BOMB DETECTION

Learning Goals

- Describe quantum operations
- Describe partial q. measurements
- Use rules of q. ops + partial meas. to analyze interferometer scenarios

Exit Tickets

not entangled

$\downarrow$ " product state

$|0\rangle_A$ and $|1\rangle_B$ combined: $|0\rangle|1\rangle = |01\rangle$

A) Inner Product   B) Tensor Product   C) Not allowed

$\langle 01 | 10\rangle =$

A) 0   B) 1

$\left(\frac{1}{\sqrt{2}}\langle 00| + \frac{1}{\sqrt{2}}\langle 10|\right)$ $\overbrace{|0\rangle \quad |1\rangle}^{|01\rangle}$

$$\left| {}_A\langle \alpha | \, {}_B\langle \beta | \, |\psi\rangle_{AB} \right| = \left| {}_{AB}\langle \psi | \, |\alpha\rangle_A \, |\beta\rangle_B \right|$$

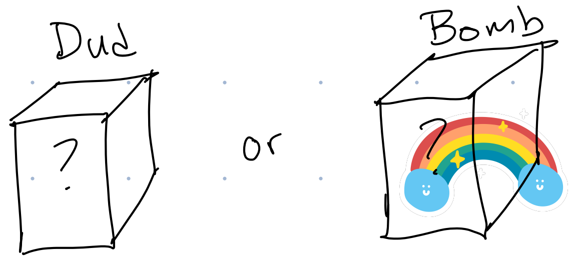
A) True B) False C) Not enough info

- When/why do we use imaginary numbers and when/why do we take complex conjugates?

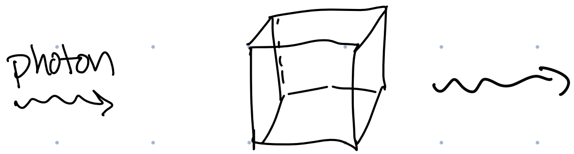
$$\boxed{\begin{array}{l} e^{ix} \neq e^{-ix} \\ e^{i\frac{\pi}{2}} = i \\ e^{-i\pi/2} = -i \end{array}}$$

- Entanglement sharing with photons for separated parties
- Why are qubits like this? ← Project?
- 100% vs 85%
- $\boxed{\text{Correlated vs. Anticorrelated}}$

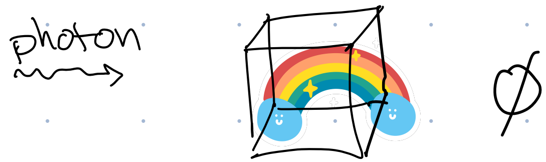
Problem: Quantum Bomb Detection (Elitzur-Vaidman)



Dud:

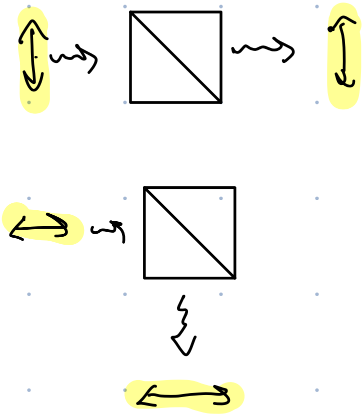


Bomb :

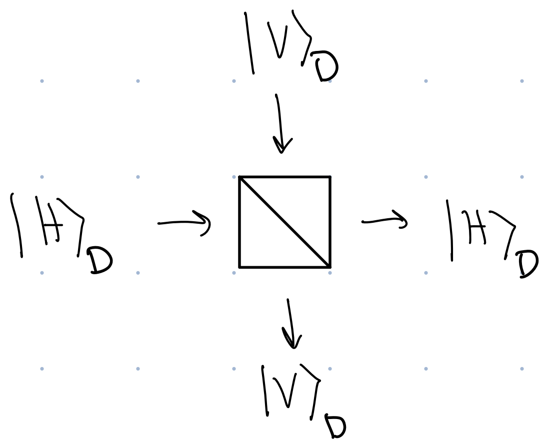


Want to give friend a rainbow bomb not a dud. How??

New Tool: Beamsplitter



- Transmits $|0\rangle_P$ (vertically polarized light)
- Reflects $|1\rangle_P$ (horiz. " ")



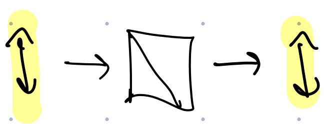
"P" for polarization

"D" for direction

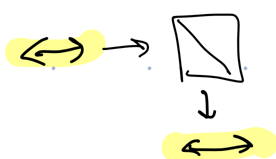
$\{|V\rangle_D, |H\rangle_D\}$ standard basis states

Behavior:

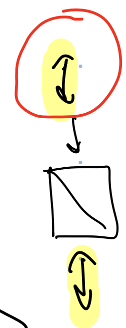
$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$



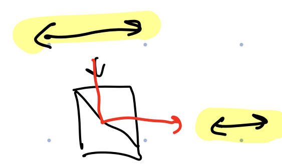
$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$



$|0\rangle_P|V\rangle_D \rightarrow |0\rangle_P|V\rangle_D$



$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$

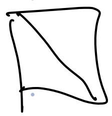


A) $|0\rangle_P|H\rangle_D$ B) $|0\rangle_P|V\rangle_D$ C) $|1\rangle_P|H\rangle_D$ D) $|1\rangle_P|V\rangle_D$

Quantum Gate

Operation that changes q. state without collapse (no info extracted)

Can describe gate via action on standard basis states



Beam-Splitter

$$
\boxed{
\begin{aligned}
|0\rangle_P|H\rangle_D &\rightarrow |0\rangle_P|H\rangle_D \\
|1\rangle_P|H\rangle_D &\rightarrow |1\rangle_P|V\rangle_D \\
|0\rangle_P|V\rangle_D &\rightarrow |0\rangle_P|V\rangle_D \\
|1\rangle_P|V\rangle_D &\rightarrow |1\rangle_P|H\rangle_D
\end{aligned}
}
$$

Mirror 

$$
\boxed{
\begin{aligned}
|H\rangle_D &\rightarrow |V\rangle_D \\
|V\rangle_D &\rightarrow |H\rangle_D
\end{aligned}
}
$$

Wave plate

θ

Only affects polarization

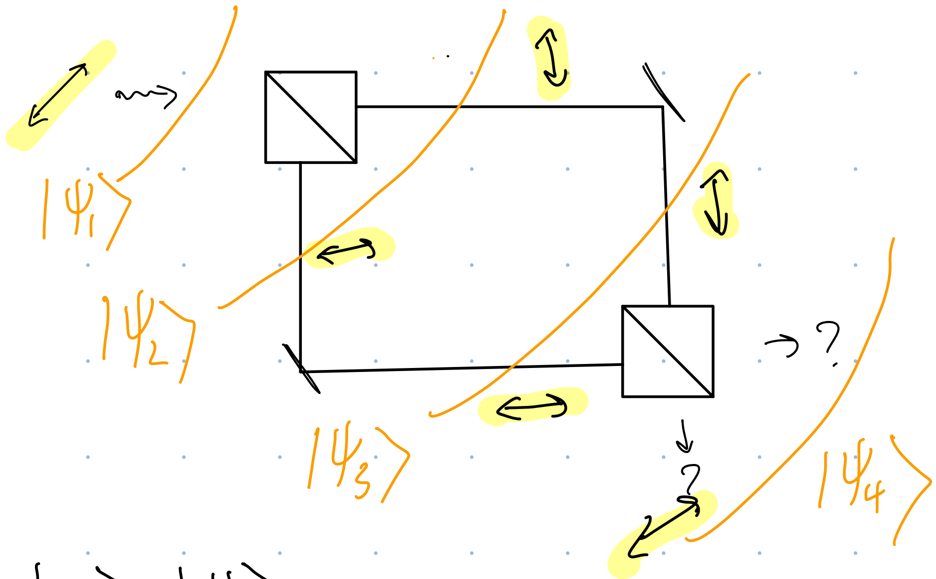
$45°$

$$
\boxed{
\begin{aligned}
|0\rangle_P &\rightarrow |+\rangle_P \\
|1\rangle_P &\rightarrow |-\rangle_P
\end{aligned}
}
$$

$90°$

$$
\boxed{
\begin{aligned}
|0\rangle_P &\rightarrow |1\rangle_P \\
|1\rangle_P &\rightarrow |0\rangle_P
\end{aligned}
}
$$

Interferometer



$\boxbslash$

$$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$$
$$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$$
$$|0\rangle_P|V\rangle_D \rightarrow |0\rangle_P|V\rangle_D$$
$$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$$

$\backslash$

$$|H\rangle_D \rightarrow |V\rangle_D$$
$$|V\rangle_D \rightarrow |H\rangle_D$$

$$|\psi_1\rangle = |+\rangle_P|H\rangle_D$$

$$|\psi_1\rangle = \left(\frac{1}{\sqrt{2}}|0\rangle_P + \frac{1}{\sqrt{2}}|1\rangle_P\right)|H\rangle_D = \frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D$$

$$|\psi_2\rangle = \boxbslash\left(\frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D\right) = \frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D$$

$$|\psi_3\rangle = \backslash\left(\frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D\right) = \frac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D$$

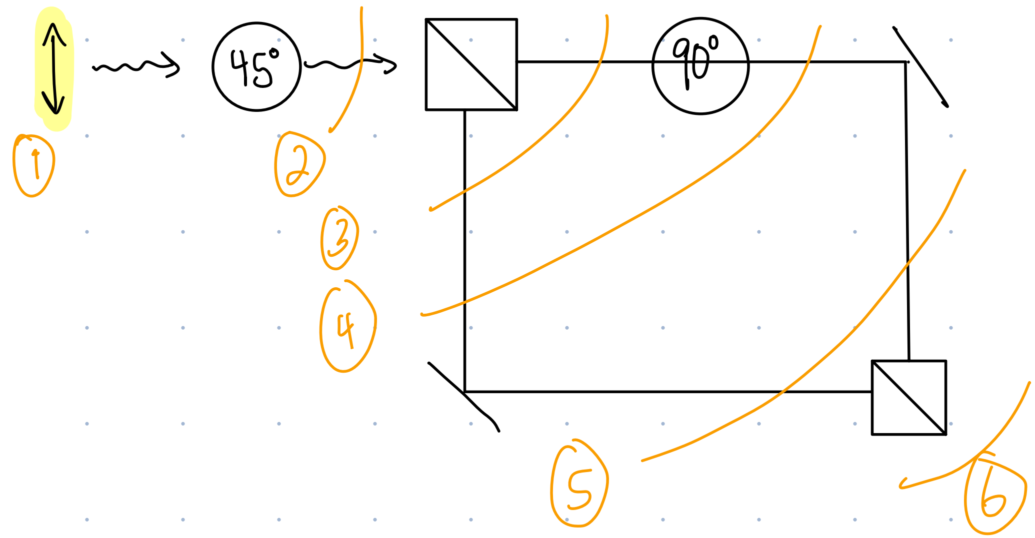
$$|\psi_4\rangle = \boxbslash\left(\frac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D\right) = \frac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D$$

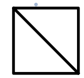
$$= \left(\frac{1}{\sqrt{2}}|0\rangle_P + \frac{1}{\sqrt{2}}|1\rangle_P\right)|V\rangle_D = |+\rangle_P|V\rangle_D$$

Group Practice

Family weekend fun? Group Style?

QI4





$|0\rangle_P|H\rangle_D \to |0\rangle_P|H\rangle_D$

$|1\rangle_P|H\rangle_D \to |1\rangle_P|V\rangle_D$

$|0\rangle_P|V\rangle_D \to |0\rangle_P|V\rangle_D$

$|1\rangle_P|V\rangle_D \to |1\rangle_P|H\rangle_D$

$|H\rangle_D \to |V\rangle_D$

$|V\rangle_D \to |H\rangle_D$

45°

$|0\rangle_P \to |+\rangle_P$

$|1\rangle_P \to |-\rangle_P$

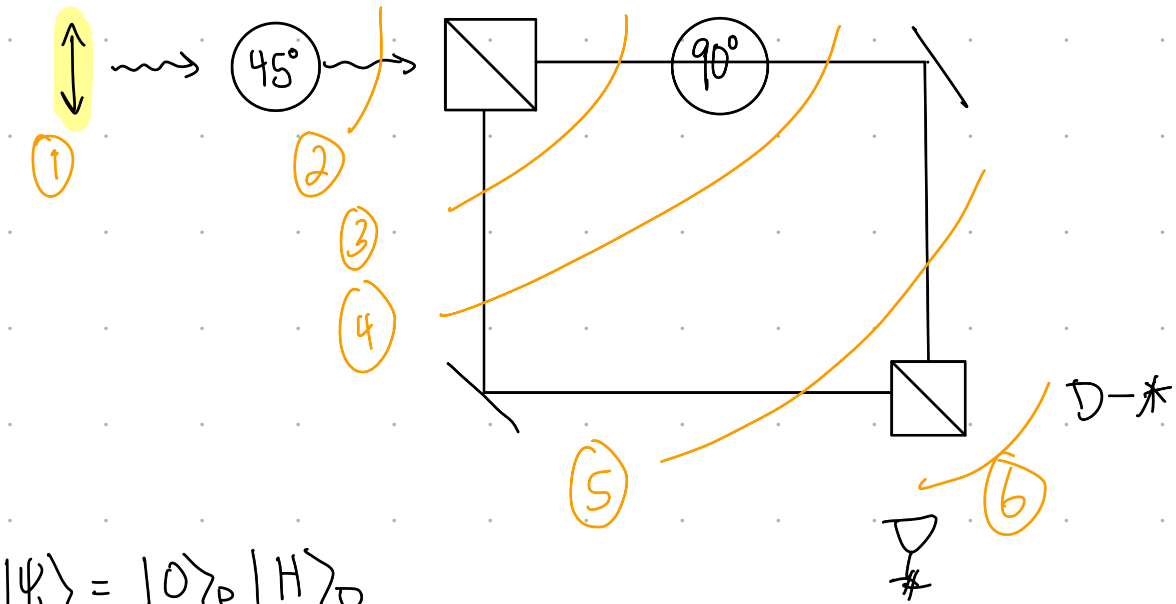
90°

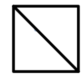
$|0\rangle_P \to |1\rangle_P$

$|1\rangle_P \to |0\rangle_P$

Group Practice

QI4





$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$

$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$

$|0\rangle_P|V\rangle_D \rightarrow |0\rangle_P|V\rangle_D$

$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$

$|H\rangle_D \rightarrow |V\rangle_D$

$|V\rangle_D \rightarrow |H\rangle_D$

45°

$|0\rangle_P \rightarrow |+\rangle_P$

$|1\rangle_P \rightarrow |-\rangle_P$

90°

$|0\rangle_P \rightarrow |1\rangle_P$

$|1\rangle_P \rightarrow |0\rangle_P$

$$|\psi_1\rangle = |0\rangle_P|H\rangle_D$$

$$|\psi_2\rangle = |+\rangle_P|H\rangle_D = \frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D$$

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D$$

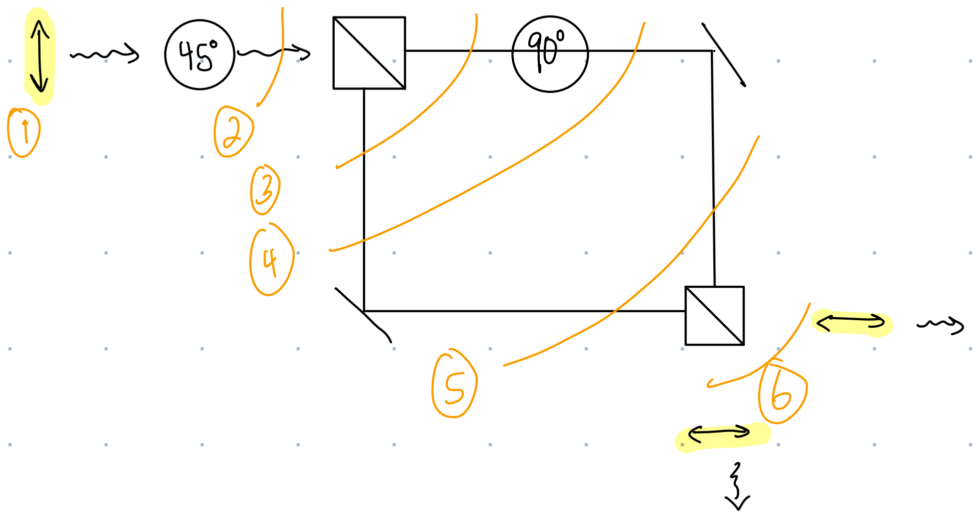
$$|\psi_4\rangle = \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D$$

$$|\psi_5\rangle = \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D$$

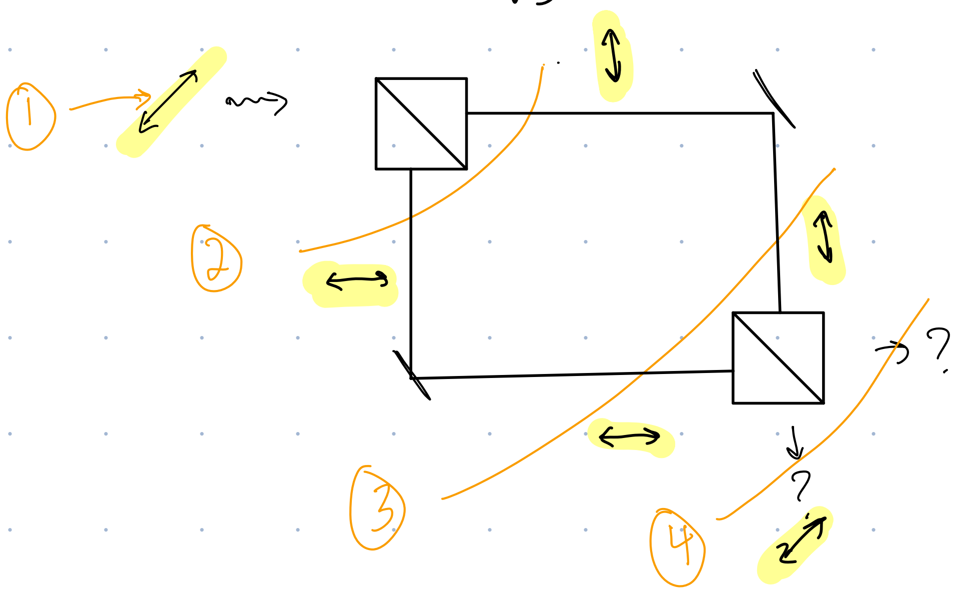
$$|\psi_6\rangle = \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D = |1\rangle_P\left(\frac{1}{\sqrt{2}}|H\rangle_D + \frac{1}{\sqrt{2}}|V\rangle_D\right)$$

$|\psi_6\rangle =$

$=$



vs



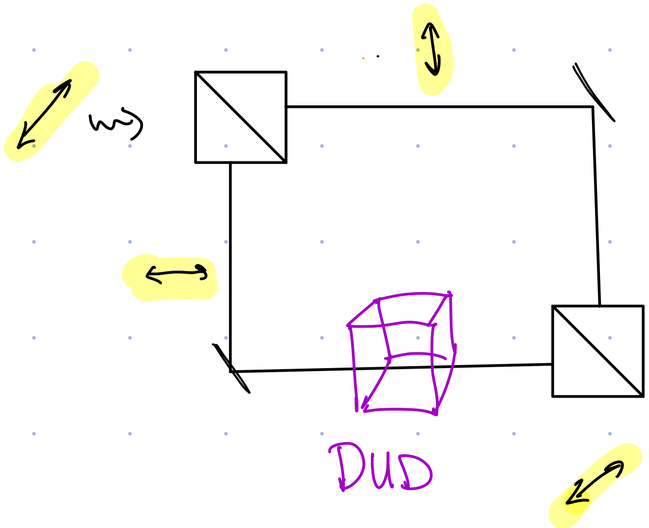
Big ideas:

- One photon takes 2 paths
- Direction & polarization are entangled
- Objects in arm affects output
-

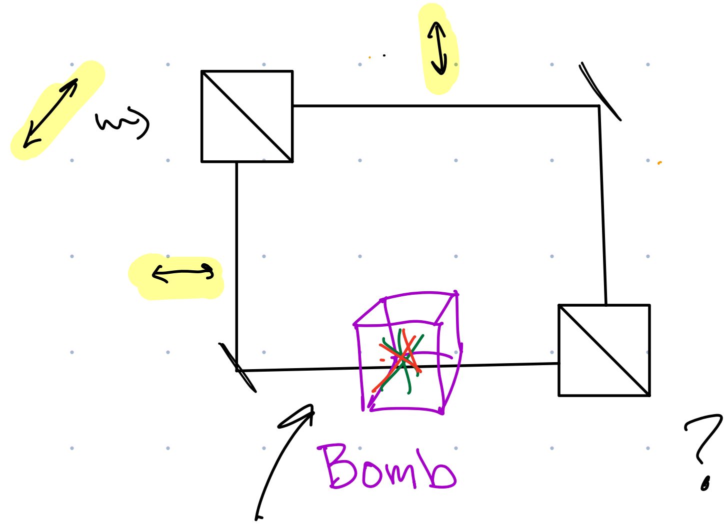
$\Downarrow$

Put potential bomb in arm

Bomb Test



vs



Same as empty interferometer

Partial Measurement

Partial Q. Measurement (in standard basis)

- State to be measured: $|\psi\rangle_{AB}$ Needs 2 parts! (A & B)
- Want to measure qubit A in $M_A = \{|0\rangle, |1\rangle\}$ No measurement of B. • Asking if A is in state $|0\rangle$ or $|1\rangle$ • Not asking B anything
- To determine outcome:
  - Write $|\psi\rangle_{AB}$ in standard basis:

$$|\psi\rangle_{AB} = \boxed{a_{00}|00\rangle_{AB} + a_{01}|01\rangle_{AB}} + a_{10}|10\rangle_{AB} + a_{11}|11\rangle_{AB}$$

  - Rewrite: Factor/group A kets: (numbers/scalars commute!)

$$|\psi\rangle_{AB} = |0\rangle_A\left(a_{00}|0\rangle_B + a_{01}|1\rangle_B\right) + |1\rangle\left(a_{10}|0\rangle_B + a_{11}|1\rangle_B\right)$$

  - Calculate outcome probabilities / collapse state
    - Outcome $|0\rangle$
      - Probability: $|a_{00}|^2 + |a_{01}|^2 = p(|0\rangle_A)$
      - Collapse state: $\frac{1}{\sqrt{p(|0\rangle_A)}}\boxed{|0\rangle\left(a_{00}|0\rangle + a_{01}|1\rangle\right)} =$

$$\frac{a_{00}}{\sqrt{p_0}}|00\rangle + \frac{a_{01}}{\sqrt{p_0}}|01\rangle$$

- Outcome $|1\rangle$
  - Probability: $|a_{10}|^2 + |a_{11}|^2 = p(|1\rangle_A)$
  - Collapse state: $\frac{1}{\sqrt{p(|1\rangle_A)}} |1\rangle\left(a_{10}|0\rangle + a_{11}|1\rangle\right) = \frac{a_{10}}{\sqrt{p(|1\rangle_A)}}|10\rangle + \frac{a_{11}}{\sqrt{p(|1\rangle_A)}}|11\rangle$

* Partial measurement destroys entanglement

$$= \frac{a_{10}}{\sqrt{p(|1\rangle_A)}}|10\rangle + \frac{a_{11}}{\sqrt{p(|1\rangle_A)}}|11\rangle$$

$$= |1\rangle\left(\frac{a_{10}}{\sqrt{p_1}}|0\rangle + \frac{a_{11}}{\sqrt{p_1}}|1\rangle\right)$$

$$\frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right) = |+\rangle|+\rangle$$

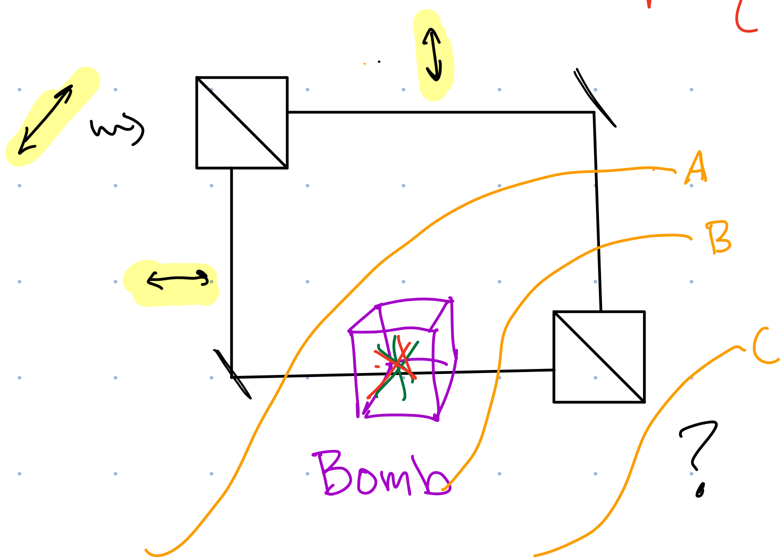
Back to the Bomb

Our two qubits are polarization + path (direction) $\left(|+\rangle_P |H\rangle_D \right)$

Is the bomb measuring the polarization qubit or path qubit?

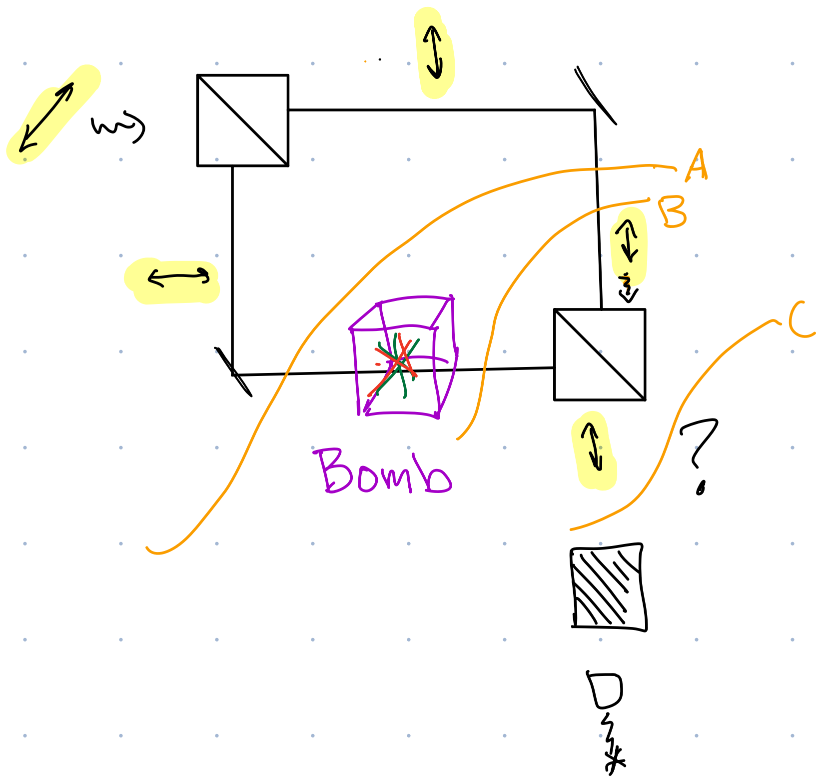
A: Polarization B: Path (direction)

$M = \{ |H\rangle_D , |V\rangle_D \}$

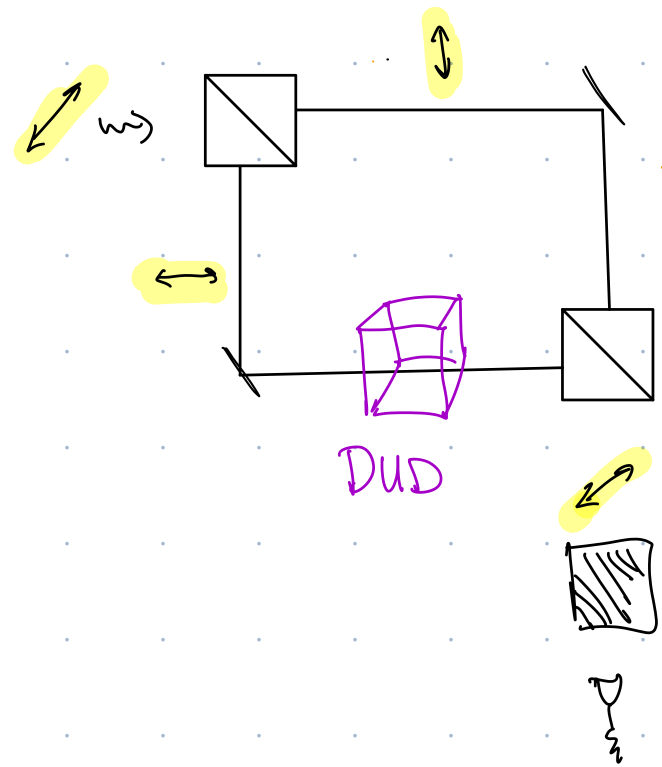


$$|\psi_A\rangle = \frac{1}{\sqrt{2}} |0\rangle_P |V\rangle_D + \frac{1}{\sqrt{2}} |1\rangle_P |H\rangle_D$$

If no explosion



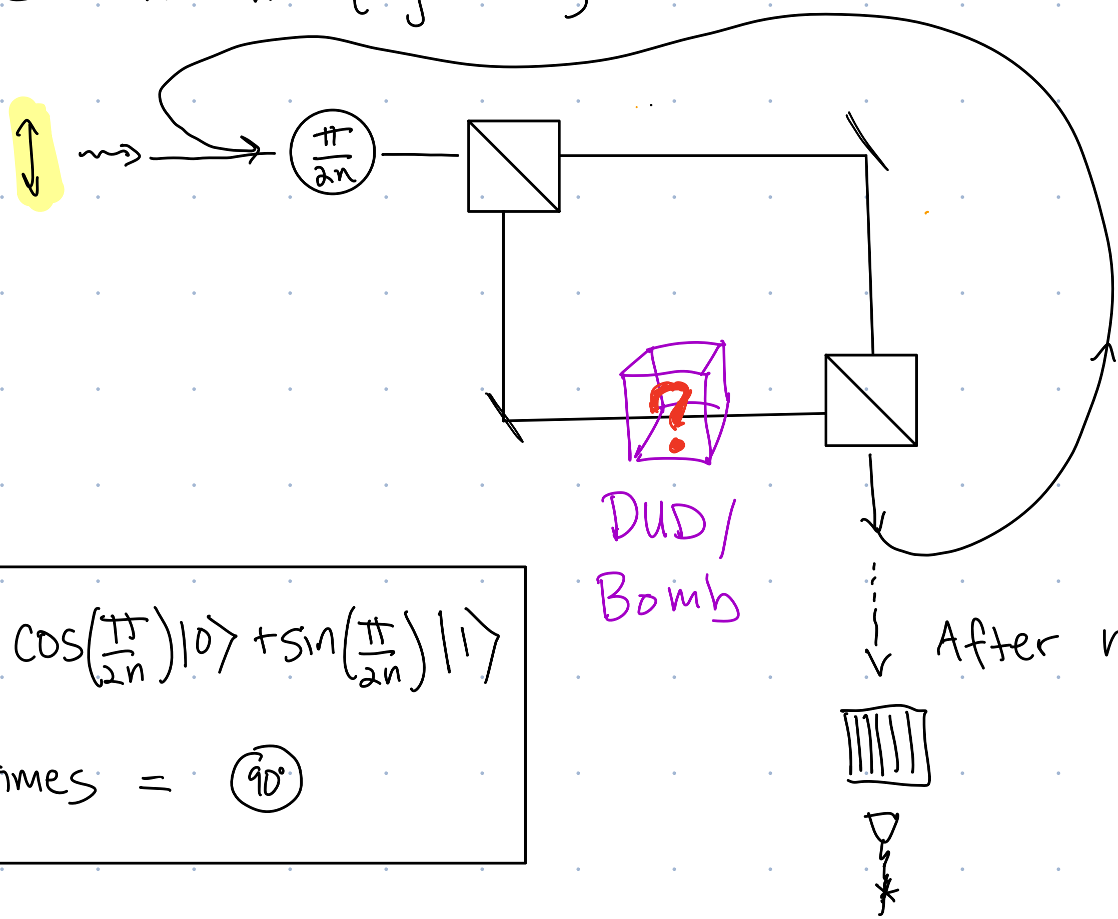
vs.



Can we do better?

Group Work: Analyze the following set up.

Choose $n \gg 1$. (Eg. 1000)



$\frac{\pi}{2n}$ $|0\rangle \to \cos\left(\frac{\pi}{2n}\right)|0\rangle + \sin\left(\frac{\pi}{2n}\right)|1\rangle$

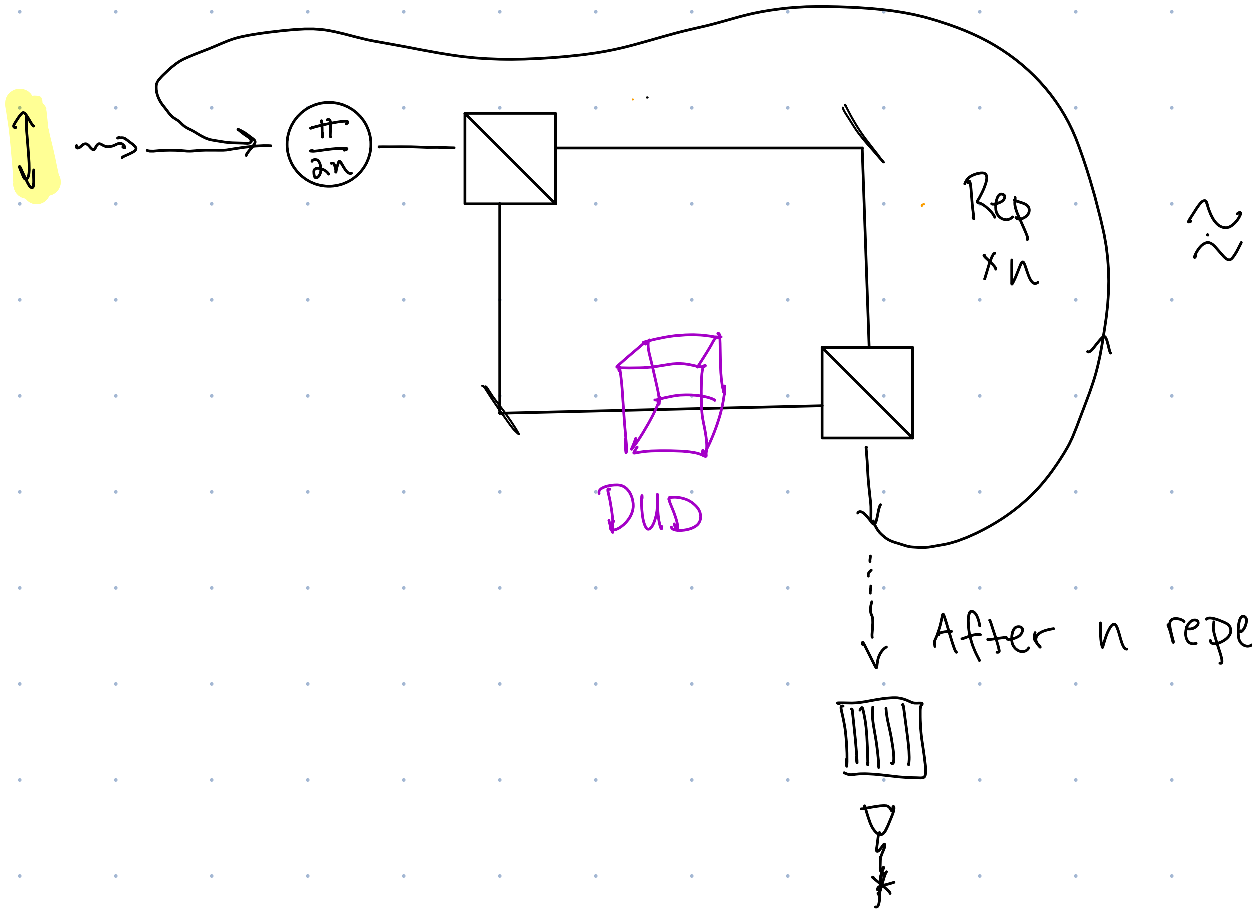
$\frac{\pi}{2n}$ x n times $=$ 90°

For small x

- $\cos(x) \approx 1 - \frac{x^2}{2}$
- $(1-x)^k \approx 1 - kx$

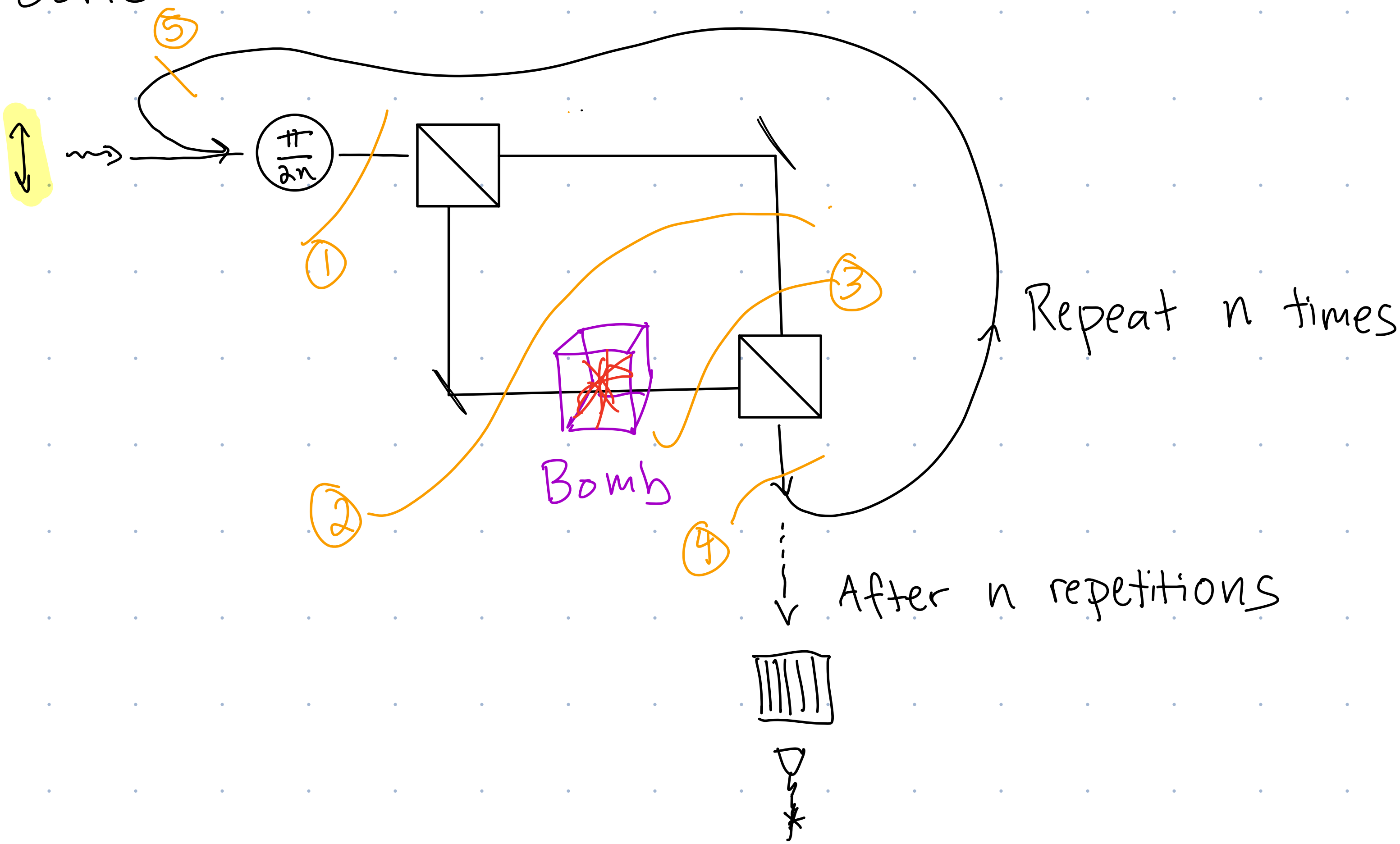
1) What happens if dud?
2) If bomb, probability of NO explosion
3) If bomb but no explosion, probability of detection?

1) If Dud:



After n repetitions

If Bomb:



$|\psi_1\rangle =$

$|\psi_2\rangle =$