REDUCTIONS

Learning Goals

- Reduce one problem to another
- Define Polynomial Time Reduction
- Describe why reductions are important

Warm-Up to [NP2]
(Reductions are a tool used in NP-Hard Proofs)

Announcements

- PS6 due dates slightly pushed back
- Ethics assignment info posted ← PS6

- Initial Off Meet + Greet
- All Intro first, cycle back
- Study group

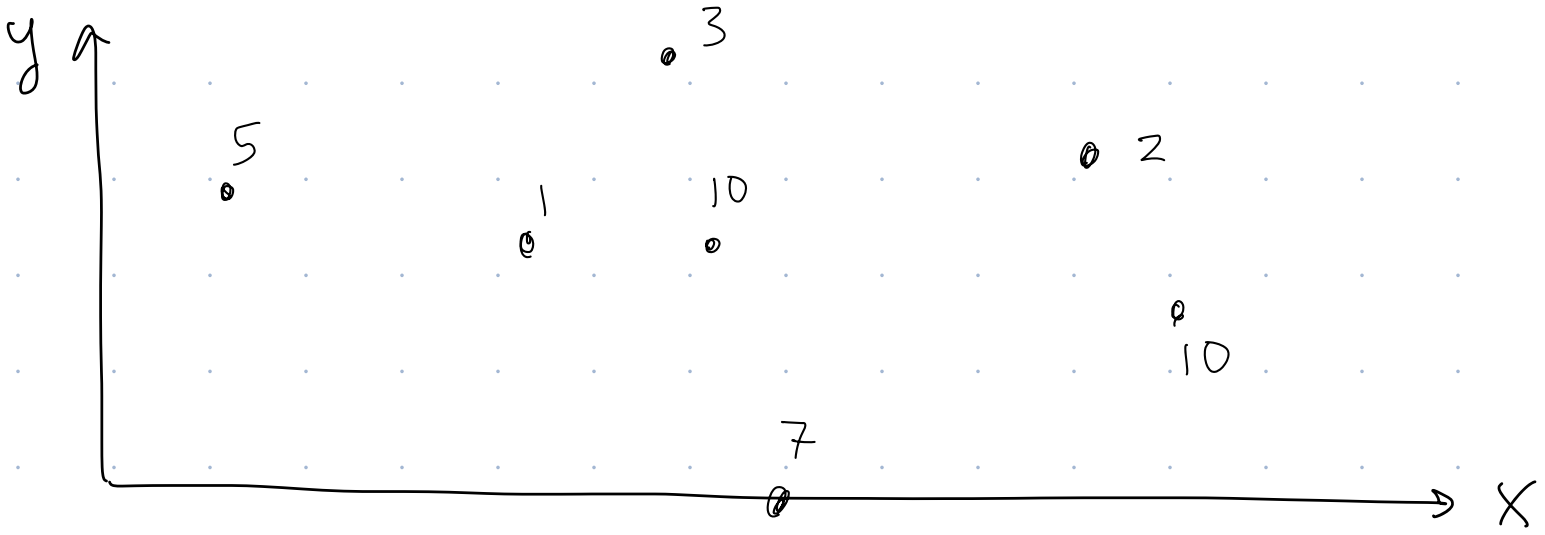
- Clopts start with begin of writer plate
- Clopts hard transition back

- Opt extra quizzes / shorter (fewer topics)

Cell Tower Scheduling

latitude, longitude ↑

Input: Array P s.t. P[i] is location of i^{th} cell tower
$n = |P|$ Array D s.t. D[i] is # of data packets to send
$n = |P|$ from i^{th} tower



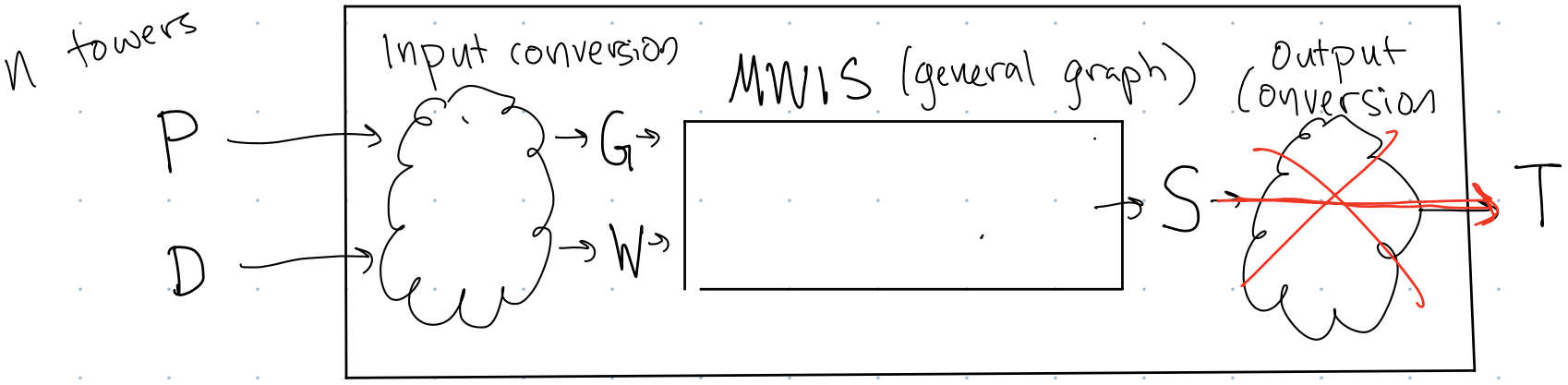
Output: Set of towers T to broadcast in the next time step.

- If 2 towers within 2 miles of each other broadcast at the same time → interference. Bad!
- Maximizes $A(T) = \sum_{i \in T} D[i]$

(Maximizes # of packets in queues of transmitting towers)

Reduction

Cell Tower Problem



1. What should input/output conversion functions be?

I.C.(P,D) ← $O(n^2)$

$V \leftarrow \{1, 2, 3, \ldots n\}$ ← $O(n)$

For $i, j \in V$ and $i \neq j$: ← $O(n^2)$

| If $d(t_i, t_j) \leq 2$, add $\{i, j\} \in E$

For $i \in V$: ← $O(n)$

| $w(i) \leftarrow D[i]$

Return $(G=(V,E), w)$

O.C.(S) ← $O(1)$ ← $O(n)$

Return S

2. Ethical Matrix (Stakeholders, Well-Being, Autonomy, Justice)

3. Runtime of conversions in terms of n?

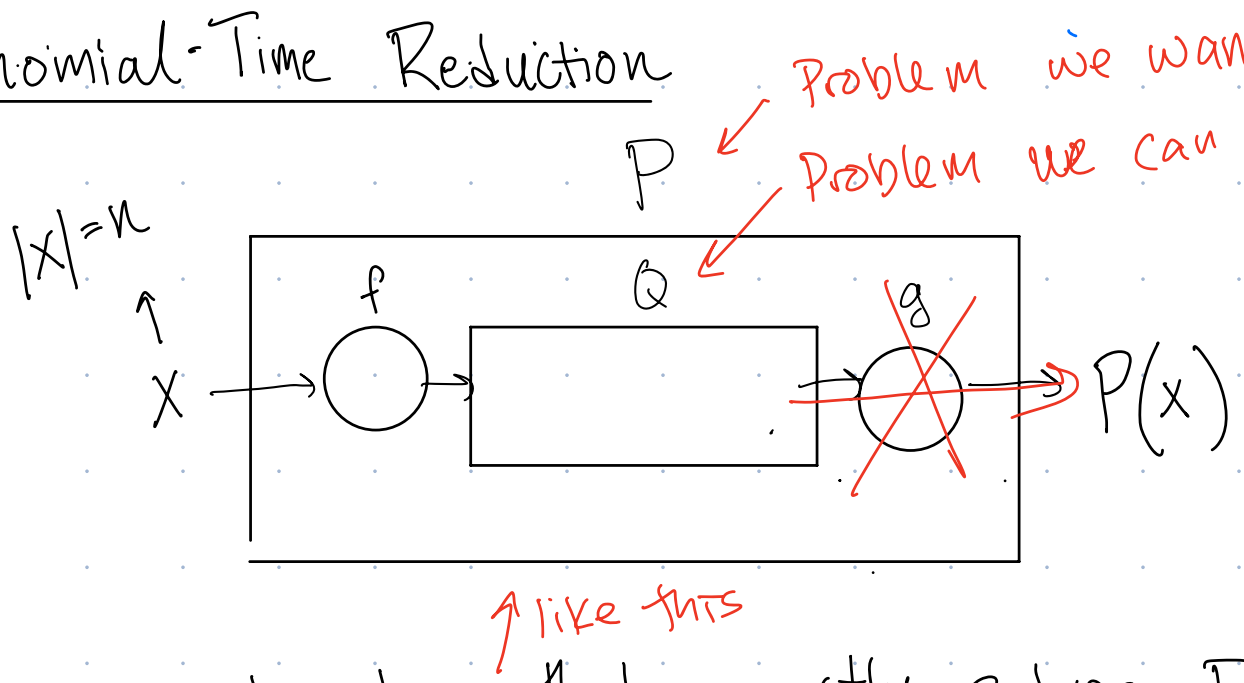
Ethical Matrix (O'Neil + Gunn)

Cell Tower

Stakeholders	Well-Being Harm? Benefit? ↑	Autonomy Choice to use? Are users informed enough to understand meaningfully take responsibility for use? ↑	Justice Alg prioritizes certain groups? Unfair treatment of different groups? Access to Tech/Alg?
Within 2 range of 2 towers	Data is slow On phones less	Move? Change Service provider?	Unfair
Urban users	Faster ↕ reverse Rotted brains	"	↕ reverse
Cell companies			
Large companies who use data			

Rural Users		"	
Real estate, potential home buyers, renters	More business in urban areas		

Polynomial-Time Reduction



Cell Tower $\leq_p$ MWIS

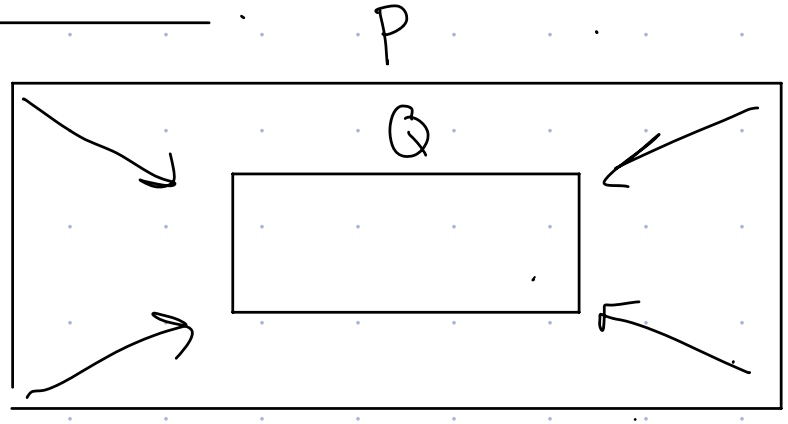
MWIS Line $\leq_p$ MWIS

def: If can create box that correctly solves P, and the runtime of f is O(poly(n)), then "P is polynomial time reducible to Q," denoted $P \leq_p Q$

↑ polynomial time

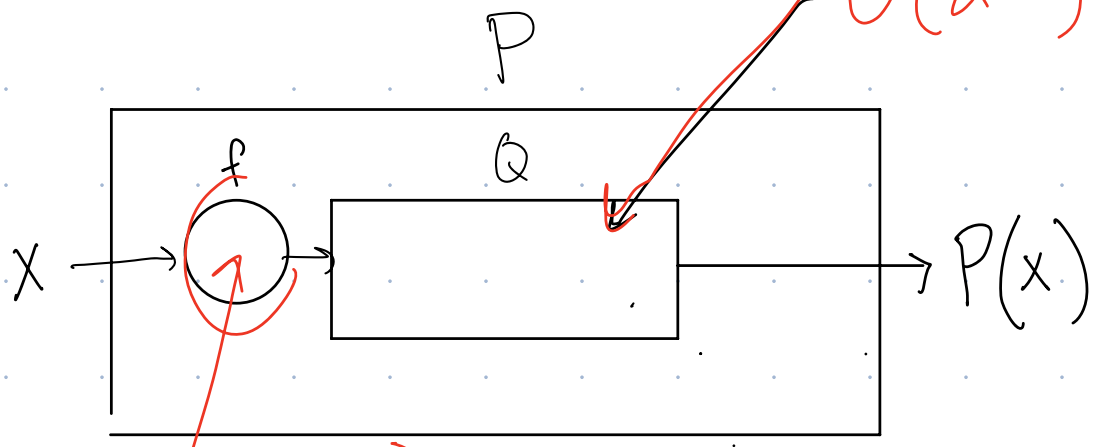
- Alg for Q is more powerful than alg for P
- Runtime of P is at most runtime of Q (ignore polynomial factors)

"P reduces to Q"



Why Polytime Reduction?

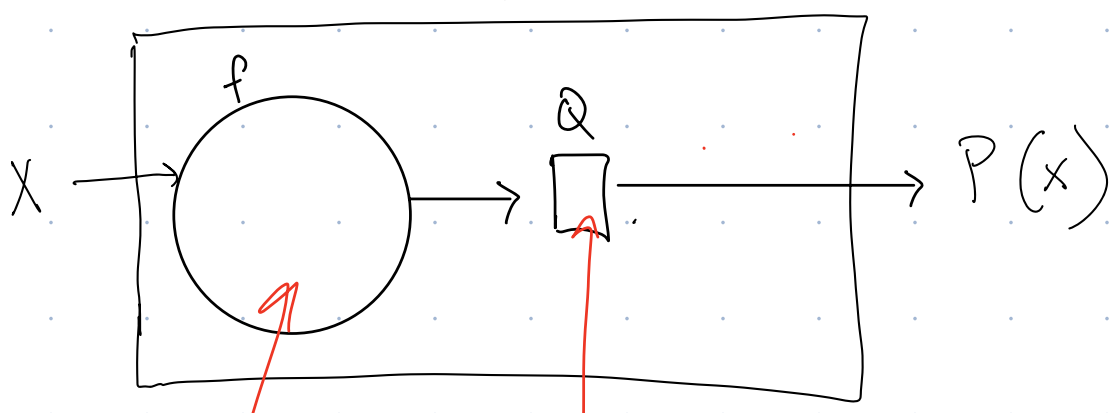
Want



$\Theta(2^n)$

$O(poly(n))$

What If:



$O(2^n)$

$O(n)$

Why think about reductions?

- Practical: If have an alg for Q, can use it for P
- Conceptual: Gives us a way to compare the difficulty of problems, resources needed to solve problems