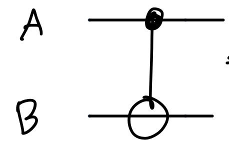


- Sources of Quantum Errors
  - control lasers not perfect
    (shape, frequency, focus, intensity)
  - imperfect vacuum
  - Stray magnetic, electric fields
  - Heat (anomalous heating)

Why won't repetition code work for quantum?

A) MAJ not reversible
B) Doesn't correct phase errors (Z)
C) No fanout because can't copy quantum states
D) There are uncountably infinitely many errors to deal with

Shor's Repetition Code

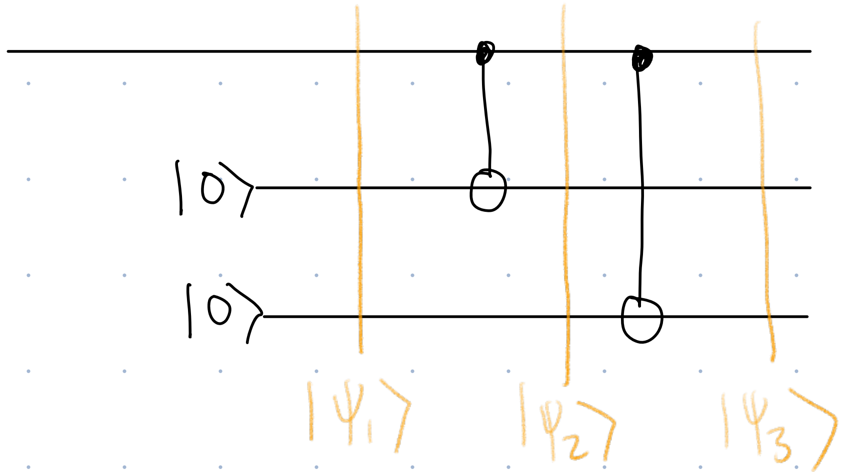


= CNOT

If $A = |1\rangle$, $\rightarrow XB$

Encoding:

$$|\psi\rangle = a|0\rangle + b|1\rangle$$



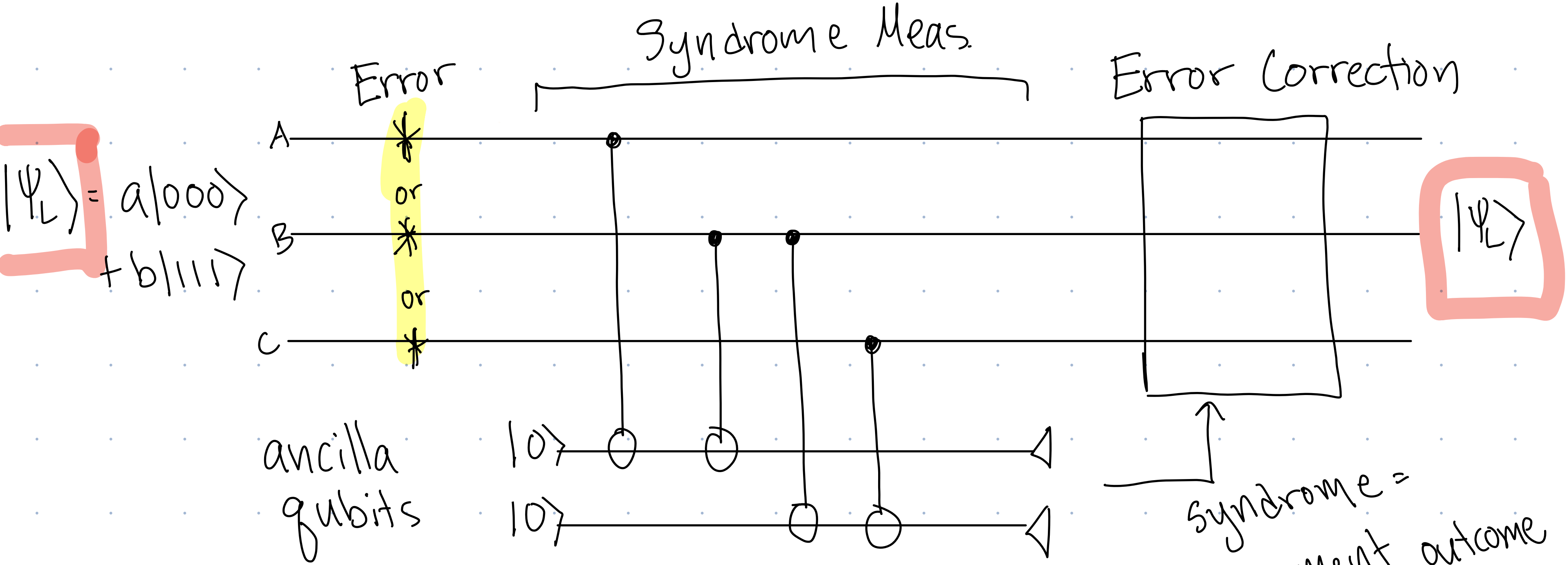
original state to protect from errors. Don't know a, b.

$$|\psi_1\rangle = (a|0\rangle + b|1\rangle)|0\rangle|0\rangle = a|000\rangle + b|100\rangle$$

$$|\psi_2\rangle = a\,CNOT_{12}|000\rangle + b\,CNOT_{12}|100\rangle$$
$$= a|000\rangle + b|110\rangle$$

$$|\psi_3\rangle = a\,CNOT_{13}|000\rangle + b\,CNOT_{13}|110\rangle$$
$$= a|000\rangle + b|111\rangle$$

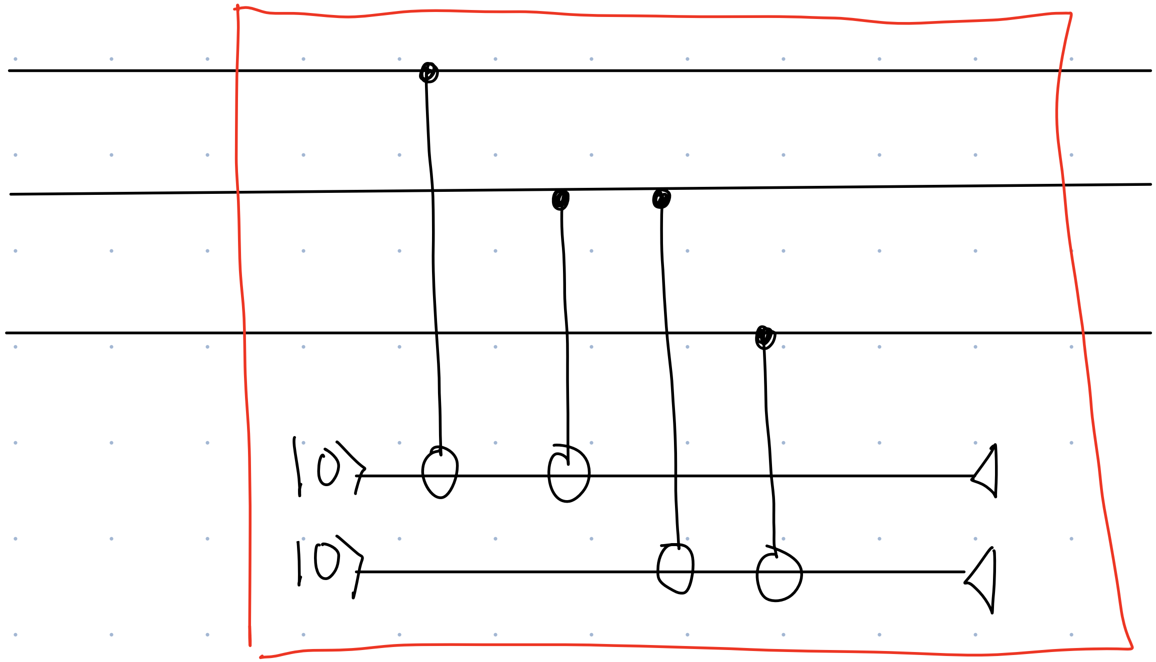
Error Correction Circuit



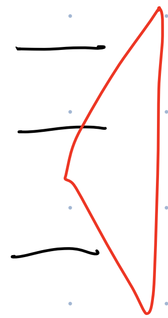
syndrome = measurement outcome of ancilla qubits

Syndrome Meas.

Projective Meas.



$\approx$



Projective Measurement

Described by a set of projectors $M = \{P_0, P_1, P_2, \dots\}$

outcomes

ex: $P_0 = |\phi_1\rangle\langle\phi_1| + |\phi_4\rangle\langle\phi_4|$

$P_1 = |\phi_3\rangle\langle\phi_3|$

$P_2 = |\phi_2\rangle\langle\phi_2| + |\phi_5\rangle\langle\phi_5| + |\phi_6\rangle\langle\phi_6|$

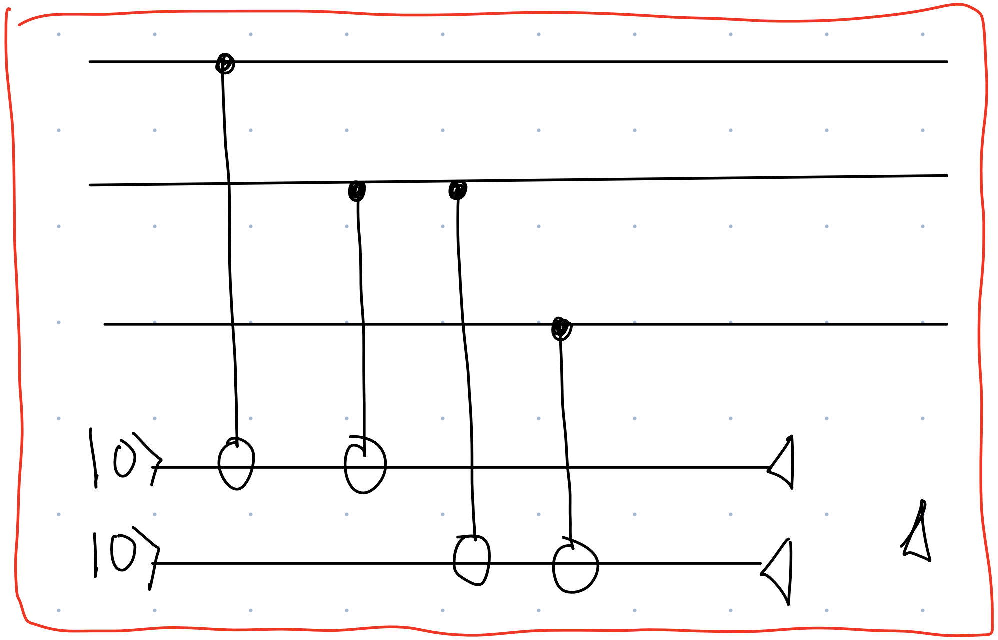
$|\phi\rangle\langle\phi|$ write as $|\phi\rangle\langle\phi|$

- $|\phi_i\rangle$ should be N-dimensional, where N is dim. of system being measured
- Technical $\sum P_i = I$

If measure $|\psi\rangle$

→ Probability of outcome P_i: $\langle\psi|P_i|\psi\rangle$

→ If outcome i: collapse to $\dfrac{P_i|\psi\rangle}{\sqrt{Pr(i) = \langle\psi|P_i|\psi\rangle}}$



$\approx$

$$M = \{ |000\rangle\langle 000| + |111\rangle\langle 111|, \quad |001\rangle\langle 001| + |110\rangle\langle 110|, \quad |010\rangle\langle 010| + |101\rangle\langle 101|, \quad |100\rangle\langle 100| + |011\rangle\langle 11| \}$$

P_0, P_1, P_2, P_3

$$\langle \psi | P | \psi \rangle$$

If measure $|\psi\rangle = \frac{1}{\sqrt{6}}|000\rangle + \sqrt{\frac{2}{6}}|001\rangle + \sqrt{\frac{3}{6}}|110\rangle$ with M, which outcomes are possible?

A) P_0

B) P_1

C) P_2

D) P_3