Learning Goals

- Describe quantum operations
- Describe partial q. measurements
- Use rules of q. ops + partial meas. to analyze both abstract and in interferometer scenarios

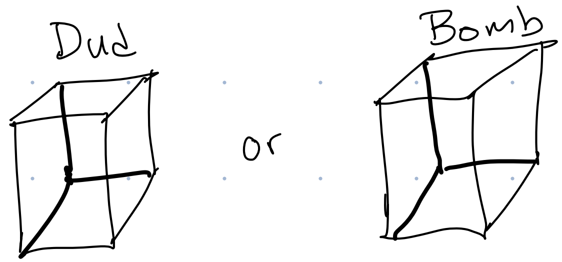
Exit Tickets:

- Local + Global Phases
- Beamsplitter … for real?
- In a quantum computation: gates versus measurement

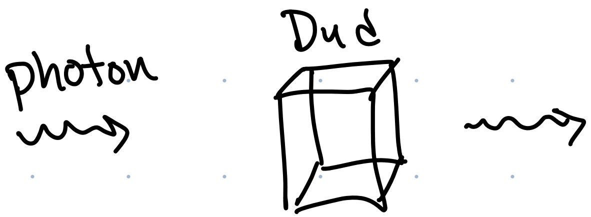
  no info ↑ (gates) info ↑ (measurement)

- Separated qubits + faster than light communication
- Qubits needed to describe a photon (fully)
- Gates help eavesdropper?

Problem: Quantum Bomb Detection (Elitzur-Vaidman)



Dud:

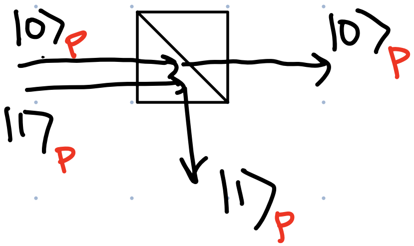


(passes through

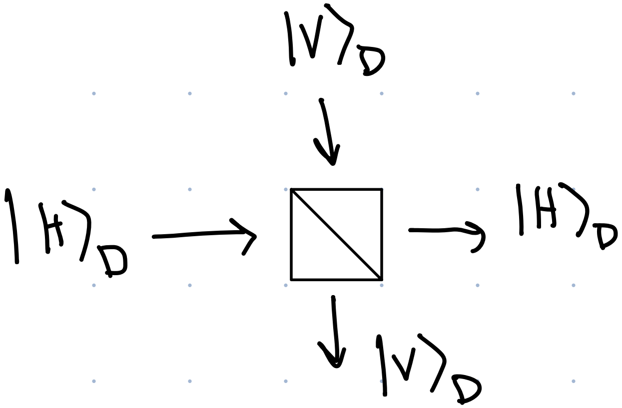
Bomb :

Want to give friend a rainbow bomb not a dud. How??

New Tool: Beamsplitter



- Transmist if photon $|0\rangle_P$
- Reflects if $|1\rangle_P$



"P" for polarization

"D" for direction

$\{|V\rangle_D, |H\rangle_D\}$ standard basis states

Behavior:

$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$

$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$

$|0\rangle_P|V\rangle_D \rightarrow \boxed{|0\rangle_P|V\rangle_D}$

$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$

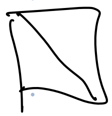
A) $|0\rangle_P|H\rangle_D$ B) $|0\rangle_P|V\rangle_D$ C) $|1\rangle_P|H\rangle_D$ D) $|1\rangle_P|V\rangle_D$

Quantum Gate

Operation that changes a quantum state without collapse.
We will describe gates using their effect on s.b. states

$|0\rangle, |1\rangle$

$|00\rangle, |01\rangle \ldots$



Beam-Splitter

$$|0\rangle_P |H\rangle_D \rightarrow |0\rangle_P |H\rangle_D$$
$$|1\rangle_P |H\rangle_D \rightarrow |1\rangle_P |V\rangle_D$$
$$|0\rangle_P |V\rangle_D \rightarrow |0\rangle_P |V\rangle_D$$
$$|1\rangle_P |V\rangle_D \rightarrow |1\rangle_P |H\rangle_D$$

Mirror

$$|H\rangle_D \rightarrow |V\rangle_D$$
$$|V\rangle_D \rightarrow |H\rangle_D$$

$\updownarrow \Rightarrow \leftrightarrow$

Wave plate

θ

Only affects polarization

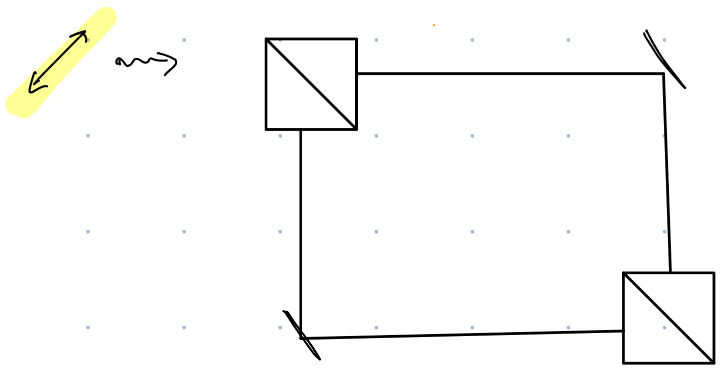
$45°$

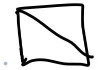
$$|0\rangle_P \rightarrow |+\rangle_P$$
$$|1\rangle_P \rightarrow |-\rangle_P$$

$90°$

$$|0\rangle_P \rightarrow |1\rangle_P$$
$$|1\rangle_P \rightarrow |0\rangle_P$$

Interferometer





$$|0\rangle_P |H\rangle_D \rightarrow |0\rangle_P |H\rangle_D$$
$$|1\rangle_P |H\rangle_D \rightarrow |1\rangle_P |V\rangle_D$$
$$|0\rangle_P |V\rangle_D \rightarrow |0\rangle_P |V\rangle_D$$
$$|1\rangle_P |V\rangle_D \rightarrow |1\rangle_P |H\rangle_D$$

$$|H\rangle_D \rightarrow |V\rangle_D$$
$$|V\rangle_D \rightarrow |H\rangle_D$$

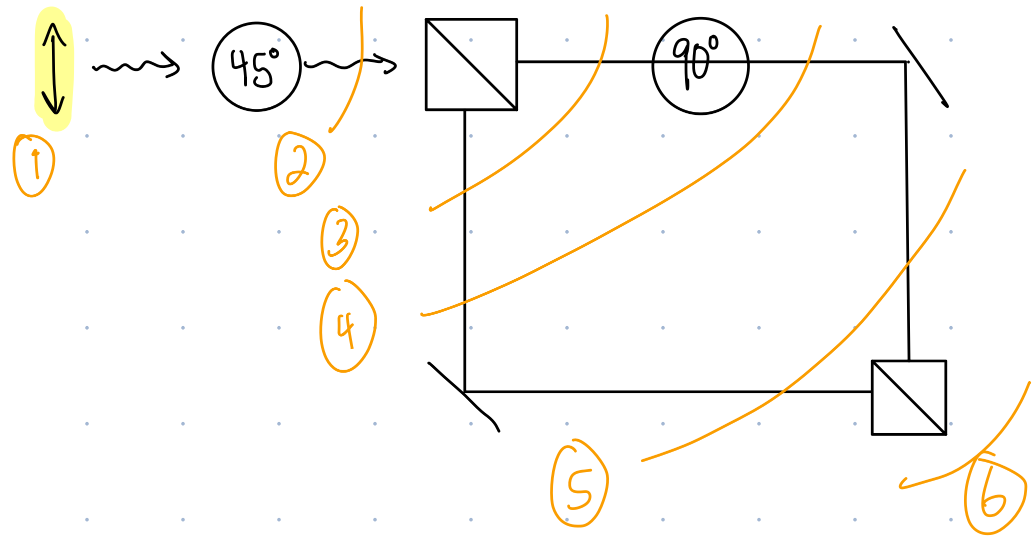
$|\psi_1\rangle =$

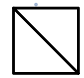
$|\psi_2\rangle =$

$|\psi_3\rangle =$

$|\psi_4\rangle =$

Group Practice





$|0\rangle_P|H\rangle_D \to |0\rangle_P|H\rangle_D$

$|1\rangle_P|H\rangle_D \to |1\rangle_P|V\rangle_D$

$|0\rangle_P|V\rangle_D \to |0\rangle_P|V\rangle_D$

$|1\rangle_P|V\rangle_D \to |1\rangle_P|H\rangle_D$

$|H\rangle_D \to |V\rangle_D$

$|V\rangle_D \to |H\rangle_D$

45°

$|0\rangle_P \to |+\rangle_P$

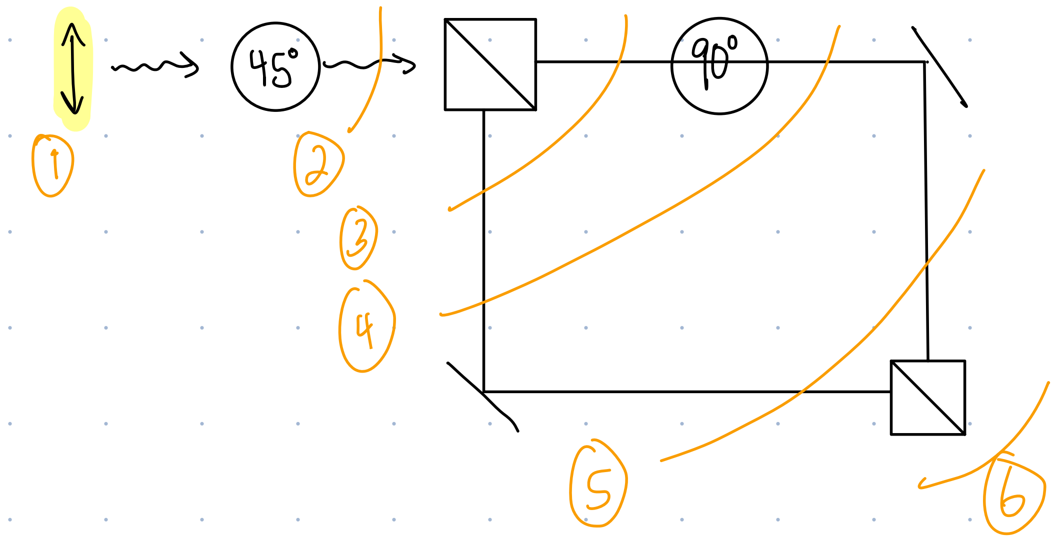
$|1\rangle_P \to |-\rangle_P$

90°

$|0\rangle_P \to |1\rangle_P$

$|1\rangle_P \to |0\rangle_P$

Group Practice



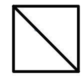
$|\psi_1\rangle =$

$|\psi_2\rangle =$

$|\psi_3\rangle =$

$|\psi_4\rangle =$

$|\psi_5\rangle =$



$$|0\rangle_P |H\rangle_D \rightarrow |0\rangle_P |H\rangle_D$$
$$|1\rangle_P |H\rangle_D \rightarrow |1\rangle_P |V\rangle_D$$
$$|0\rangle_P |V\rangle_D \rightarrow |0\rangle_P |V\rangle_D$$
$$|1\rangle_P |V\rangle_D \rightarrow |1\rangle_P |H\rangle_D$$

$$|H\rangle_D \rightarrow |V\rangle_D$$
$$|V\rangle_D \rightarrow |H\rangle_D$$

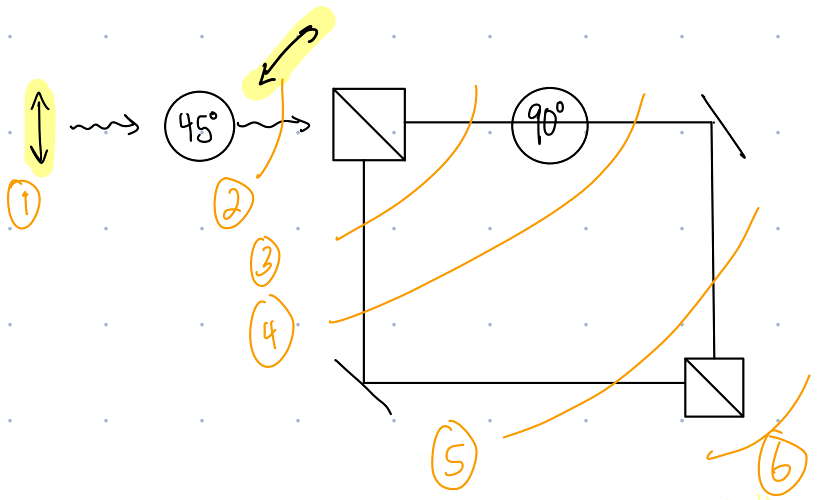
45°

$$|0\rangle_P \rightarrow |+\rangle_P$$
$$|1\rangle_P \rightarrow |-\rangle_P$$

$$|0\rangle_P \rightarrow |1\rangle_P$$
$$|1\rangle_P \rightarrow |0\rangle_P$$

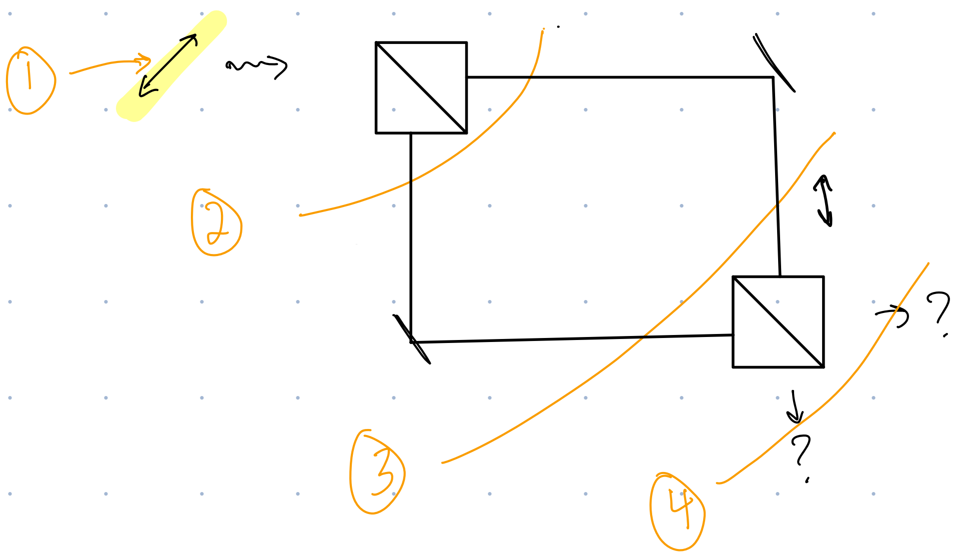
$|\psi_6\rangle =$

$=$

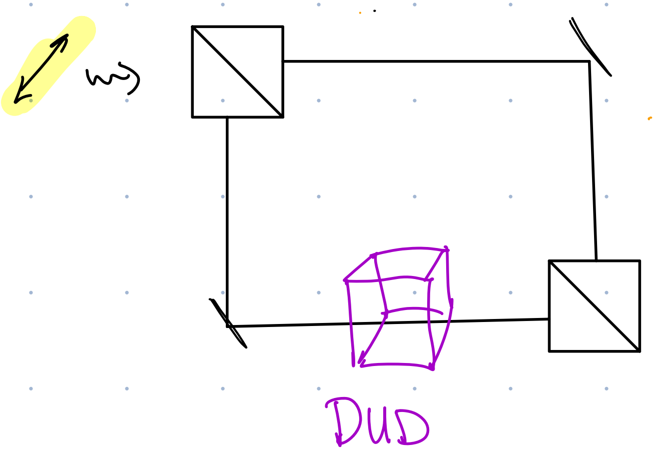


Big ideas:

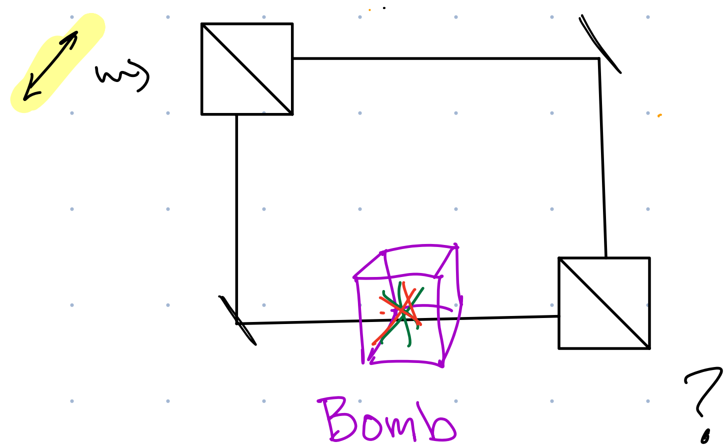
VS



Bomb Test



vs



Intuition:

Partial Q. Measurement (in standard basis)

-
-

- To determine outcome:
  -
  -
  - Calculate outcome probabilities / collapse state
    - Outcome $|0\rangle$

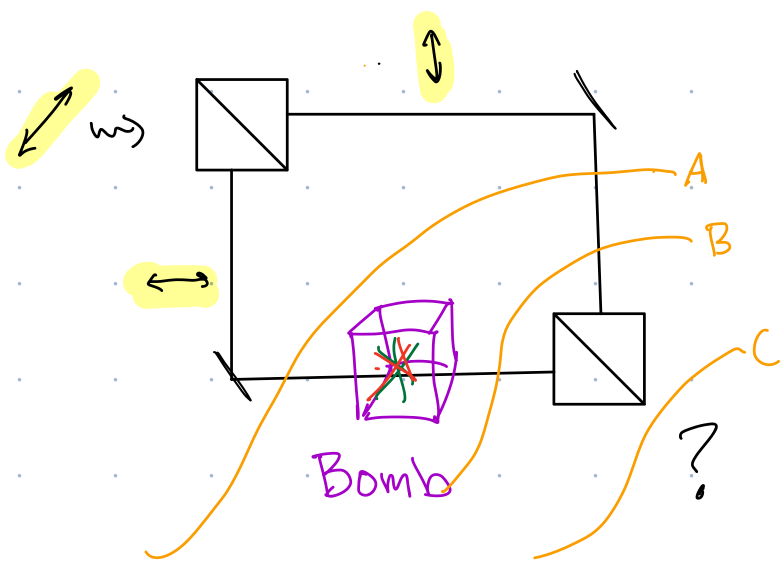
- Outcome 117

Back to the Bomb

Our two qubits are polarization + path (direction) ex: $\left(|+\rangle_P |H\rangle_D \right)$

Is the bomb measuring the polarization qubit or path qubit?

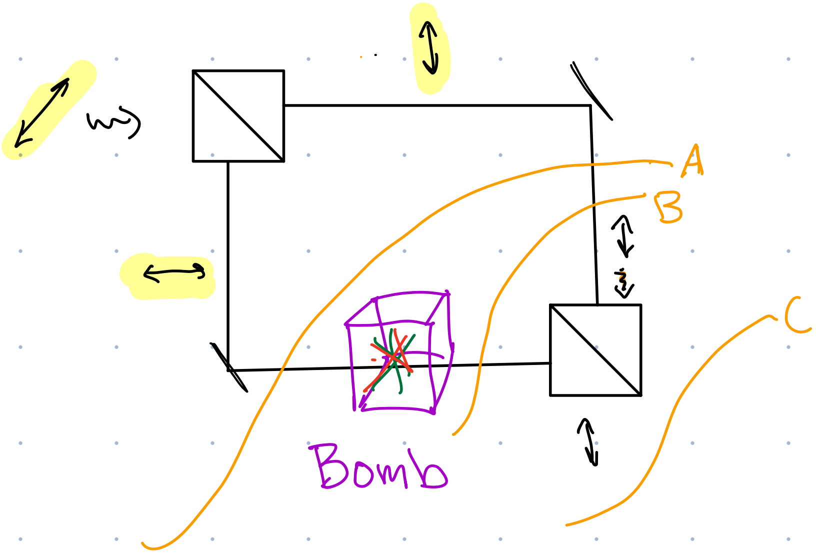
A: Polarization B: Path (direction)



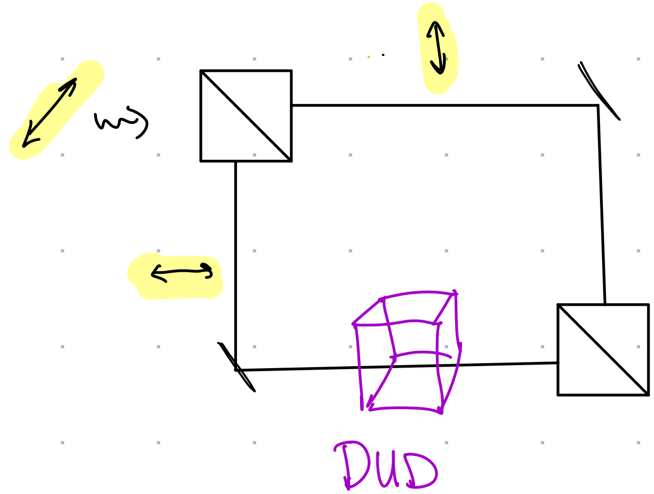
- Outcome $|H\rangle_D$:

- Outcome $|V\rangle_D$:

If no explosion

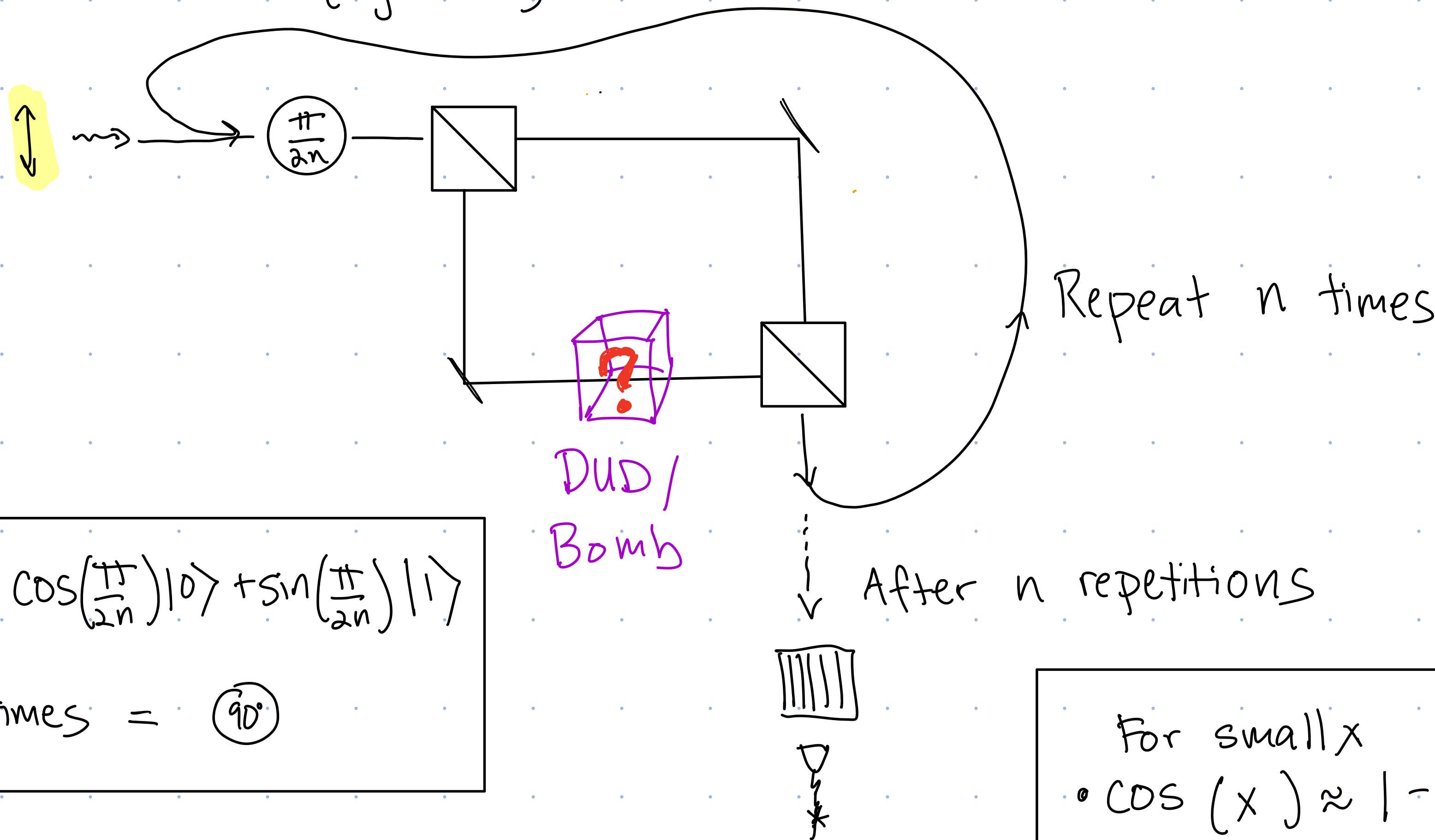


vs.



Group Work: Analyze the following set up.

Choose $n \gg 1$. (Eg. 1000)



$\frac{\pi}{2n}$ $|0\rangle \to \cos\left(\frac{\pi}{2n}\right)|0\rangle + \sin\left(\frac{\pi}{2n}\right)|1\rangle$

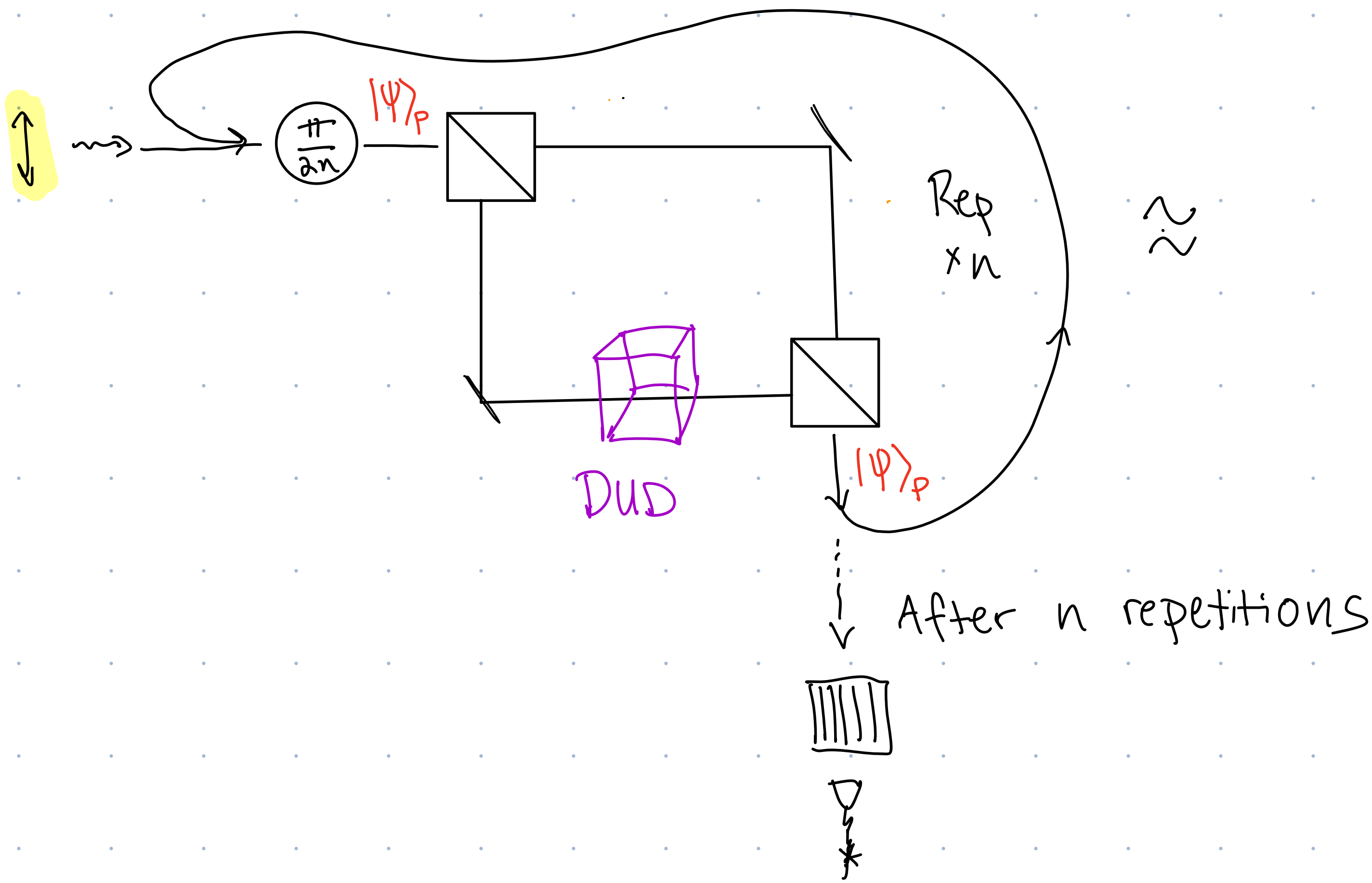
$\frac{\pi}{2n}$ x n times = $90°$

For small x

- $\cos(x) \approx 1 - \frac{x^2}{2}$
- $(1-x)^k \approx 1 - kx$

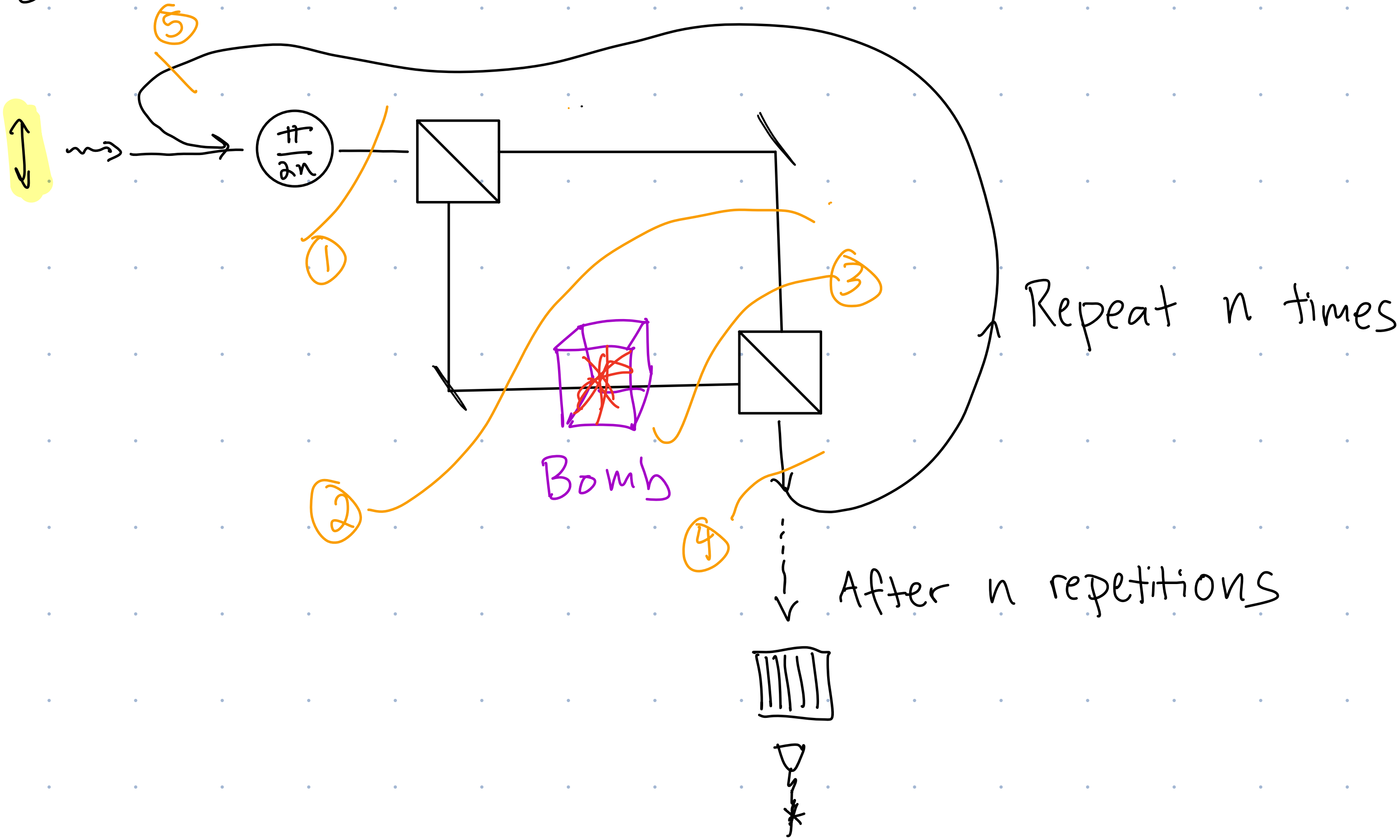
1) What happens if dud?
2) If bomb, probability of NO explosion
3) If bomb but no explosion, probability of detection?

1) If Dud:



$\frac{\pi}{2n}$
$|\psi\rangle_p$
Rep
$\times n$
DUD
$|\psi\rangle_p$
$\approx$
After n repetitions

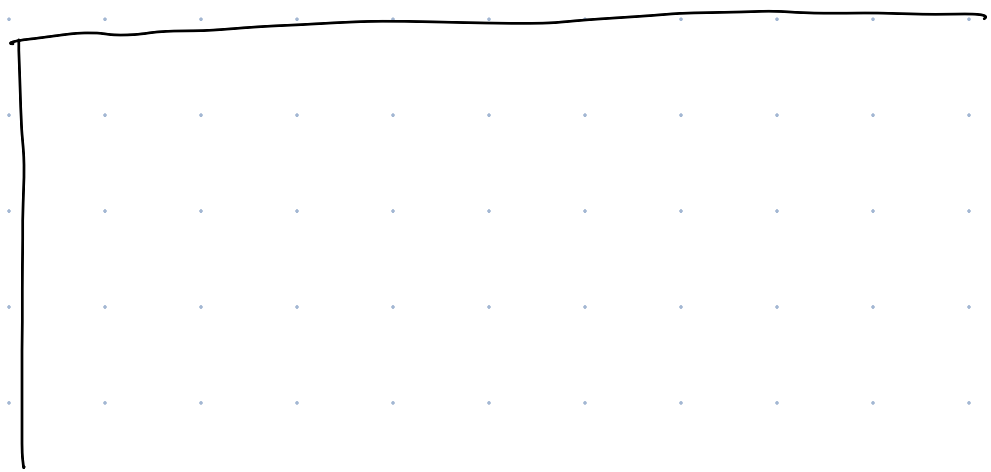
If Bomb:

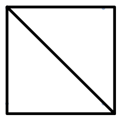


$|\psi_1\rangle =$

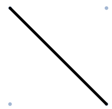
$|\psi_2\rangle =$

Prob of outcome $|V\rangle$ is

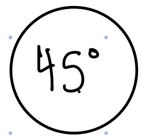




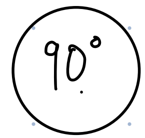
Beamsplitter



Mirror



45° Waveplate



90° Waveplate