Learning Goals

- Describe quantum operations
- Describe partial q. measurements
- Use rules of q. ops + partial meas. to analyze both abstract and in interferometer scenarios

- ## Exit Tickets:

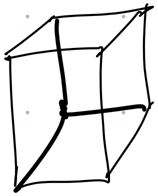
- Local + Global Phases
- Beamsplitter ... for real?
- In a quantum computation: gates versus measurement

  gates ↑ no info; measurement ↑ info

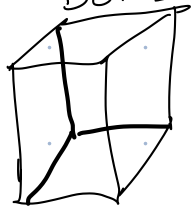
- Separated qubits + faster than light communication
- Qubits needed to describe a photon (fully)
- Gates help eavesdropper?

Problem: Quantum Bomb Detection (Elitzur-Vaidman)

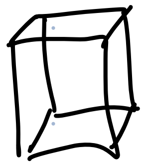
Dud | Bomb



or



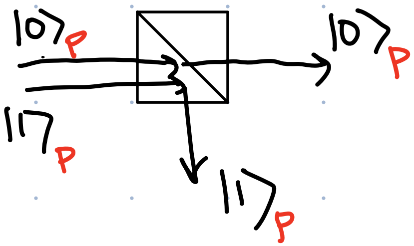
Dud

Dud: photon  (passes through

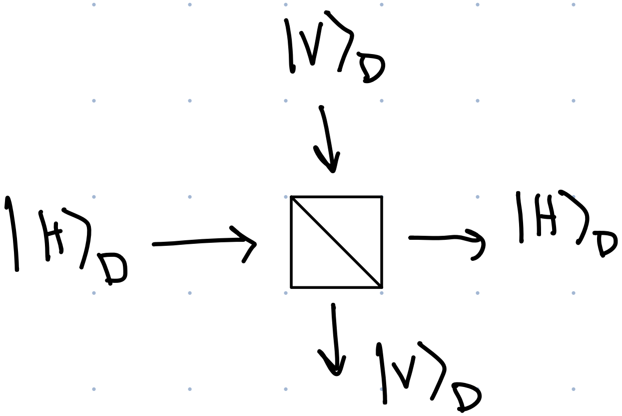
Bomb: photon  ϕ

Want to give friend a rainbow bomb not a dud. How??

New Tool: Beamsplitter



- Transmist if photon $|0\rangle_P$
- Reflects if $|1\rangle_P$



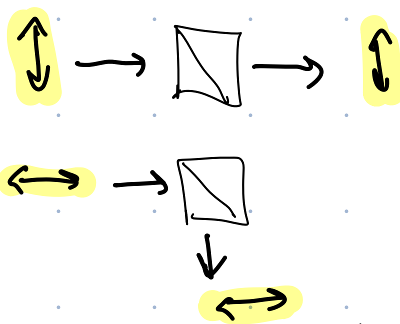
"P" for polarization
"D" for direction

$\{|V\rangle_D, |H\rangle_D\}$ standard basis states

Behavior:

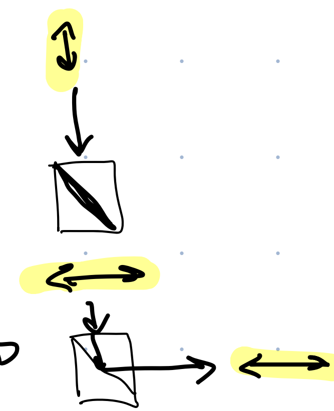
$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$

$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$



$|0\rangle_P|V\rangle_D \rightarrow \boxed{|0\rangle_P|V\rangle_D}$

$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$



A) $|0\rangle_P|H\rangle_D$ B) $|0\rangle_P|V\rangle_D$ C) $|1\rangle_P|H\rangle_D$ D) $|1\rangle_P|V\rangle_D$

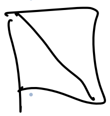
Quantum Gate

Operation that changes a quantum state without collapse.

We will describe gates using their effect on s.b. states

$|0\rangle, |1\rangle$

$|00\rangle, |01\rangle \ldots$

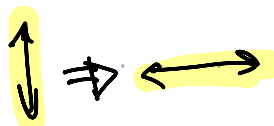


Beam-Splitter

$$|0\rangle_P |H\rangle_D \rightarrow |0\rangle_P |H\rangle_D$$
$$|1\rangle_P |H\rangle_D \rightarrow |1\rangle_P |V\rangle_D$$
$$|0\rangle_P |V\rangle_D \rightarrow |0\rangle_P |V\rangle_D$$
$$|1\rangle_P |V\rangle_D \rightarrow |1\rangle_P |H\rangle_D$$

Mirror

$$|H\rangle_D \rightarrow |V\rangle_D$$
$$|V\rangle_D \rightarrow |H\rangle_D$$



Wave plate

θ

Only affects polarization

$45°$

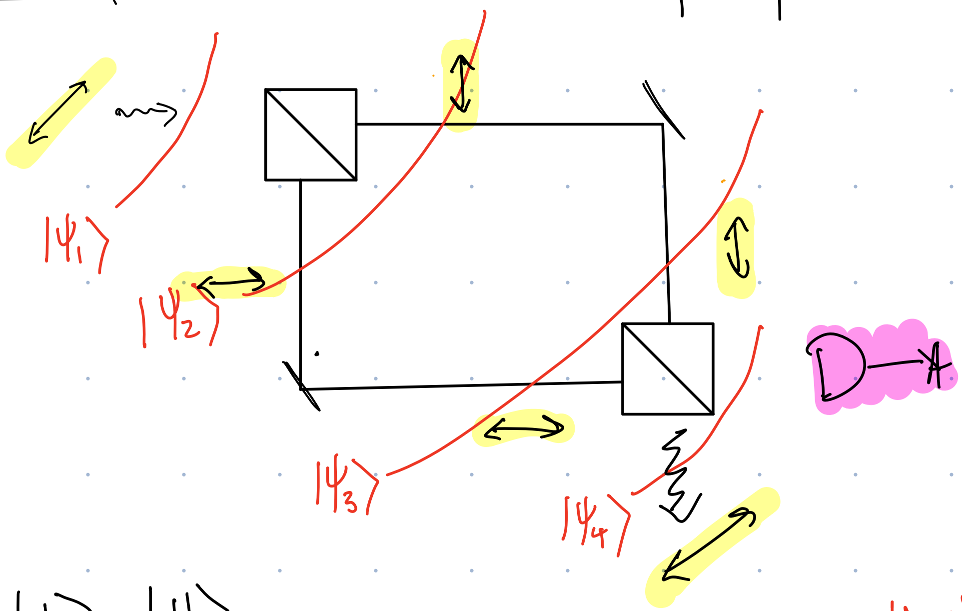
$$|0\rangle_P \rightarrow |+\rangle_P$$
$$|1\rangle_P \rightarrow |-\rangle_P$$

$90°$

$$|0\rangle_P \rightarrow |1\rangle_P$$
$$|1\rangle_P \rightarrow |0\rangle_P$$

Interferometer

"Superposition"



$\boxbslash$

$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$
$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$
$|0\rangle_P|V\rangle_D \rightarrow |0\rangle_P|V\rangle_D$
$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$

$\backslash$

$|H\rangle_D \rightarrow |V\rangle_D$
$|V\rangle_D \rightarrow |H\rangle_D$

$|\psi_1\rangle = |+\rangle_P|H\rangle_D$

$|\psi_2\rangle = \boxbslash\,|\psi_1\rangle = \boxbslash\,|+\rangle_P|H\rangle_D = \boxbslash\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)|H\rangle_D =$ Writing in standard basis

$= \boxbslash\left(\frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D\right)$ Distributing

$= \frac{1}{\sqrt{2}}\boxbslash\,|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}\boxbslash\,|1\rangle_P|H\rangle_D$ " commuting w/ constant

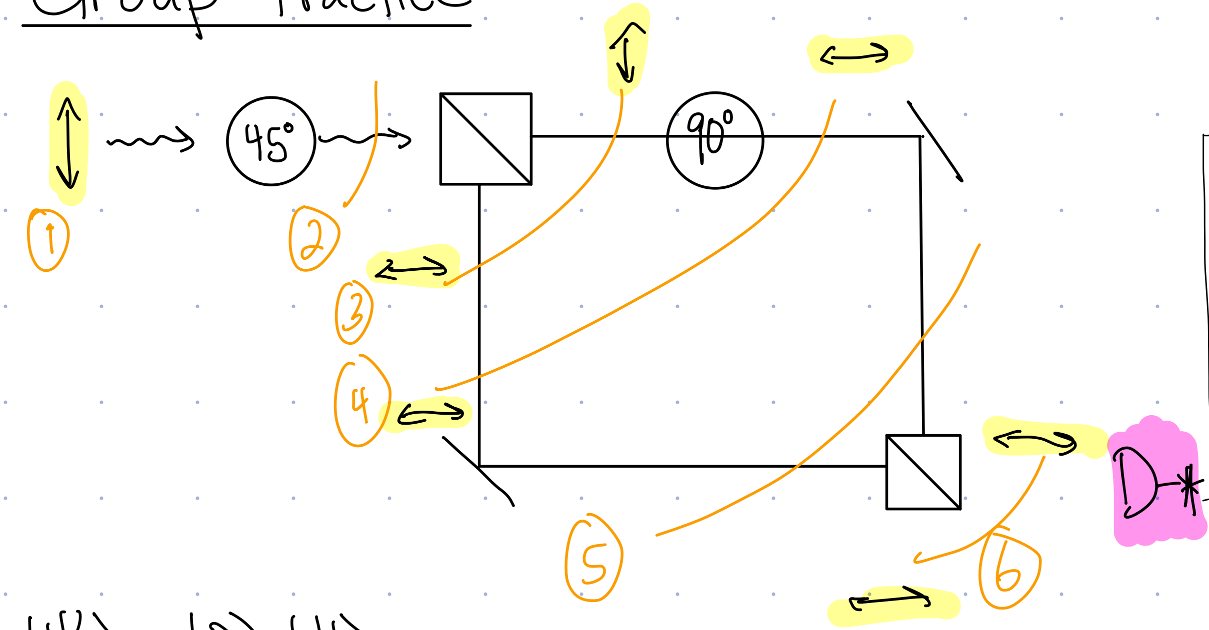
$= \frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D$

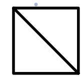
$|\psi_3\rangle = \backslash\left(\frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D\right) = \frac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D$

$|\psi_4\rangle = \boxbslash\left(\frac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D\right) = \frac{1}{\sqrt{2}}\boxbslash\,|0\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}\boxbslash\,|1\rangle_P|H\rangle_D$

$= \frac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D = \left(\frac{1}{\sqrt{2}}|0\rangle_P + \frac{1}{\sqrt{2}}|1\rangle_P\right)|V\rangle_D = |+\rangle|V\rangle$

Group Practice





$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$

$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$

$|0\rangle_P|V\rangle_D \rightarrow |0\rangle_P|V\rangle_D$

$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$

$|H\rangle_D \rightarrow |V\rangle_D$

$|V\rangle_D \rightarrow |H\rangle_D$

45°

$|0\rangle_P \rightarrow |+\rangle_P$

$|1\rangle_P \rightarrow |-\rangle_P$

90°

$|0\rangle_P \rightarrow |1\rangle_P$

$|1\rangle_P \rightarrow |0\rangle_P$

$$|\psi_1\rangle = |0\rangle_P|H\rangle_D$$

$$|\psi_2\rangle = |+\rangle_P|H\rangle_D$$

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}|0\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D$$

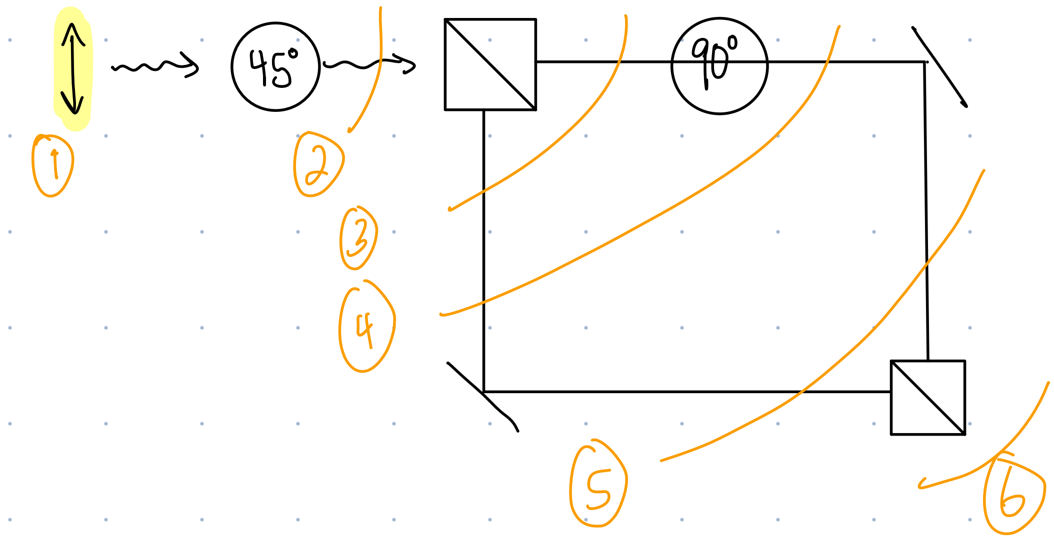
$$|\psi_4\rangle = \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D$$

$$|\psi_5\rangle = \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D$$

$$|6\rangle = \frac{1}{\sqrt{2}}\boxed{\diagdown}\,|1\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D$$

$$= \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|V\rangle_D = |1\rangle_P\left(\frac{1}{\sqrt{2}}|H\rangle_D + \frac{1}{\sqrt{2}}|V\rangle_D\right)$$

Group Practice



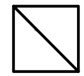
$|\psi_1\rangle =$

$|\psi_2\rangle =$

$|\psi_3\rangle =$

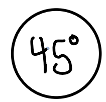
$|\psi_4\rangle =$

$|\psi_5\rangle =$



$$|0\rangle_P|H\rangle_D \rightarrow |0\rangle_P|H\rangle_D$$
$$|1\rangle_P|H\rangle_D \rightarrow |1\rangle_P|V\rangle_D$$
$$|0\rangle_P|V\rangle_D \rightarrow |0\rangle_P|V\rangle_D$$
$$|1\rangle_P|V\rangle_D \rightarrow |1\rangle_P|H\rangle_D$$

$$|H\rangle_D \rightarrow |V\rangle_D$$
$$|V\rangle_D \rightarrow |H\rangle_D$$

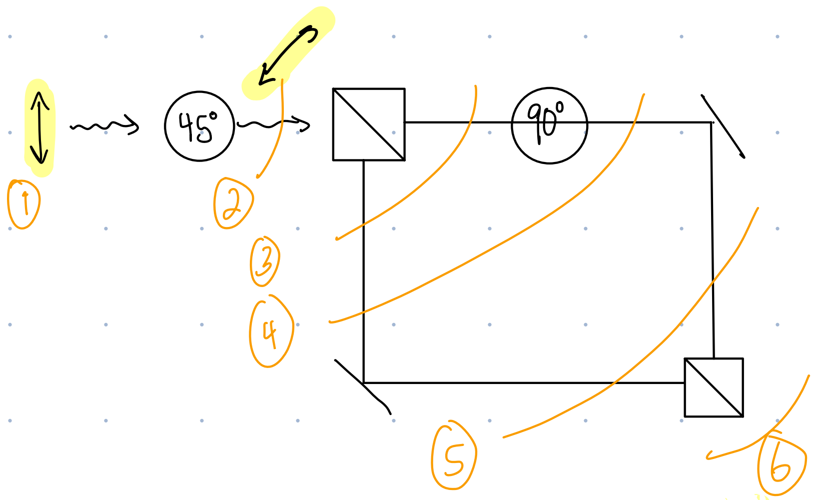


$$|0\rangle_P \rightarrow |+\rangle_P$$
$$|1\rangle_P \rightarrow |-\rangle_P$$

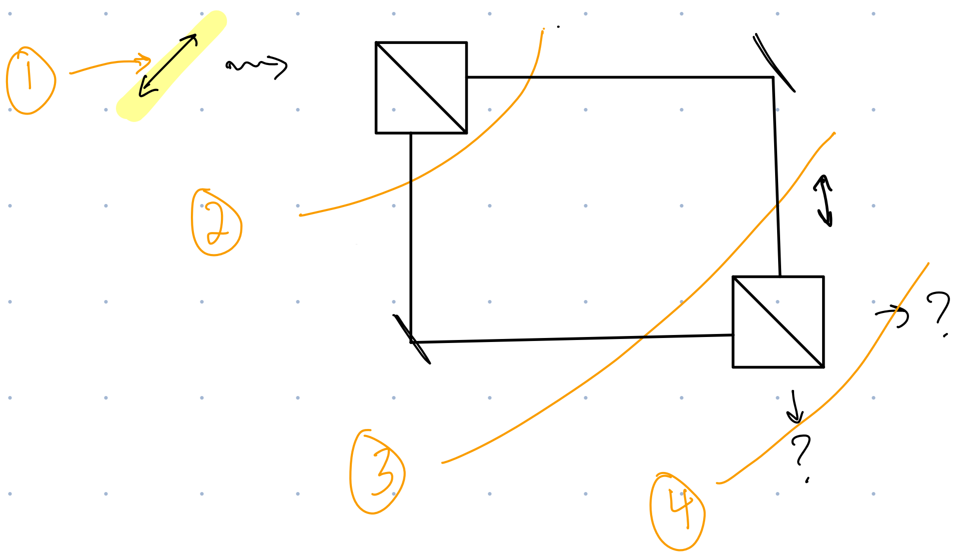
$$|0\rangle_P \rightarrow |1\rangle_P$$
$$|1\rangle_P \rightarrow |0\rangle_P$$

$|\psi_6\rangle =$

$=$



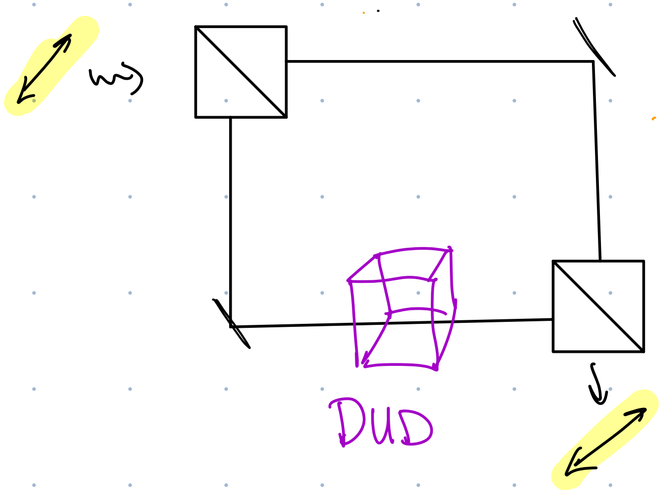
VS



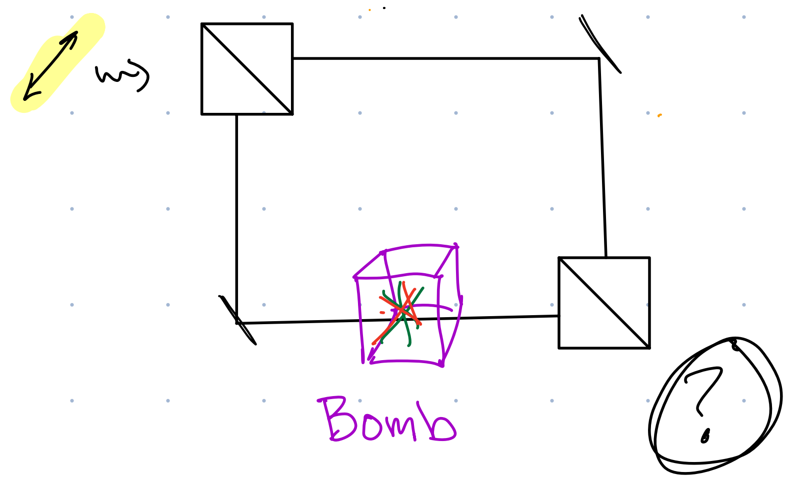
Big ideas:

- One photon takes 2 paths (superposition)
- Path + polarization are entangled
- Objects in arm affect the output photon state

Bomb Test



vs



Intuition:

- Will bomb always explode?
- Bomb makes a measurement:
  -  → photon in lowarm
  -  → photon in upper arm

Partial Q. Measurement (in standard basis)

- State to be measured $|\psi\rangle_{AB}$
- Want to measure qubit A with $M_A = \{|0\rangle, |1\rangle\}$

- To determine outcome:

REWRITE

- Write $|\psi\rangle_{AB}$ in standard basis:

$$|\psi\rangle_{AB} = a_{00}\underbrace{|00\rangle_{AB}}^{|0\rangle|0\rangle} + a_{01}\overbrace{|01\rangle_{AB}}^{|0\rangle|1\rangle} + a_{10}|10\rangle_{AB} + a_{11}|11\rangle_{AB}$$

- Factor/Group A kets:

$$|\psi\rangle_{AB} = |0\rangle_A\left(a_{00}|0\rangle_B + a_{01}|1\rangle_B\right) + |1\rangle_A\left(a_{10}|0\rangle_B + a_{11}|1\rangle_B\right)$$

- Calculate outcome probabilities/collapse state
  - Outcome $|0\rangle$
    - Probability: $P(|0\rangle_A) = |a_{00}|^2 + |a_{01}|^2$
    - Collapse: $\frac{1}{\sqrt{P(|0\rangle_A)}}|0\rangle_A\left(a_{00}|0\rangle_B + a_{01}|1\rangle_B\right) = \frac{a_{00}}{\sqrt{P(|0\rangle)}}|00\rangle_{AB} + \frac{a_{01}}{\sqrt{P(|0\rangle)}}|01\rangle_{AB}$

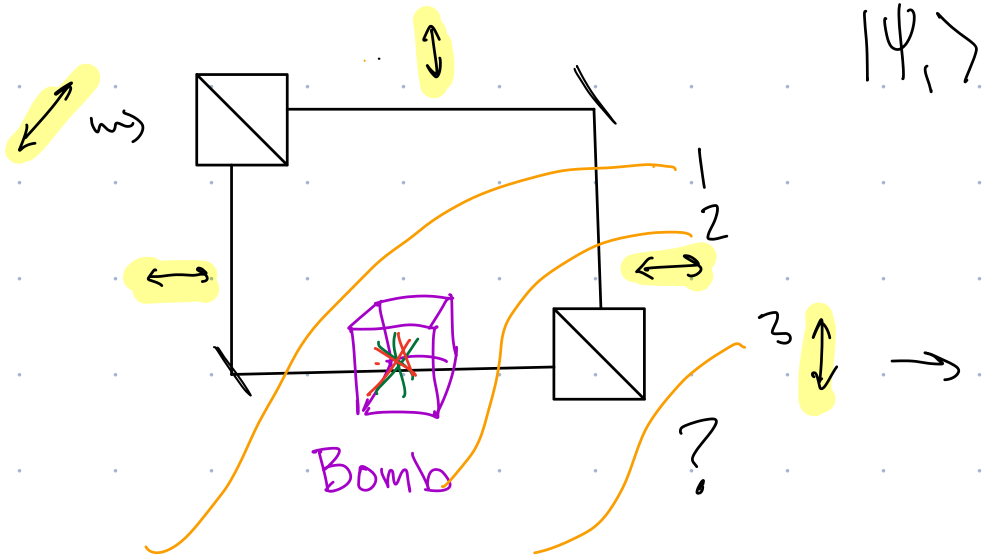
- Outcome $|1\rangle$
  - Probability : $P(|1\rangle_A) = |a_{10}|^2 + |a_{11}|^2$
  - Collopse: $\dfrac{a_{10}}{\sqrt{p(|1\rangle)}}|10\rangle_{AB} + \dfrac{a_{11}}{\sqrt{p(|1\rangle)}}|11\rangle_{AB}$

Back to the Bomb

Our two qubits are polarization + path (direction) ex: $\left(|+\rangle_P|H\rangle_D\right)$

Is the bomb measuring the polarization qubit or path qubit?

A: Polarization B: Path (direction)



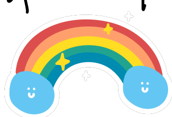
$$|\psi_1\rangle = \frac{1}{\sqrt{2}}|0\rangle_P|V\rangle_D + \frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_D + 0|0\rangle_P|H\rangle_D + 0|1\rangle_P|V\rangle_D$$

$$= \left(\frac{1}{\sqrt{2}}|0\rangle_P + 0|1\rangle_P\right)|V\rangle_D + \left(\frac{1}{\sqrt{2}}|1\rangle_P + 0|0\rangle_P\right)|H\rangle_D$$

- Outcome $|H\rangle_D$:

Prob: $\left|\frac{1}{\sqrt{2}}\right|^2 + |0|^2 = \frac{1}{2}$

Collapses: $\frac{1}{\sqrt{\frac{1}{2}}}\left(\frac{1}{\sqrt{2}}|1\rangle_P + 0|0\rangle_P\right)|H\rangle_D$



Explosion

- Outcome $|V\rangle_D$:

Prob: $\left|\frac{1}{\sqrt{2}}\right|^2 + |0|^2 = \frac{1}{2}$

Collapse: $\frac{1}{\sqrt{\frac{1}{2}}}\left(\frac{1}{\sqrt{2}}|0\rangle_P + \cancel{0|1\rangle_P}\right)|V\rangle_D$

$= |0\rangle_P|V\rangle_D$ No explosion

If no explosion