

DIVIDE + CONQUER: CLOSEST POINTS

Learning Goals

- Analyze runtime of D&C alg [DC1]
- Create a D&C alg [DC2]
- Benchmark algorithm with brute force / easier problem
- Build intuition with easier problems
- Analyze ethics of alg using ethical matrix [Eth 1]

Announcements

- CS Seminar: Fri @ 12:20. 75 Shannon 102 J-Term research
- Exam: 20 min, DC1, DC2, No cheat sheet.
- Ethical matrix "correctness"

CS302 - Short In-Class Exam

Put your answers entirely in the boxes corresponding to that question. If you need additional space, put a note *within* the corresponding box saying that the work continues on scratch paper, and clearly label any additional pages you submit with the problem number and your name.

This exam should be completed on your own.

Please write and sign the honor code in the box. (I have neither given nor received unauthorized aid on this assessment.)

Possibly helpful:

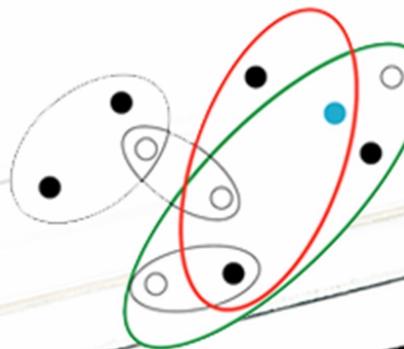
The tree formula is as follows. If

$$T(n) = \begin{cases} O(1) & \text{if } n \leq c \\ aT(n/b) + O(n^d) & \text{else} \end{cases} \quad (1)$$

for constants a , b , c , and d , read off the values for a , b , and d from your recurrence relation, and use the following expressions to determine the big-Oh behavior of T :

$$T(n) = \begin{cases} O(n^d \log_b n) & \text{if } a = b^d \\ O(n^d) & \text{if } a < b^d \\ O(n^{\log_b a}) & \text{if } a > b^d \end{cases} \quad (2)$$

COMPUTER SCIENCE



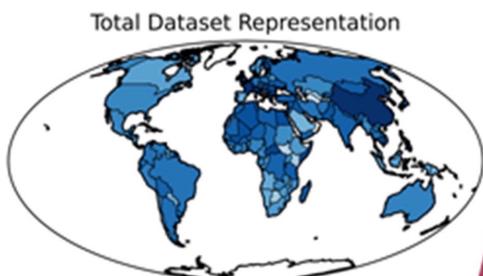
J-Term

Research Talks

Frannie Cataldo
Community Detection in Edge-Copying Hypergraphs

Elise Scott
A discussion of Global and Multilingual
Bias in Large Language Models

75 Shannon St 102 - 2/20/2026 @ 12:00



10⁰ 10¹
Number of Individuals

SCIENCE!
KNOWLEDGE!
INDIAN FOOD!

Warm-ups

• What are the meanings of

Well-Being
Harm / Benefit

Informed?
Autonomy
Choice?

Fairness?

Justice $\epsilon > 0$

• Why can we replace $O(n \log n)$ with $O(n^{1+\epsilon})$?
And what should ϵ be? $\epsilon = 1, \epsilon = 2$?

\downarrow
 $O(n^{1.01})$

\downarrow
 $O(n^2)$

\downarrow
 $O(n^3)$

Ethical Matrix "correctness"? Used in practice?

When finished?

Warm-Up:

DC-CP-ID (P) (Divide + Conquer)

B.C.

If $n \leq 3$

Return Brute Force (P)

$\} O(1)$

Sort (P)

$\cdot \text{min}_1 \leftarrow \text{DC-CP-ID}(P[1: \lfloor \frac{n}{2} \rfloor])$

$\cdot \text{min}_2 \leftarrow \text{DC-CP-ID}(P[\lfloor \frac{n}{2} \rfloor + 1: n])$

$\cdot \text{min}_3 \leftarrow P[\lfloor \frac{n}{2} \rfloor + 1] - P[\lfloor \frac{n}{2} \rfloor]$

$\cdot \text{return } \text{min} \{ \text{min}_1, \text{min}_2, \text{min}_3 \}$

← Review! ~~Runtime Recurrence?~~

Why Brute Force?

BruteForce 2D (P) (check every pair)

Input: array P of size n

$\cdot \text{min} \leftarrow \infty$

$\cdot \text{for } i \leftarrow 1 \text{ to } n-1:$

$\cdot \text{for } j \leftarrow i+1 \text{ to } n:$

$\cdot \text{if } \text{dist}(p_i, p_j) < \text{min}, \text{ then } \text{min} \leftarrow \text{dist}(p_i, p_j)$

$\cdot \text{return } \text{min}$

$O(n) = O(1) \quad O(n^2) =$

• Rule for base case number (e.g. $n \leq 3$)?

• Which is better in pseudocode:
 $A[1: n]$ or $A[0: n-1]$

Valid?	Divide?
	$n=4$ ✓
	$n=3$ X
	$n=2$ X
	$n=1$ X

Warm-Up Problems

$\&\&$ and
 \wedge

Which is better in pseudocode:

A) $x \leftarrow 10$
if $x = 5$, then do
| ...

B) $x = 10$
if $(x == 5)$,
{ ... }

What is the big-Oh of $10x^2 + 5x^3 \log_{10} x$?

A) $O(x^2)$ B) $O(x^3)$ C) $O(x^3 \log_{10} x)$ D) $O(x^3 \log x)$

When do you use $T(\cdot)$ to characterize runtime of a line of pseudocode?

Why is final result of tree formula $O(\cdot)$?

Problems where brute force runtime = sophisticated runtime?
(You've heard their name!)

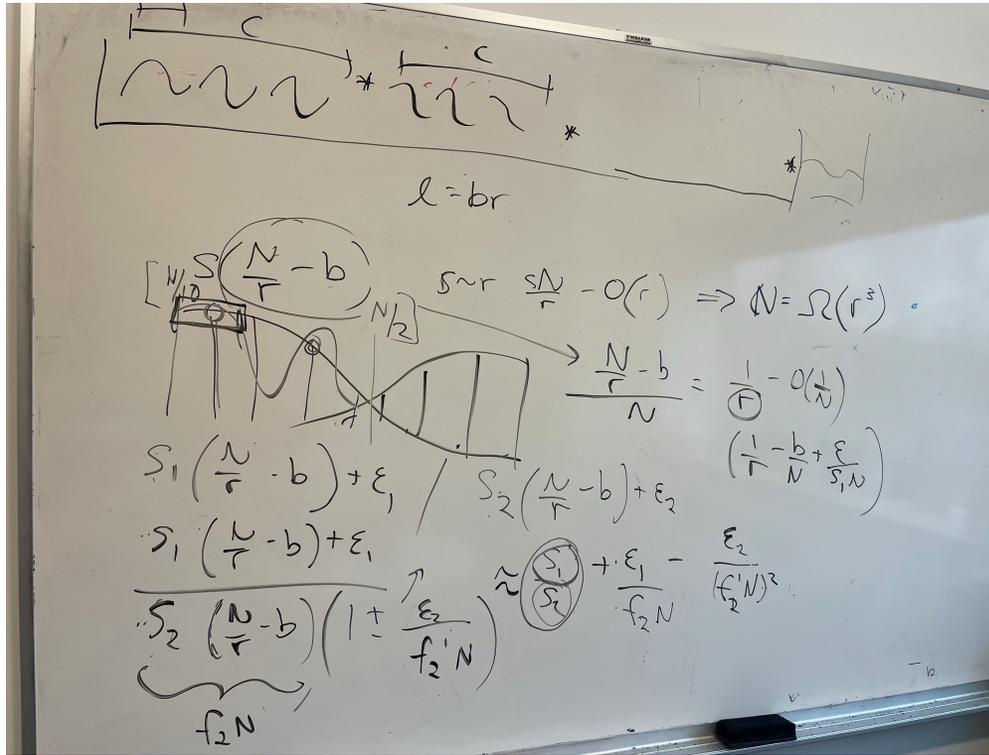
Can you prove you've found most efficient alg?

Denies

Syllabus Q's

A Little More About Me:

1/13



1/11



12/4

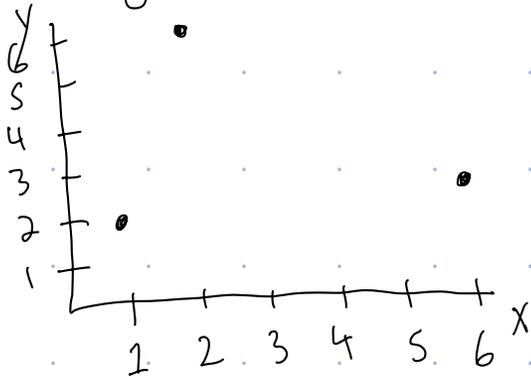


Closest Points Problem

Input: Array of 2-D points:

$$P = \boxed{(1, 2) \mid (2, 6) \mid (6, 3) \mid \dots}$$

$$|P| = n$$



Output: Distance b/t 2 closest points

$$\hookrightarrow d(p_1, p_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Applications:

- air traffic control
- robotics
- stereo \rightarrow 3D

Algorithms + Ethics

Algorithm is essentially a mathematical object.

But once it gets implemented for a particular task, has ethical implications.

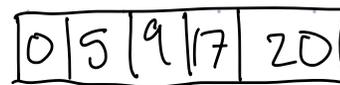
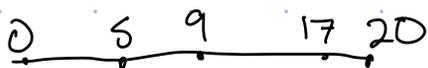
Closest Points 2D

Before designing a sophisticated algorithm, try to benchmark

- Want better than "Brute Force"

- Can't do better than 1-D

Input: $P = [5, 9, 0, 20, 17]$



3. Runtime

BruteForce 2D(P) (check every pair) [Input: array P of size n]

- $\text{min} \leftarrow \infty$ $O(1)$
- for $i \leftarrow 1$ to $n-1$: $\leftarrow O(n)$
 - | for $j \leftarrow i+1$ to n : $\leftarrow O(n)$
 - | | if $\text{dist}(p_i, p_j) < \text{min}$, then $\text{min} \leftarrow \text{dist}(p_i, p_j)$ $\leftarrow O(1)$
- return min $\leftarrow O(1)$

$O(n^2)$

$\left. \left. \left. O(1) \right\} O(n) \right\} O(n^2) \right.$

2.

↳

Closest Pts 1D(P) (For loop)

- Sort(P) $\leftarrow O(n \log n)$
- $\text{min} \leftarrow \infty$ $O(1)$
- for $i \leftarrow 1$ to $n-1$: $\leftarrow O(n)$
 - | if $\text{dist}(p_i, p_{i+1}) < \text{min}$, then $\text{min} \leftarrow \text{dist}(p_i, p_{i+1})$ $\leftarrow O(1)$
- return min $O(1)$

$O(n \log n)$

$\left. \left. \left. O(1) \right\} O(n) \right. \right.$

1.

go/cs302groups

DC-CP-ID (P) (Divide + Conquer)

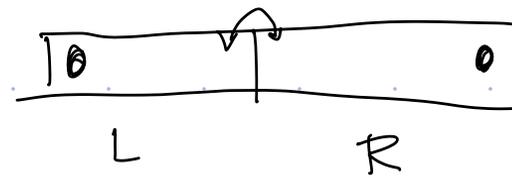
B.C.

If $n \leq 3$

| Return Brute Force (P)

Sort (P)

- $min_1 \leftarrow DC-CP-ID(P[1 : \lfloor \frac{n}{2} \rfloor])$
- $min_2 \leftarrow DC-CP-ID(P[\lfloor \frac{n}{2} \rfloor + 1 : n])$
- $min_3 \leftarrow P[\lfloor \frac{n}{2} \rfloor + 1] - P[\lfloor \frac{n}{2} \rfloor]$
- return $\min \{ min_1, min_2, min_3 \}$



[D&C 1,2]

Input : Integer array A of length n

Output: Sorted array

// Base Case

1 if $n == 1$ then

2 | return A ;

3 end

// Divide and Conquer

4 $A_1 = \text{MergeSort}(A[1 : n/2]);$

5 $A_2 = \text{MergeSort}(A[n/2 + 1 : n]);$

// Combine

Base Case (When too small to divide further?)

Divide input into equal sized pieces +
Recursively solve each piece

How solve whole problem if can solve parts

Base Case!

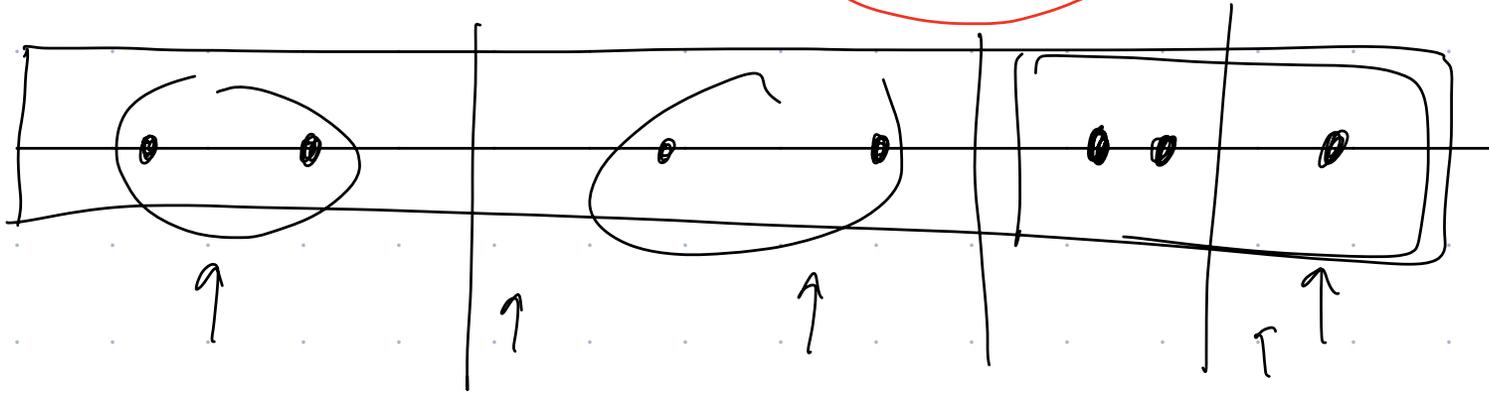
What size set of pts should trigger base case?

A) 0

B) ≤ 1

C) ≤ 2

D) ≤ 3



DC-CP-ID(P) (Divide + Conquer)

B.C.

If $n \leq 3$

Return Brute Force (P)

Sort(P)

- $\text{min}_1 \leftarrow \text{DC-CP-ID}(P[1 : \lfloor \frac{n}{2} \rfloor])$
- $\text{min}_2 \leftarrow \text{DC-CP-ID}(P[\lfloor \frac{n}{2} \rfloor + 1 : n])$
- $\text{min}_3 \leftarrow P[\lfloor \frac{n}{2} \rfloor + 1] - P[\lfloor \frac{n}{2} \rfloor]$
- return $\min \{ \text{min}_1, \text{min}_2, \text{min}_3 \}$

Runtime?

$O(n \log n)$

$T(\frac{n}{2}) + O(1)$

$T(\frac{n}{2}) + O(1)$

$O(1)$

$O(1)$

no constant
 $n^d \neq n \log n$

$$T(n) = \begin{cases} O(1) & \text{if } n \leq 3 \\ \underline{2T(\frac{n}{2})} + O(n \log n) & \text{if } n \geq 4 \end{cases}$$

$O(n^{1+\epsilon})$

$$a = 2, \quad b = 2, \quad c = 3, \quad d = 1 + \epsilon$$

$$n \log n = O(n^{1+\epsilon})$$

$$\epsilon = 10^{-10}$$

ϵ is any constant

$$\epsilon > 0$$

Strategy to improve runtime:

1. find slow parts in recursive call
2. Remove from recursion or improve

$$P \leftarrow \text{Sort}(P)$$
$$\text{return DC-CP-ID-presort}(P)$$

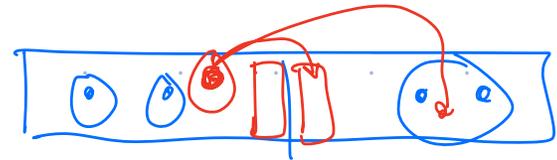
DC-CP-ID-preSort(P)

B.C.

If $n \leq 3$

Return Brute Force (P)

~~Sort(P)~~

$$\text{min}_1 \leftarrow \text{DC-CP-ID}(P[1: \lfloor \frac{n}{2} \rfloor])$$
$$\text{min}_2 \leftarrow \text{DC-CP-ID}(P[\lfloor \frac{n}{2} \rfloor + 1: n])$$
$$\text{min}_3 \leftarrow P[\lfloor \frac{n}{2} \rfloor + 1] - P[\lfloor \frac{n}{2} \rfloor]$$
$$\text{return } \min \{ \text{min}_1, \text{min}_2, \text{min}_3 \}$$


Ethical Matrix (O'Neil + Gunn)

Group Challenge

Air Traffic Control Improvement

Harm?
Benefit?

Choice?
Informed enough to meaningfully take responsibility for interaction/choice

Unfair treatment of different subgroups

Stakeholders	Well-Being	Autonomy	Justice
Passengers	Flight Times ✓	No choice if most fly?	\$ / commercial \$ / private
Airlines	Safety ✓ Prices ✓ Piece of Mnd ~	Choice to Fly? \$	
Manufacturers	Crowded Traffic X		
Pilots			
Insurance Cos			
Shareholders			
ATC			

Airport
Employees

Ppl. live
near airports

Tourism Economies

Tourism Companies

Wildlife

Everybody
(Environment)

Ethical matrix does not tell you what to do!

Tool for thinking about consequences, both + and -.

→

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