Learning Goals

- Define NP-complete and NP-Hard Problems and describe their importance
- Describe parts of NP-Complete Proof
- Practice proving a problem is NP complete (Hamiltonian Path)

Types of Problems

Easy
(Polynomial time)

- Search
- Sort
- Multiplication
- Closest Points
- Greedy Scheduling
- MWIS on a line
- Matrix Mult.

Puzzles / NP

Crossword
Sudoku
Delivery rt ≤ 100 miles
Protein Folding
Factor larger numbers
Primality Testing

Quantum
Reg.

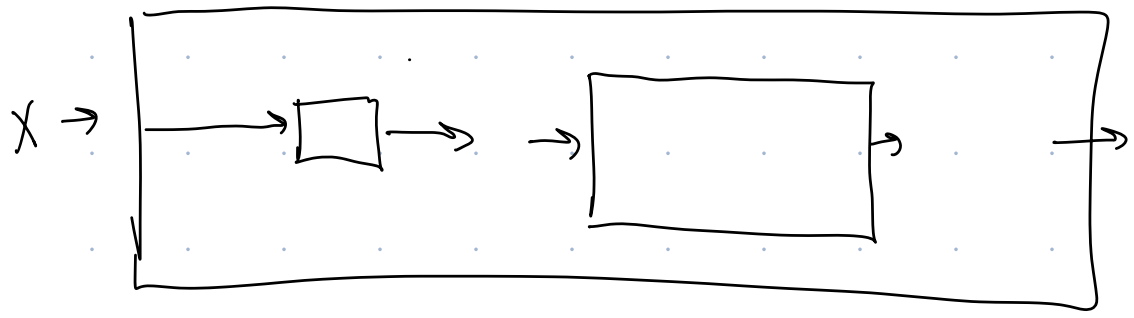
Question: How do we identify the hardest problems in NP?

→ Empirical:

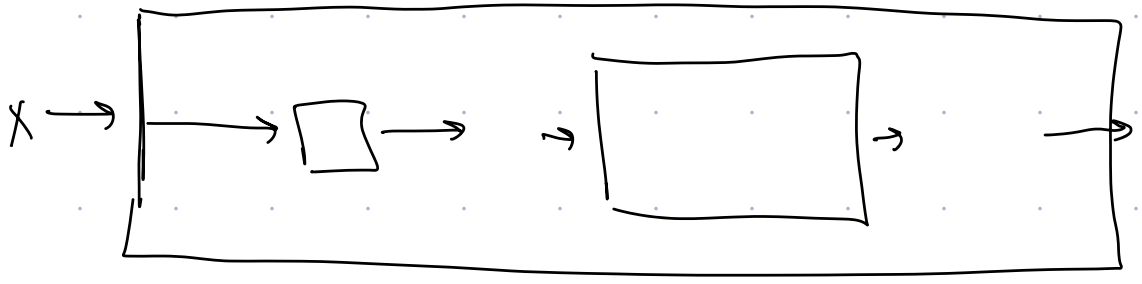
→ Analytical:

NP-Hard

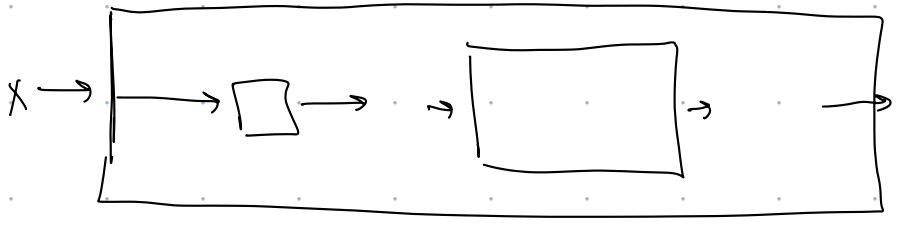
Ex:



Also

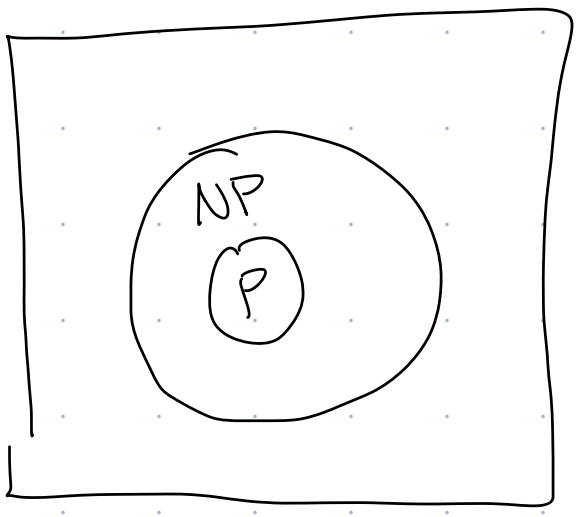


Also



→

BUT



def:

Fact 1:

Lemma 1:

Theorem :

Pf :

Formal Definition of Polytime Reduction

def:

-

-

Lemma: 3SAT $\leq_p$ Ham-Path

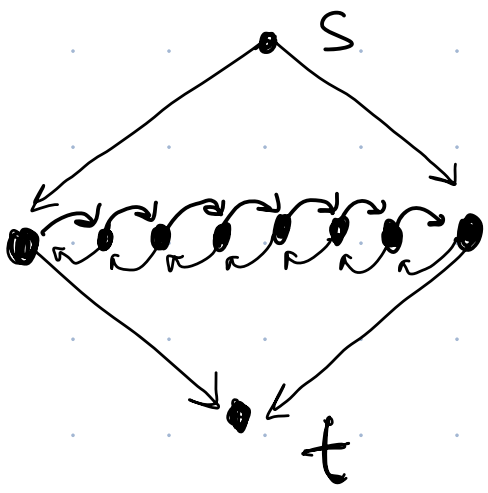
Strategy: 1.

2.

3.

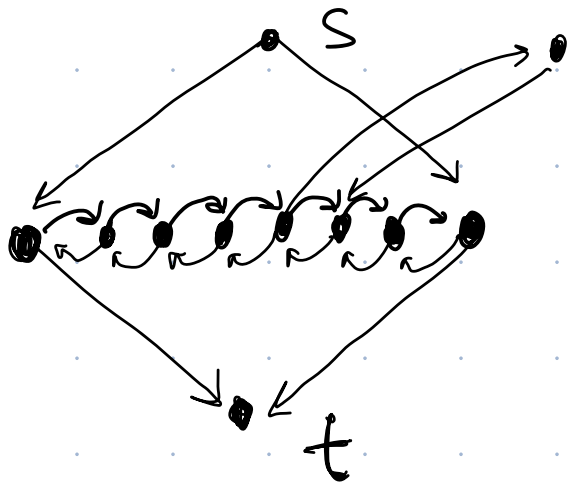
1.

How many Hamiltonian Paths are in this graph?

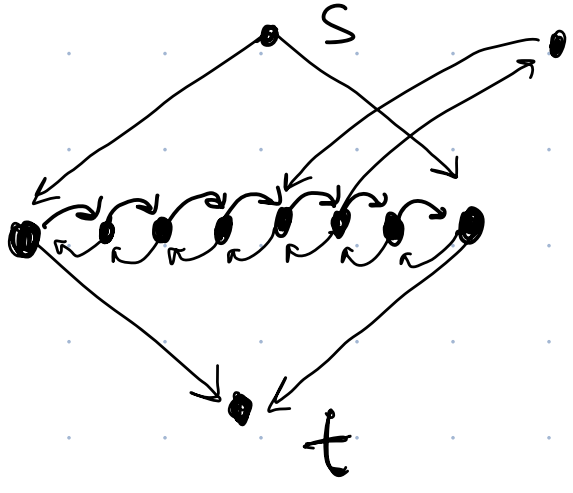


A. 2 B. 3 C. 49 D. $\binom{7}{2}$

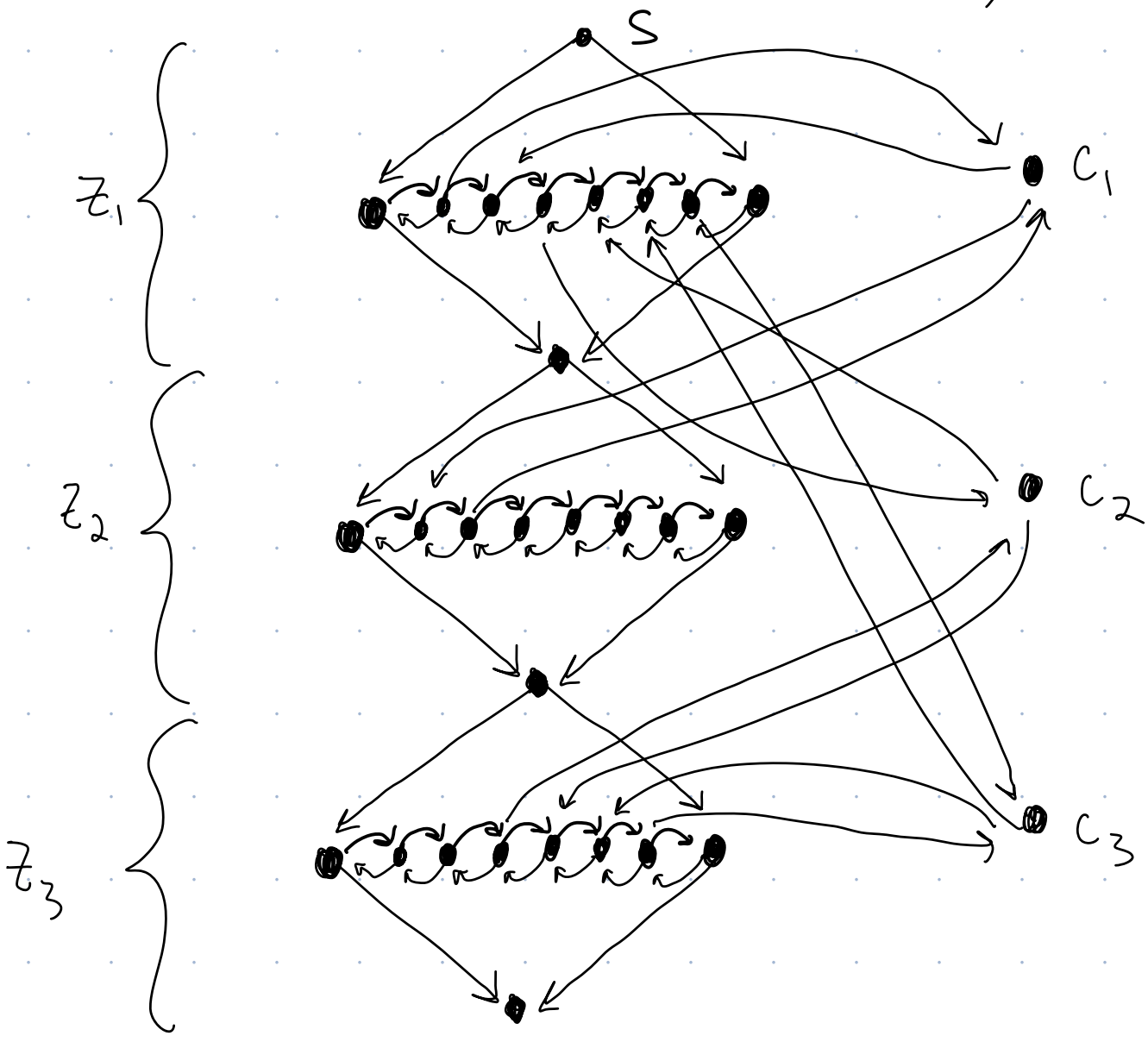
How many Hamiltonian Paths are in this graph?



A. 0 B. 1 C. 2 D. 3



$X = (z_1 \vee \neg z_2) \wedge (z_1 \vee z_3) \wedge (\neg z_1 \vee \neg z_3)$



Group Work

1. Encode $(z_1) \wedge (\neg z_1 \vee z_2) \wedge (\neg z_1 \vee \neg z_2)$ into Ham-Path instance. Show get a No Instance.
2. Runtime of $f_{3SAT \to HAM\text{-}PATH}$? (Create adj matrix for graph)
3. $3SAT(x) = Yes$ iff $HAMPATH(f_{3SAT\text{-}HAMPATH}(x)) = Yes$

1. $(z_1) \wedge (\neg z_1 \vee z_2) \wedge (\neg z_1 \vee \neg z_2)$

2.

3.

$3SAT(x) = Yes$ iff $HAMPATH(f_{3SAT-HAMPATH}(x)) = Yes$

$\Rightarrow$

Note:

Lemma 1: If $Q \in$ NP-Hard and $Q \leq_p R$ then $R \in$ NP-Hard.

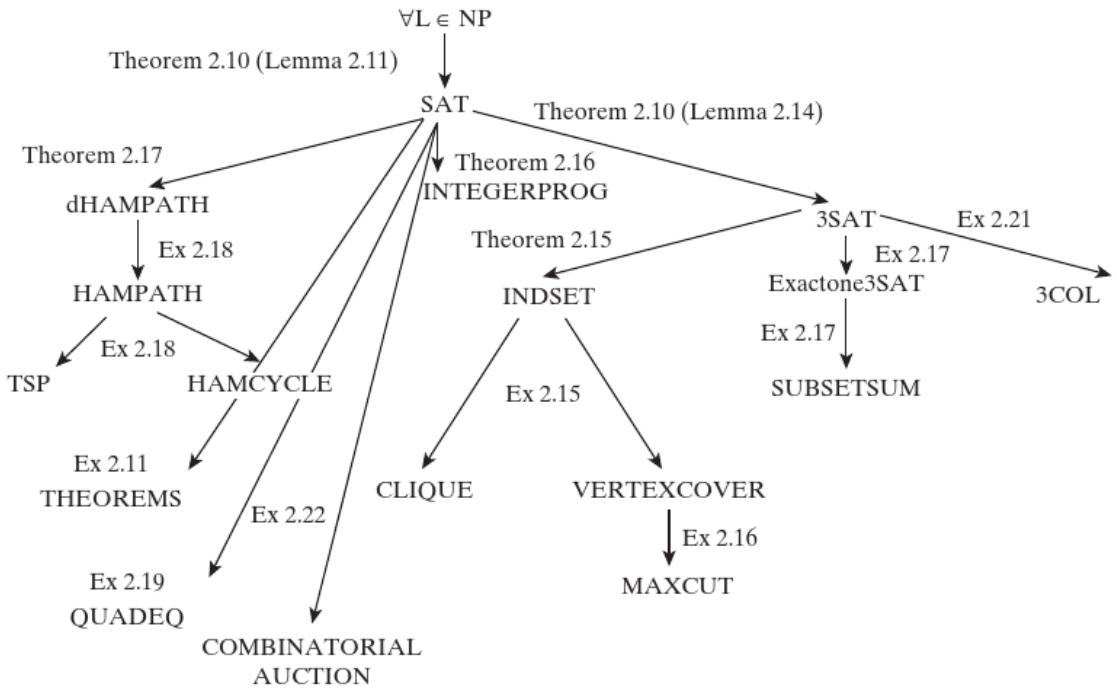


Figure 2.4. Web of reductions between the **NP**-completeness problems described in this chapter and the exercises. Thousands more are known.

(Arora + Boaz, Computational Complexity)