# Learning Goals

- Design + analyze a quant. algorithm
- Read circuit diagrams
- Describe time + query complexity

Deutsch's algorithm analyzes a 1-bit function f:

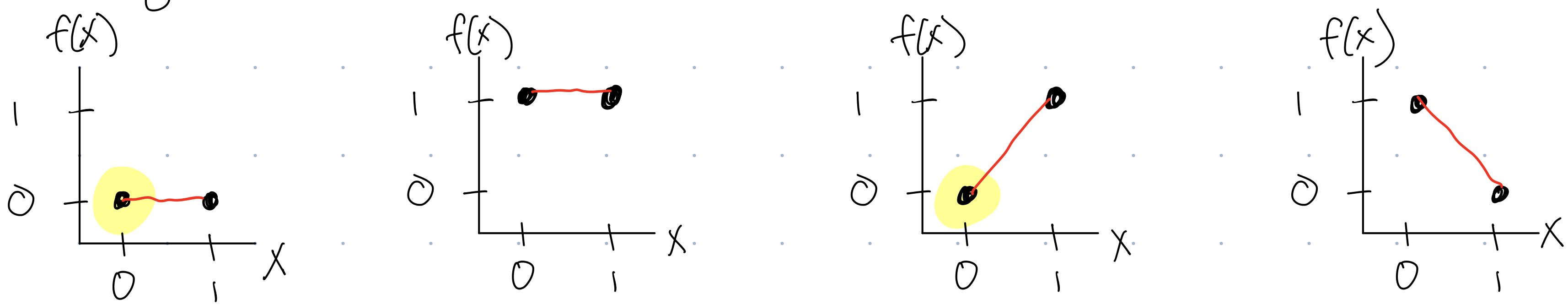
| $x$ | $f(x)$ |
|---|---|
| 0 | $f(0)$ |
| 1 | $f(1)$ |

$$f(0), f(1) \in \{0,1\}$$

ex: $f(x) = \begin{cases} 1 & \text{if it will rain at time } x \\ 0 & \text{if it will not rain at time } x \end{cases}$

$x = \begin{cases} 0 \rightarrow \text{day} \\ 1 \rightarrow \text{night} \end{cases}$

<u>Only 4 1-bit Functions:</u>



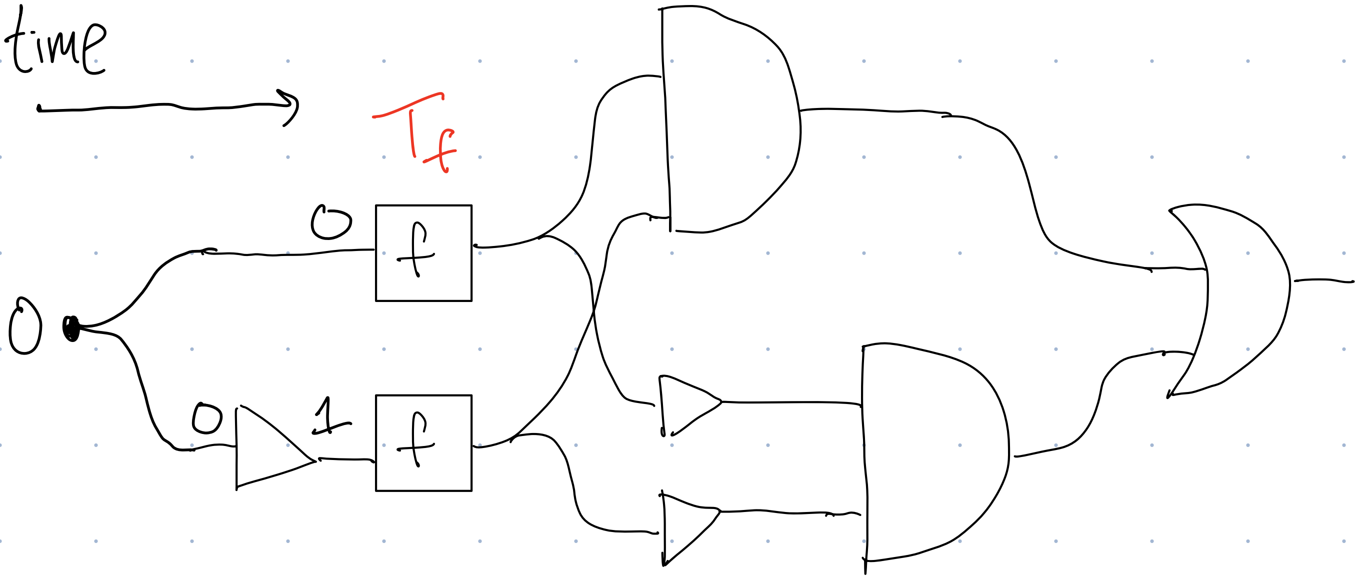

flat (even)

sloped (balanced)

**Deutsch's Problem**: Given query access to f, determine if flat or sloped

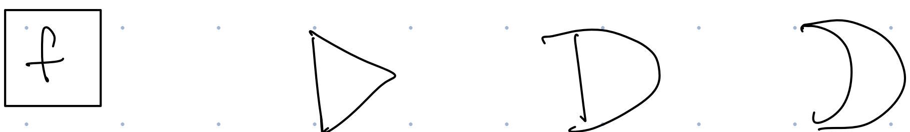
## A Classical Algorithm (circuit diagram)

time →




- time goes left → right
- wires are bits
- shapes are gates

1 ⇒ flat
0 ⇒ sloped



"query to f"    NOT    AND    OR

\# of times the gate f is used

**Query Complexity** = 2    **Time Complexity** = time to run

$2T_f + 6$ // $T_f + 3$

We would like to determine if $f$ is flat or sloped using as few queries as possible.

What is the minimum number of classical queries to $f$ needed to determine flat/sloped

A) 0 B) 1 C) 2

# Deutsch's Problem

- Classical Query Complexity: 2
- Quantum Query Complexity: 1

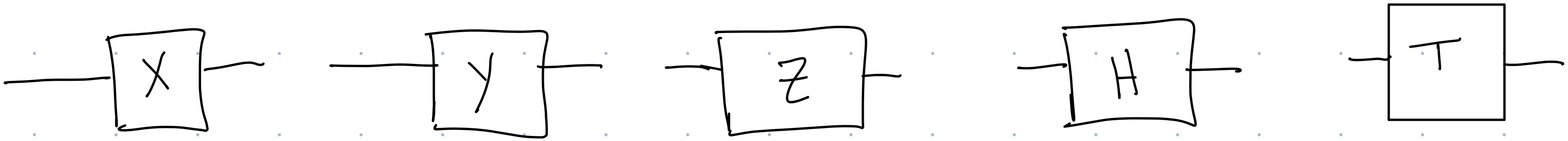
Why do we care?!

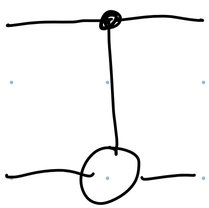
# Quantum Gates (Circuit picture)

- time L → R
- wires are qubits
- Gates + measurements are shapes

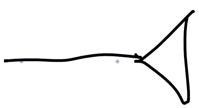
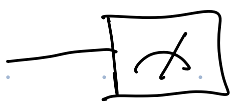
$|0\rangle$ 

Initialize each qubit $|0\rangle$ (usually)



CNOT

 or 

measure in standard basis

$M = \{|0\rangle, |1\rangle\}$

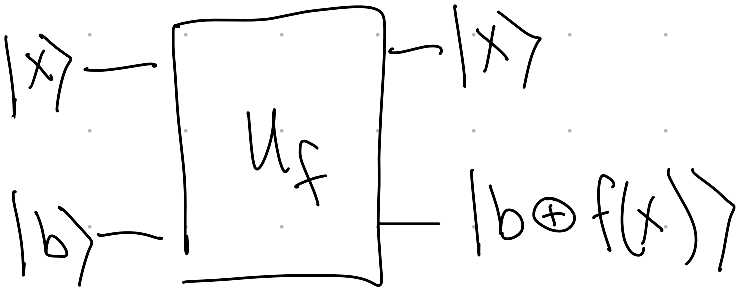
Need gate for $f$. What about:

$$|0\rangle \to |f(0)\rangle$$
$$|1\rangle \to |f(1)\rangle$$

Explain why this is not an allowed gate.

* State → State
* Reversible

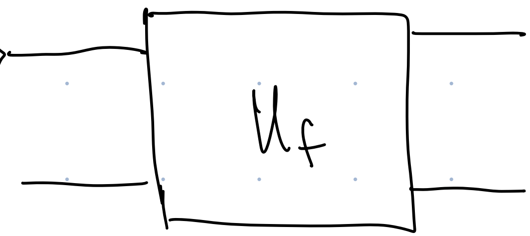
Instead:



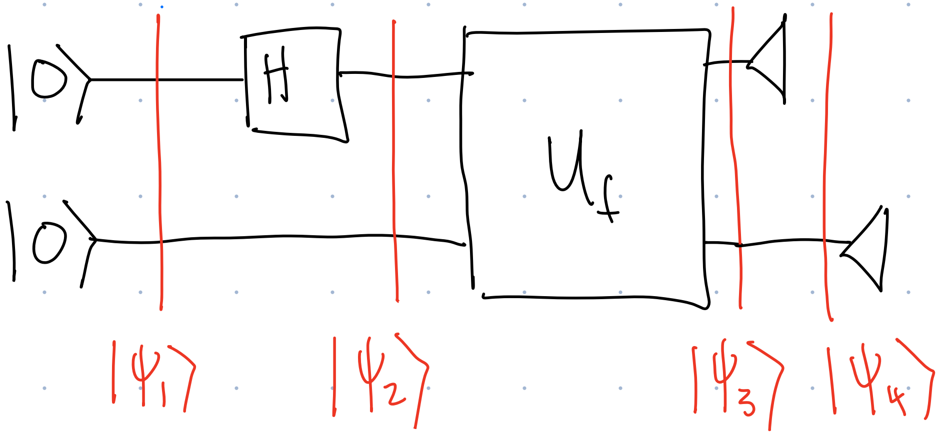

If $x=0$ get info about $f(0)$
If $x=1$ get info about $f(1)$

Idea – Superposition!

$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$



# Analyzing a Quantum Circuit




$|\psi_1\rangle = |0\rangle|0\rangle$

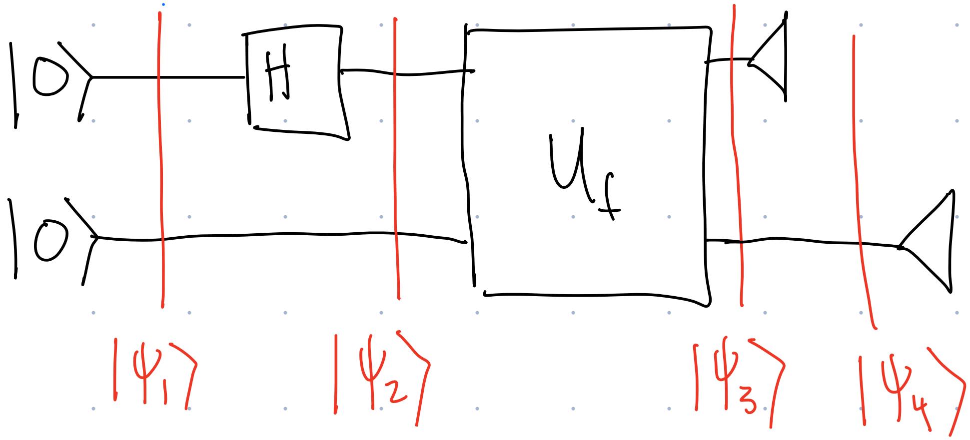
$|\psi_2\rangle =$

$|\psi_3\rangle =$

$=$

$=$

$=$




Partial Measurement:

Outcome $|0\rangle$

- Prob
- Collapse

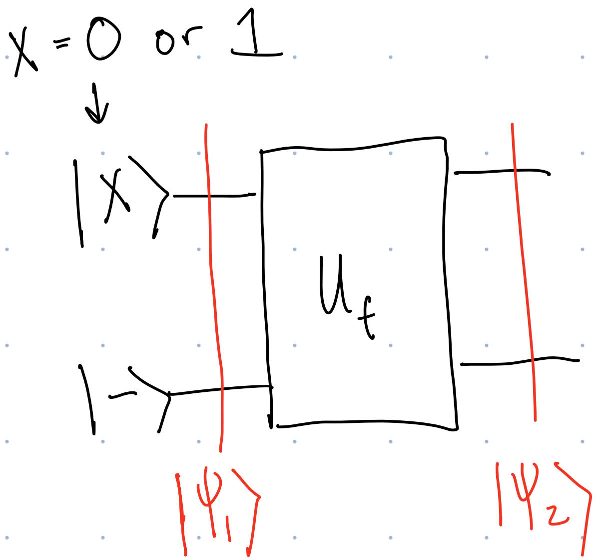
Outcome $|1\rangle$

- Prob
- Collapse

## Mini-Hype Lesson

You need more than superposition to get a quantum advantage.

## Group Exercise:

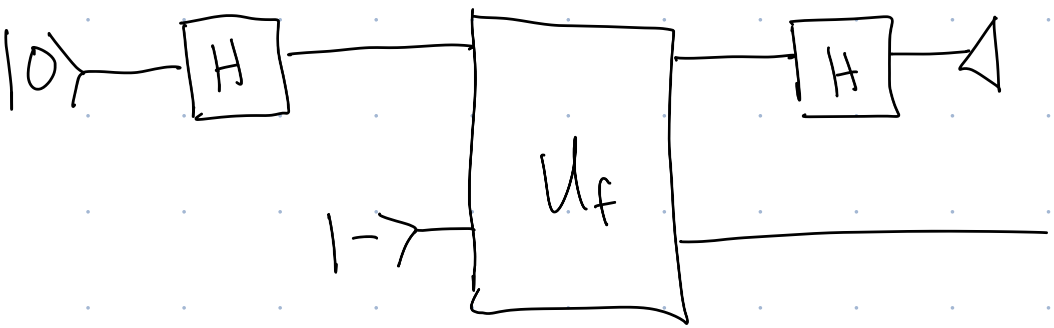



Show that $|\psi_2\rangle = (-1)^{f(x)} |x\rangle |-\rangle$

If $f(x) = 0$

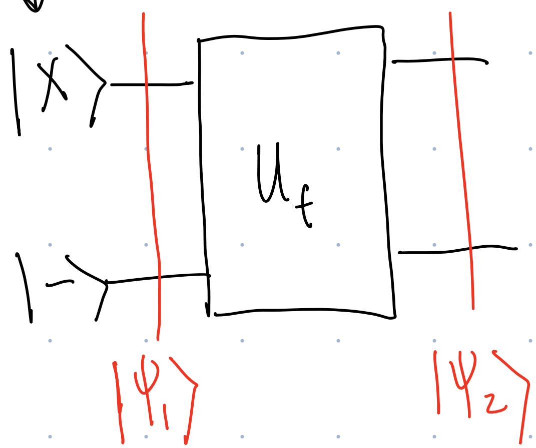
If $f(x) = 1$

Analyze outcome of this circuit:



# Solution

$x = 0$ or $1$



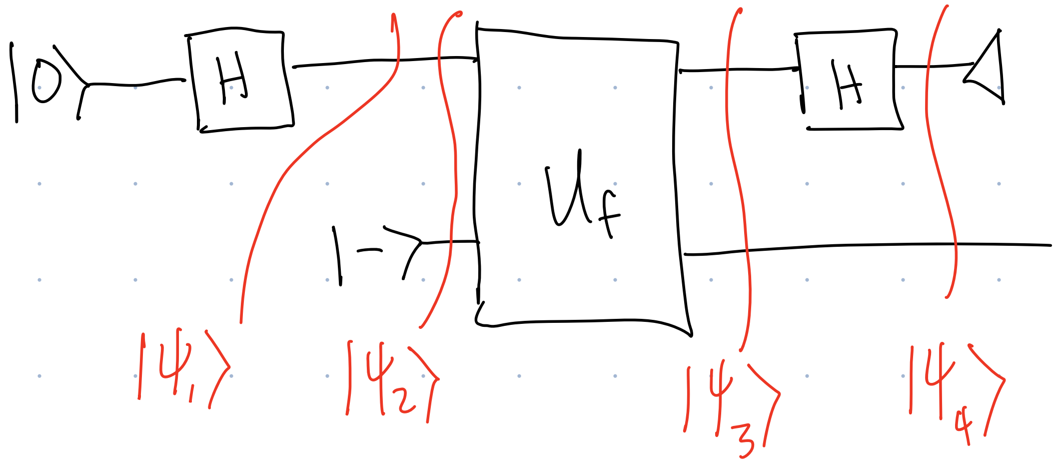

Show that $|\psi_2\rangle = (-1)^{f(x)} |x\rangle|-\rangle$

$|\psi_2\rangle = U_f |x\rangle|-\rangle$

## New rule for $U_f$:

# Phase Kickback

$U_f : |x\rangle|-\rangle \rightarrow (-1)^{f(x)}|x\rangle|-\rangle$




$|\psi_1\rangle =$

$|\psi_2\rangle =$

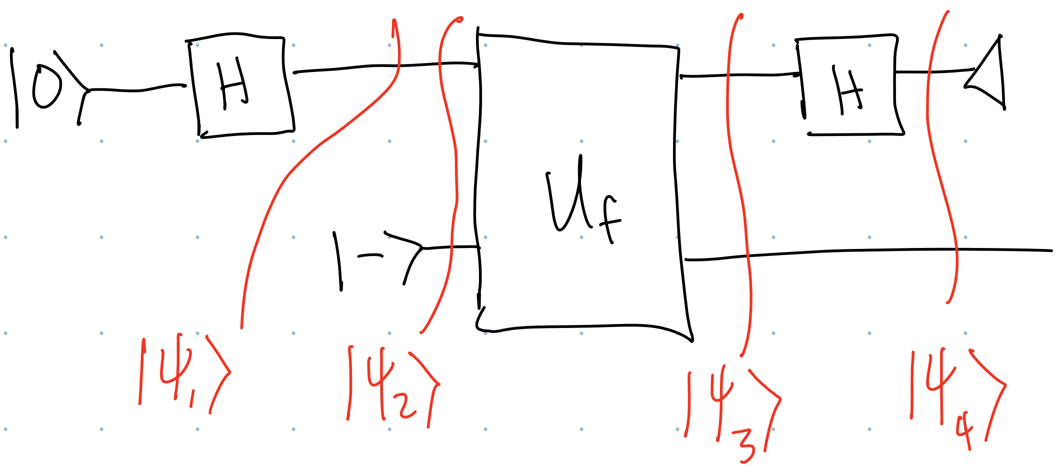
$|\psi_3\rangle =$

$=$

$=$

$=$

Deutsch's Alg:



# Phase Kickback