Learning Goals V. Define NP-complete and NP-Hard Problems and describe their Importance V. Describe parts of NP-complete Proof Practice proving a problem is NP complete (Hamiltonian Path)

·Distributed Bellman-Ford

Types of Problems Easy Puzzles/NP Crossword (Polynomial time) Sudoku · Search E 100 miles Delivery rt · Sort · Multiplication Protein Folding · Closest 'Points Factor larger numbers · Greedy Scheduling Primality Testing · MWIS on a line · Matrix Mult. Question How do we identify the hardest problems in NP? -> Empirical: Keep trying to find alg... but can't ... HARD -> Analytical' Prove a problem is hard . [Possible!]

NP-Hard def: A problem QENP-Hard if for every problem RENP, REpQ. Ex: Halting Problem E NP-Hard Yes, No les, No 5-SAT Boolean X -> -> T Halt. Prob. form. 1 >HP(x') -> 3-SAT(x)  $\rightarrow$   $\times$ runs in poly program MWIS (general graph /YES-NO) Alsobraph FMWIS->HP Also program  $x \rightarrow f_{R} \rightarrow HP$  $X \rightarrow X' \rightarrow HP(x') \rightarrow HP(x')$ R(X)

NP-Hard problem are Marder/require more resources > than NP problems, b/c if could solve, then would have power to solve any NP problem. NP-Hard BUT Sampling Halting ProbleM BOR NR def: & NP-Hard & NP-Complete Hardest problems in NP: (Traveling Salesperson, MWIS, Negative cycle avoiding shortest path) def: QENP-Complete  $\widehat{\mathbb{Q}}$ ENP and BENP-Hard

Formal Definition of Polytime Reduction def: R≤pQ ("Ris polytime reducible to Q") if / ] f<sub>R→G</sub>: 20,13\* → 20,13\* s.t. " ] constant CRAQ s.t. runtime of fRAQ on input x is  $O(|x|^{c_{R \to Q}})$  (Polytime) •  $\forall x \in g_{0,1}g^*$ ,  $R(x) = \forall es iff Q(f_{R,Q}(x)) = \forall es$ (Correctly convert input)

Lemma: 3SAT = p Ham - Path Strategy Describe f35AT > Ham-Path runs in Polytime fosat > Ham-path 2. Show 3. Show X is 35AT-Yes iff f3SAT-Ham-Path(X) Is a Ham-Path-Yes  $\chi = \left(\frac{2}{2}, \sqrt{2} \frac{2}{2}, \sqrt{2} \frac{2}{3}\right) \wedge \left(\frac{2}{2}, \sqrt{2} \frac{2}{3}\right) \rightarrow \left(\frac{2}{3}, \sqrt{2}\right) \rightarrow \left(\frac{2}{3}, \sqrt{2}\right)$