Learning Goals

- ✓ Describe P and NP informally, and why these ideas are important
- Define NP
- Prove a problem is in NP

Announcements

Exit Tickets

Types of Problems

1 million dollars!

Easy

(Polynomial time)

- Search
- Sort
- Matrix Mult.
- Find closest pair

Puzzles

Sudoku

Crossword

} no known polynomial alg.

Factoring large numbers

$n \stackrel{?}{=} a \times b$

Quantum

n prime?

regular

Hard

Chess: what is the next best move

Halting Problem

Can mathematically characterize Easy / Puzzle

P (Polynomial Time)

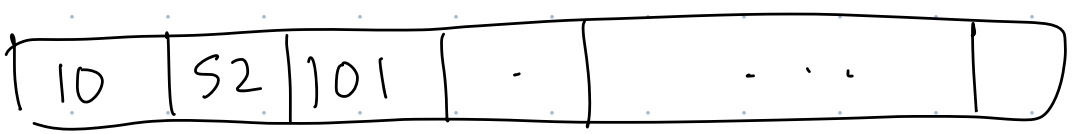
Informal: A problem is in P if it can be solved in polynomial time.

NP (Non-deterministic polynomial time)

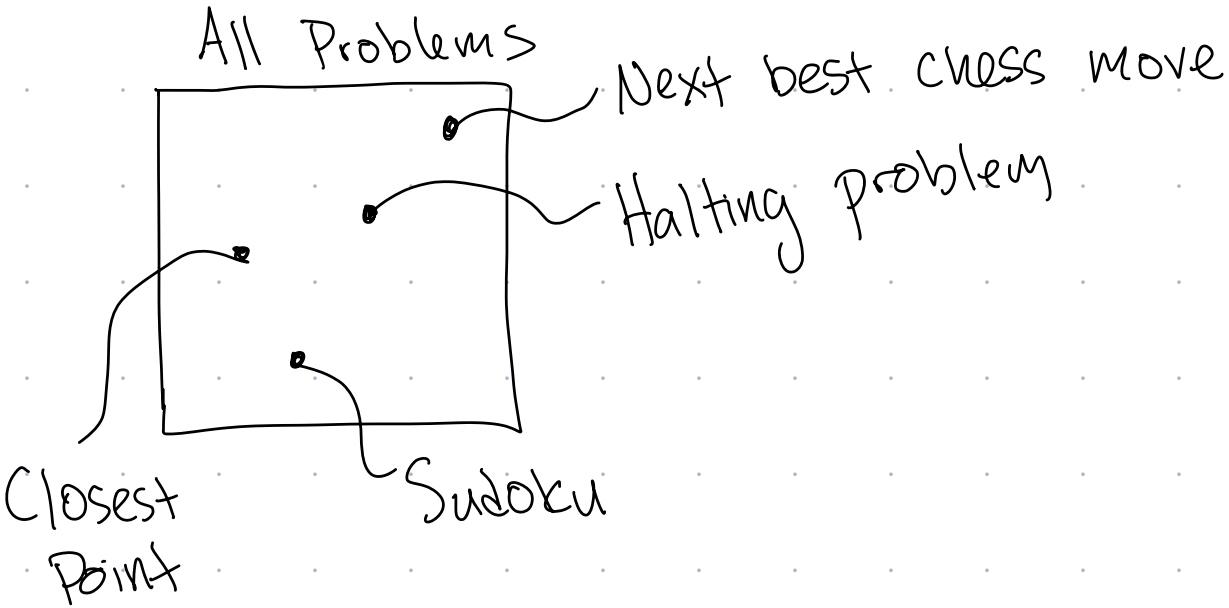
Informal: A problem is in NP if a possible solution can be checked/verified in polynomial time.

Polynomial Time

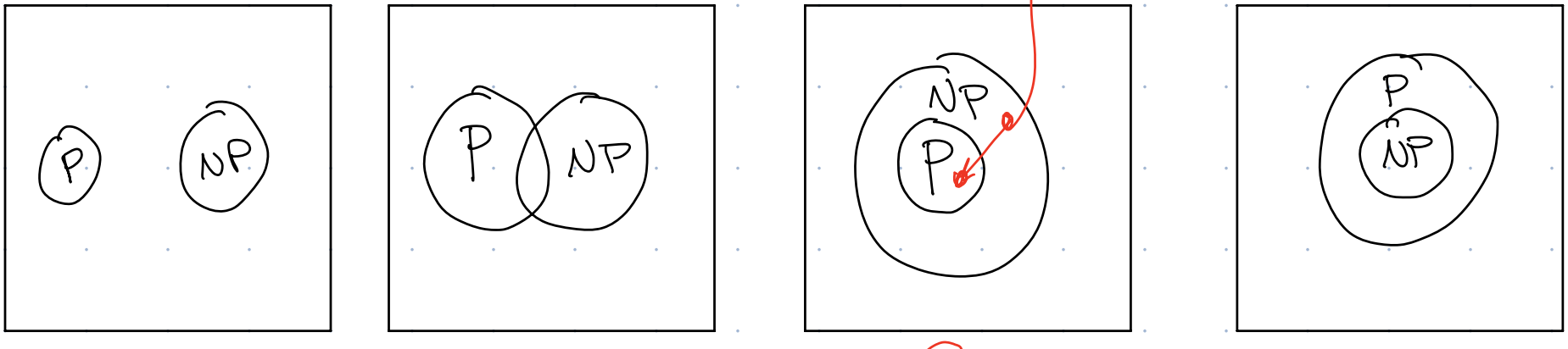
- $O(n^c)$ time for a constant c, where n = # of bits used to describe input.



size: m
largest int: K
$n = m \log_2 K$



Which picture is correct?



NP problems are YES-NO: $Q(x) = \begin{cases} \text{Yes} \\ \text{No} \end{cases}$ (Q = name of problem)

example of NP problem: 3SAT → $3SAT(x) = \text{Yes}$

3SAT: X is a Yes instance if it describes a Boolean formula that is an AND of ORs, each clause has at most 3 literals and there is a satisfying assignment.

↳ e.g. $z_1 = T,\ z_2 = F \dots$ etc. s.t. X is True

Instance: (OR, AND)

$$X = \underbrace{(z_1 \vee z_2 \vee \neg z_3)}_{\text{clause}} \wedge (\neg z_1 \vee \neg z_3 \vee z_4) \wedge (z_2 \vee z_4) \wedge \dots$$

Otherwise X is a No instance

$z_1, z_2, \dots z_n \Leftrightarrow$ variables

$z_1, \neg z_1, z_2, \neg z_2 \dots \Leftrightarrow$ literals

Is $3SAT \in NP$?

Questions to Ask Yourself to Prove $Q \in NP$

① What info would convince me that x is a yes for Q

$$(z_1 = T,\ z_2 = F,\ z_3 = F, \ldots) \leftarrow y$$

② If given info from ① how could I <u>quickly</u> check if x is a yes for Q

* You do not have to find y
* Only need to verify y is a solution

Proof that 3SAT ∈ NP

algorithm, instance, potential solution

- Let $M(x, y)$ be the algorithm that
  1. Check that x is an AND of OR with at most 3 literals in each clause
  2. Check y is an assignment of T, F to each variable
  3. Check that assignment in y makes x true

  And outputs 1 if all checks pass, and 0 otherwise
- $M(x, y)$ runs in
  1. Read through x. Size of x is $O(|x|)$ → Time $O(|x|)$
  2.
  3.