ENTANGLEMENT + CHSH

Goals

- Describe entangled states + product states
- Determine if a 2-qubit state is entangled
- Describe why entanglement helps us win CHSH
- Analyze 2-qubit systems

Announcements

- In class exam Thursday QI1, QI2 (15 min, no notes)
- Review where to find notes on Canvas
- OH Today 12:30-1:30

conv: meas in bra

$$|\langle \psi | \phi \rangle| = |\langle \phi | \psi \rangle|$$

Exit Tickets

Exit Tickets

- How many qubit states are there?

measurement outcome

A) 2   B) 4 $|0\rangle, |1\rangle$ $|+\rangle, |-\rangle$ $|\circlearrowright\rangle$ $|\circlearrowleft\rangle$ C) COUNTABLY ∞ D UNCOUNTABLY ∞

- Does this represent a qubit state?

$$|\psi\rangle = \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle$$

$$\left|\frac{1}{2}\right|^2 + \left|\frac{1}{2}\right|^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \neq 1$$

A) YES   B) NO   C) Maybe   D) IDK

- When $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ vs $\cos\theta|0\rangle + \sin\theta|1\rangle$?

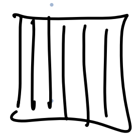
$45°$

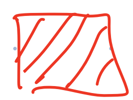
- Does orientation/order of $|0\rangle$, $\langle 1|$ matter?

$|0\rangle\langle 1|$

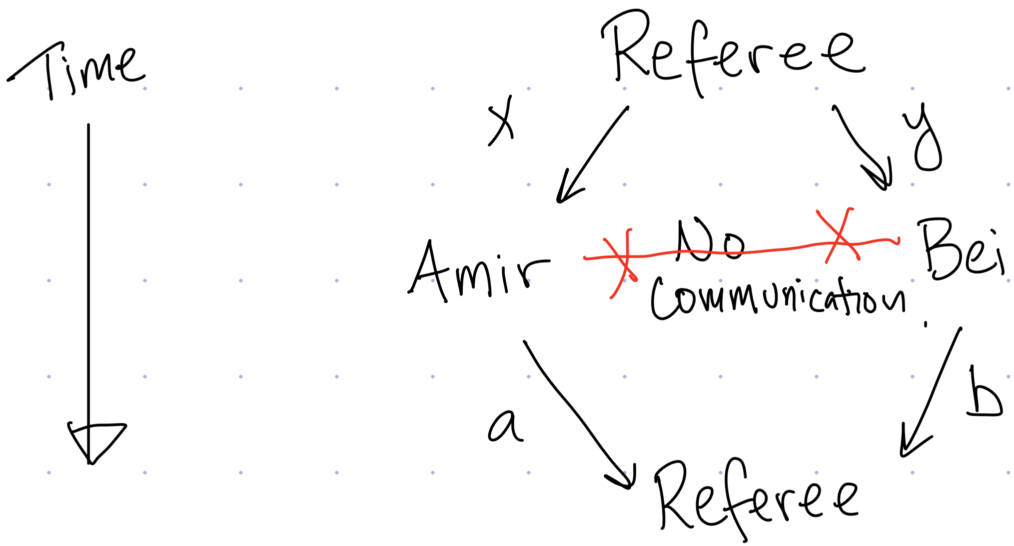
$|0\rangle|1\rangle$

$\langle 0|1\rangle$

- Bra physical?   Bra name?
-  vs $\{|0\rangle, |1\rangle\}$
- No sines/cosines on exam (or will provide)

 $\{|+\rangle, |-\rangle\}$

CHSH Game (Clauser, Horne, Shimony, Holt) (Bell Inequality)



$a, b, x, y \in \{0, 1\}$

Amir + Bei win:

x,y	winning condition
00, 01, 10	$a = b$
11	$a \neq b$

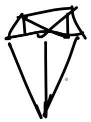
PSet: show w/o quantum best is 75% win rate

What if we give them each a qubit?

Diamond nitrogen vacancy qubit

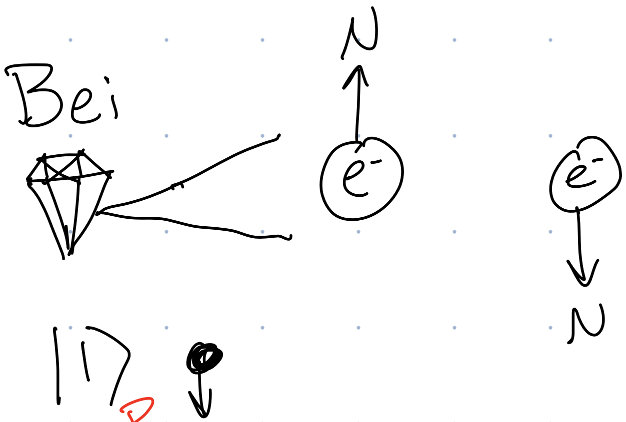
~ 1 second coherence

Amir



$|0\rangle_A$

Bei



$|1\rangle_B$

How do we represent the combined state of both qubits?

✓ A: $|0\rangle \otimes |1\rangle$ — Tensor Product

✓ B: $|0\rangle_A |1\rangle_B$ — implied tensor product

✓ C: $\overset{A\,B}{|01\rangle}_{AB}$ — Combine into 1 ket

✓ D: $\overset{A\,B}{|01\rangle}$

1st qubit "A" qubit is $|0\rangle$

inner product

$\langle 0| \rightarrow\leftarrow |1\rangle \rightsquigarrow \langle 0|1\rangle \rightarrow 0$ (number)

$|0\rangle \rightarrow\leftarrow |1\rangle \Rightarrow |01\rangle$

tensor product   Bigger kets

"Standard basis states"

Generic 2 Qubit State:

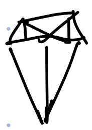
$$|\psi\rangle_{AB} = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$$

amplitudes

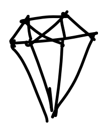
$$\sum_{i \in \{0,1\}^2} |a_i|^2 = 1 \qquad a_i \in \mathbb{C}$$

Strategy:

Amir



Bei

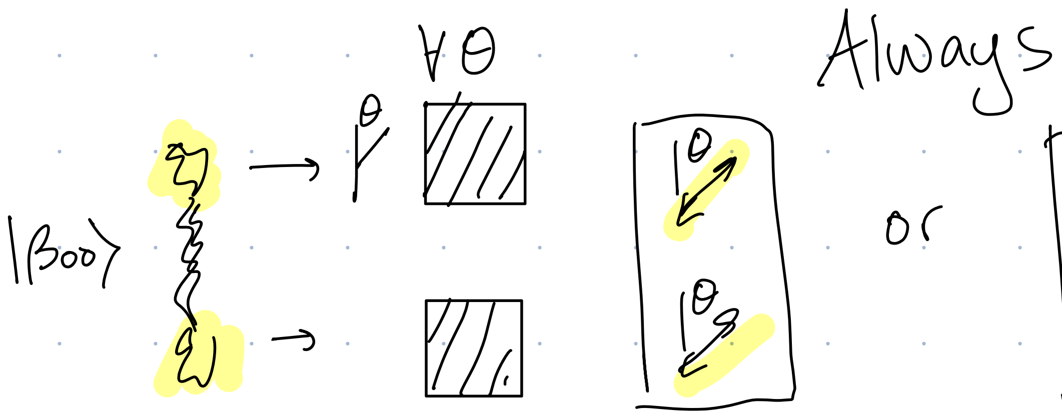


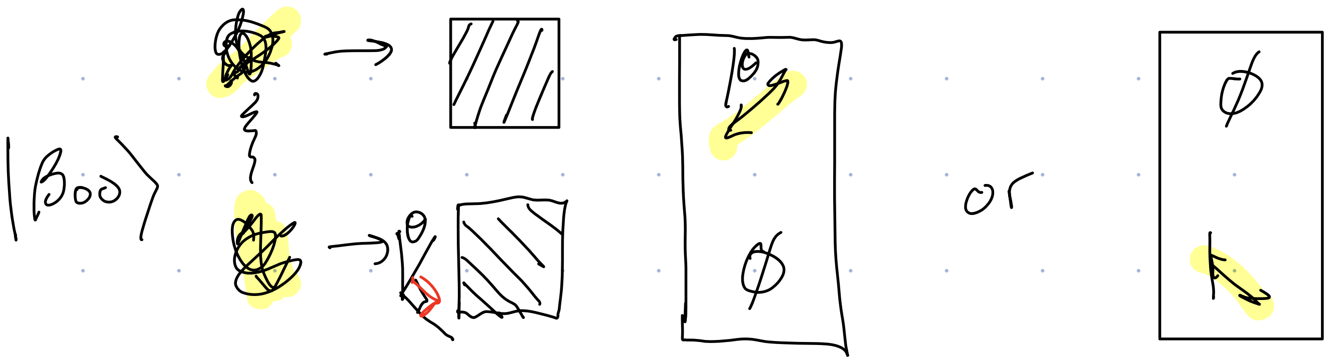
$$a_{00} = a_{11} = \frac{1}{\sqrt{2}}$$

$$a_{01} = a_{10} = 0$$

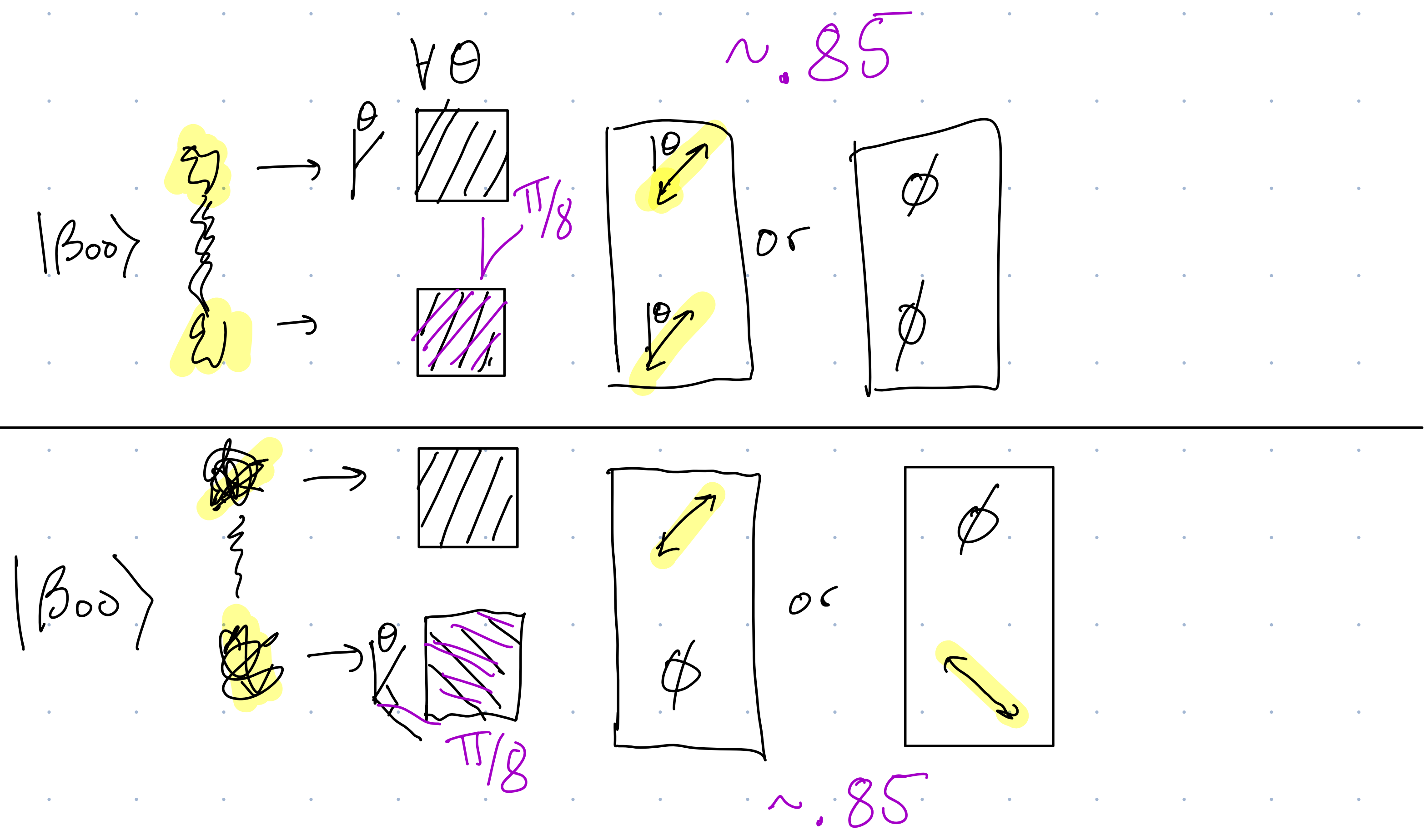
$$|\beta_{00}\rangle_{AB} = \frac{1}{\sqrt{2}}|00\rangle_{AB} + \frac{1}{\sqrt{2}}|11\rangle_{AB}$$

$\frac{1}{\sqrt{2}}|00\rangle_{AB} + \frac{1}{\sqrt{2}}|11\rangle_{AB}$ is Entangled!





$\frac{1}{\sqrt{2}}|00\rangle_{AB} + \frac{1}{\sqrt{2}}|11\rangle_{AB}$ is <u>Entangled</u>!



Fav. Media

CHSH Game

Referee

x ↙ ↘ y

Amir → ~~X~~ No comms ~~X~~ ← Bei

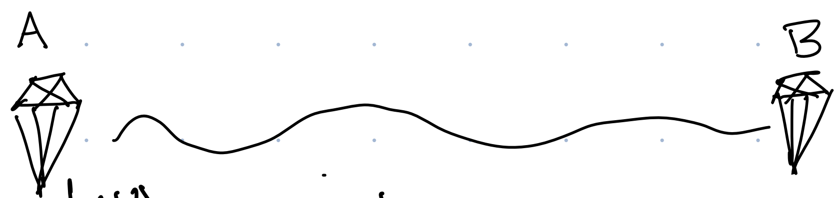
a ↘ ↙ b

Referee

Amir + Bei win

x, y	if
00, 01, 10	$a = b$
11	$a \neq b$

Strategy

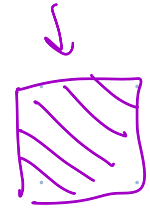


$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |11\rangle_{AB}\right)$$

Success Prob? Why? ~85%

Amir Receives	Amir does measurement
$x = 0$	0: → ↕ $a = 0$; → ϕ $a = 1$
$x = 1$	$\pi/4$: → ⤢ $a = 0$; → ϕ $a = 1$

Bei Receives	Bei does measurement
$y = 0$	$\pi/8$: → ⤢ $b = 0$; → ϕ $b = 1$
$y = 1$	$-\pi/8$: → ⤡ $b = 0$; → ϕ $b = 1$

$-\frac{\pi}{4}$ 

Quantum Entanglement

def:

A state $|\psi\rangle_{AB}$ is a product state if $\exists\ |\psi_1\rangle, |\psi_2\rangle$ s.t. $|\psi\rangle_{AB} = |\psi_1\rangle_A |\psi_2\rangle_B$

A state $|\psi\rangle_{AB}$ is entangled if $\nexists\ |\psi_1\rangle, |\psi_2\rangle$ s.t. $|\psi\rangle_{AB} = |\psi_1\rangle_A |\psi_2\rangle_B$

There are valid 2-qubit states that can't be described as A system in a state + B system in a state.

Similar to classical correlation $\Longrightarrow$

$$Pr(00) = \frac{1}{2}$$

$$Pr(11) = \frac{1}{2}$$

Qubit A — $|\psi_1\rangle_A = a_0|0\rangle + a_1|1\rangle$

Qubit B — $|\psi_2\rangle_B = b_0|0\rangle + b_1|1\rangle$

$|0\,1\rangle$

Combined: $|\psi\rangle_{AB} = |\psi_1\rangle_A \otimes |\psi_2\rangle_B = |\psi_1\rangle|\psi_2\rangle = \cancel{|\psi_1\psi_2\rangle}$

mult

$$= (a_0|0\rangle + a_1|1\rangle)_A (b_0|0\rangle + b_1|1\rangle)_B$$

$$= a_0b_0|0\rangle_A|0\rangle_B + a_0b_1|0\rangle_A|1\rangle_B + a_1b_0|1\rangle_A|0\rangle_B + a_1b_1|1\rangle|1\rangle$$

$$= a_0b_0|00\rangle_{AB} + a_0b_1|01\rangle_{AB} + a_1b_0|10\rangle_B + a_1b_1|11\rangle$$

Let $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

Prove $|\beta_{00}\rangle$ is entangled.

$a|0\rangle + b|1\rangle$

Pf: Assume for contradiction $|\beta_{00}\rangle$ is not entangled.

$$\boxed{\text{A state } |\psi\rangle_{AB} \text{ is not entangled if } \exists\, |\psi_1\rangle, |\psi_2\rangle \text{ s.t. } |\psi\rangle_{AB} = |\psi_1\rangle_A |\psi_2\rangle_B}$$

$ad = 0 \Rightarrow a = 0$ or $d = 0$

$c|0\rangle + d|1\rangle$

① If have 2-qubit state $|\psi\rangle_{AB}$ and measure A qubit with $M_A = \{|\alpha_0\rangle, |\alpha_1\rangle\}$ and B qubit with
$M_B = \{|\beta_0\rangle, |\beta_1\rangle\}$

③

Get outcome with prob.

State collapses to

② How many measurement outcomes are there?

A) 2 (M_A and M_B) B) 4 ($|\alpha_0\rangle|\beta_0\rangle, |\alpha_0\rangle|\beta_1\rangle, \ldots$)

C) Not enough information

① If have 2-qubit state $|\psi\rangle_{AB}$ and measure A qubit with $M_A = \{|\alpha_0\rangle, |\alpha_1\rangle\}$ and B qubit with $M_B = \{|\beta_0\rangle, |\beta_1\rangle\}$

③ Get outcome $|\alpha_i\rangle_A |\beta_j\rangle_B$ with prob. $\left| \langle \alpha_i|_A \langle \beta_j|_B |\psi\rangle_{AB} \right|^2$, state collapses to $|\alpha_i\rangle_A |\beta_j\rangle_B$

$$\left| \langle \psi|_{AB} |\alpha_i\rangle_A |\beta_j\rangle_B \right|^2 = \left| \langle \alpha_i|_A \langle \beta_j|_B |\psi\rangle_{AB} \right|^2$$

④ Suppose Amir + Bei cannot communicate, but share $|\psi\rangle_{AB}$ (unknown state). If Amir measures $M_A = \{|0\rangle, |1\rangle\}$ and Bei measures $M_B = \{|+\rangle, |-\rangle\}$ and they get outcome $|0\rangle_A |+\rangle_B$, what does Amir know about the two qubits after the measurements if no communication?

A) Nothing

B) A is in $|0\rangle$, B unknown

C) A is in $|0\rangle$, B in $|+\rangle$

Example: $|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

$$\left| {}_A\langle\alpha_i| \, {}_B\langle\beta_j| \, |\psi\rangle_{AB} \right|^2$$

$$M_A = \{|+\rangle, |-\rangle\} \quad M_B = \{|+\rangle, |-\rangle\}$$

Probability of $|+\rangle_A |-\rangle_B$?

Practice: Start PS3 #4a