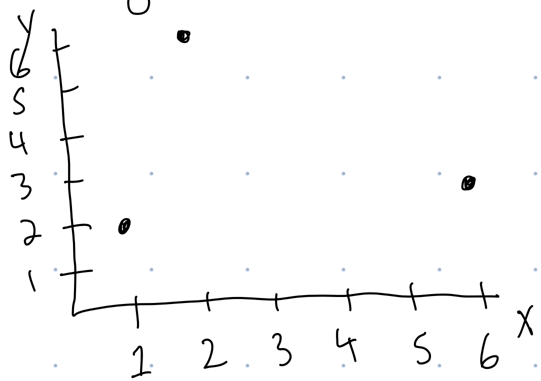
Closest Points Problem

Input: Array of 2-D points: P = | (1,2) | (2,6) | (6,3) | ...



Output: Distance b/t 2 closest points

↳ $d(p_i, p_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$

Assume:
x,y coordinats are unique for each pt

~~| (1,2) | (1,6) |~~

| (1,2) | (2,1) |

Applications:
- air traffic control
- robotics
- stereo imaging

Algorithms + Ethics

Algorithm is essentially a mathematical object.
But once it gets implemented for a particular task, has ethical implications

Ethical Matrix (O'Neil + Gunn)

Where travel + why?

Air Traffic Control Improvement

Harm? Benefit?

Choice to use? Are users informed enough to understand meaningfully take responsibility for use?

Unfair treatment of different groups?

Stakeholders	Well-Being	Autonomy	Justice
Airplane Passengers	Safety, Cheaper (?) Environment	A - transport, Zoom	Equal access (rich / poor) (geographic)
Airline Cos	\$, safer	No choice (international)	Difference by country, by region
Employees, ATC	Automation → No job Efficiency, less stress	No choice safer	Who is to blame if error
Civilians, Bystanders	Noise pollution, Make war easier, environment, help economy/tourists	No choice, opt out (vote)	(Un) Equal impact on environment, near airport more affected

Army			
People of color			
Non-binary folks			

Ethical matrix does not tell you what to do!

Tool for thinking about consequences, both + and −.

→ How can I mitigate negatives? → Modify → Aid

→

Closest Points 2D

Before designing a sophisticated algorithm, try to benchmark

- Want better than "Brute Force"
- Can't do better than 1D

Brute Force (check every pair)

- min ← ∞ $O(1)$
- for i ← 1 to n-1 $O(n)$ } $O(n^2)$
  - for j ← i+1 to n $O(n)$
    - if dist(p_i, p_j) < min, then min ← dist(p_i, p_j) $O(1)$
- return min

$O(n^2)$

Closest Pts 1D

5	9	0	20	17

0 5 9 17 20

0	5	9	17	20

- Sort(P) $O(n\log n)$ +
- min ← ∞ $O(1)$
- for i = 1 to n-1: $O(n)$
  - if dist(p_i, p_{i+1}) < min, then min ← dist(p_i, p_{i+1}) $O(1)$
- return min

- Sort(P) $O(n\log n)$

n

$O(1)$ ← } $O(n)$ } $O(n\log n)$

$O(1)$

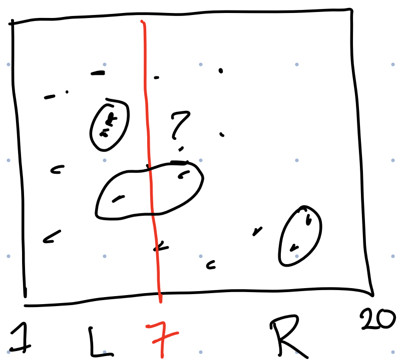
Ethical Matrix: Race? Include the right stakeholders / how many? Weakness? When to use? What to do after fill out?

CloPts(P) (Divide + Conquer 2D Closest Pts)

Base Case : Later

Sort by x

Divide :



(1,5)	(2,2)	(4,0)	(10,20)	(12,3)	(20,6)

L R

midline: 7

Conquer :

$\delta_1 \leftarrow$ CloPts(L)

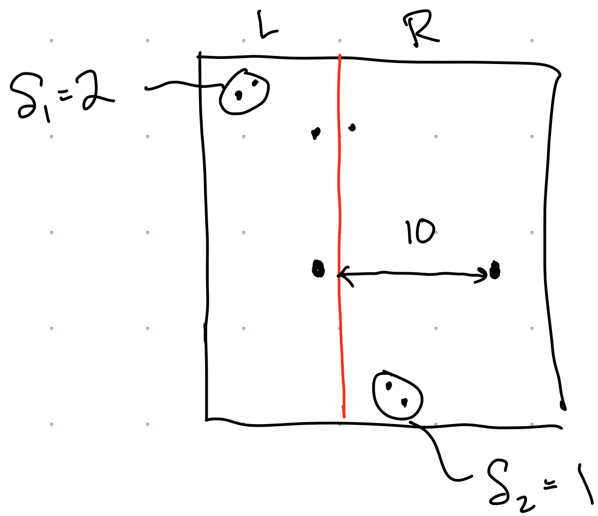
$\delta_2 \leftarrow$ CloPts(R)

$\delta \leftarrow \min(\delta_1, \delta_2)$

Combine

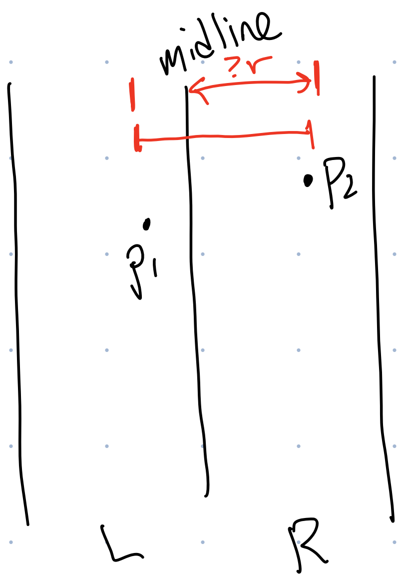
Combine:

Let's think about this:



Lemma: If p_2 in R has difference in x-coord. of more than $r = \delta$ from midline, then for any point $p_1 \in L$, $d(p_1, p_2) > r = \delta$

Proof Sketch: (Basic ideas, but need more English in real proof)



$$d(p_1, p_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$(x_1 - x_2)^2 = \Big(\overset{>r}{\text{diff in x-coord from } x_2 \text{ to midline}} + \underset{>0}{\text{diff '' '' midline to } x_1}\Big)$$

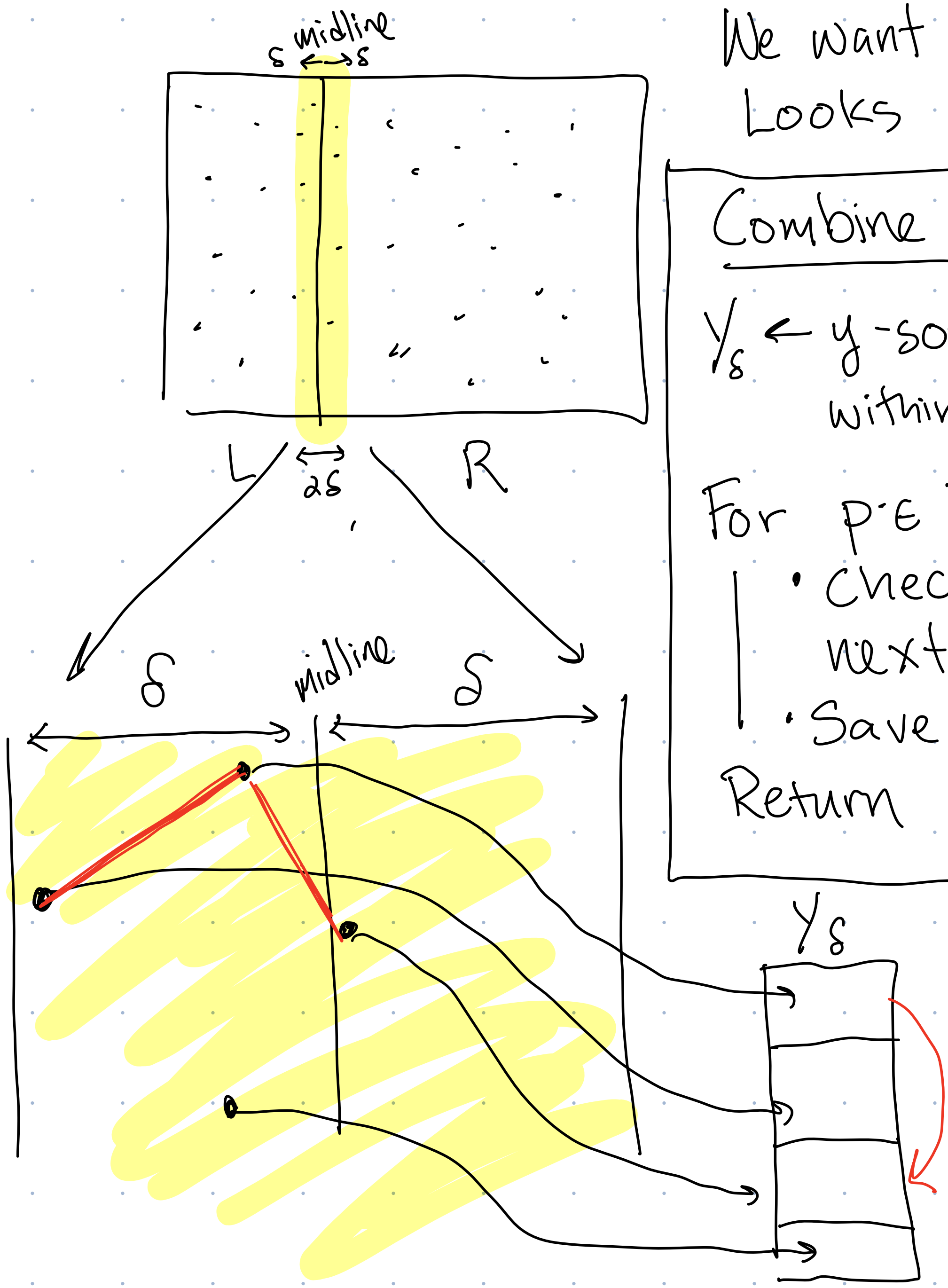
$$> r^2$$

$$(y_1 - y_2)^2 > 0$$

$$\Downarrow$$

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} > \sqrt{r^2}$$

$$d(p_1, p_2) > r$$



We want to check points in this region,
Looks like a line!

Combine Step

$Y_\delta \leftarrow$ y-sorted list of pts in P
within δ of midline

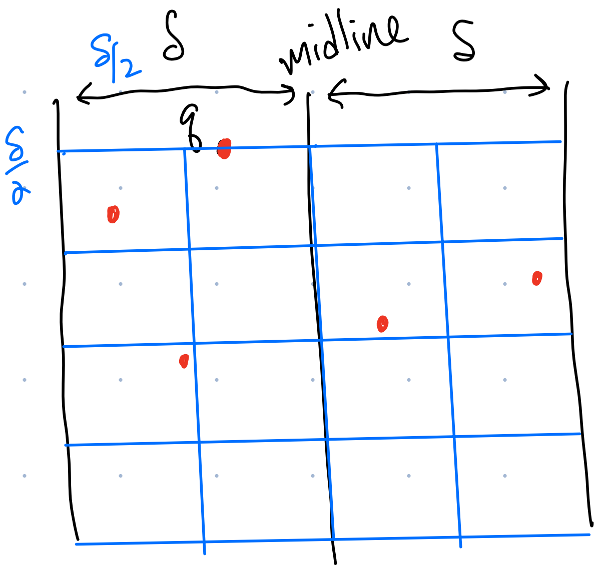
For $p \in Y_\delta$
- Check check distance from p to next <u>2+</u> pts
- Save if smallest found

Return smallest distance found

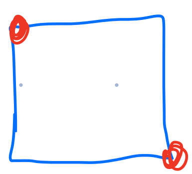
What goes here?

Lemma: Only need to look at next 7 pts in Y_S

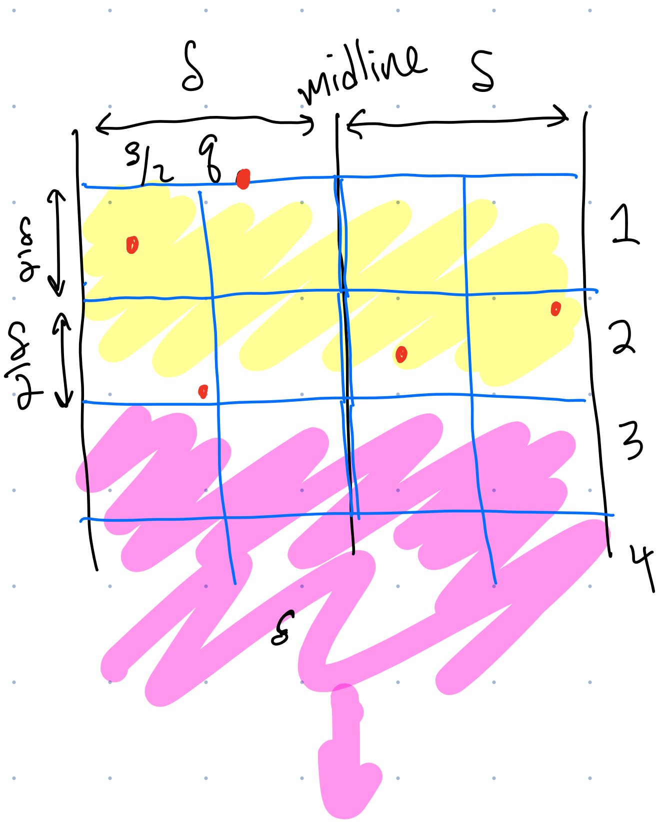
Pf: Imagine dividing region w/in δ of midline into $\frac{\delta}{2} \times \frac{\delta}{2}$ squares starting at current pt (q). Each square can contain at most one point. To see this, for contradiction, suppose there are 2 pts in a square. The pts have largest distance when on opp. corners. In that case, their distance is



$$\sqrt{\left(\frac{\delta}{2}\right)^2 + \left(\frac{\delta}{2}\right)^2} = \sqrt{\frac{\delta^2}{4} + \frac{\delta^2}{4}} = \sqrt{\frac{\delta^2}{2}} = \frac{\delta}{\sqrt{2}} < \delta$$



Each box is in L or R, so any 2 pts in a box must have distance $\geq \delta$, a contradiction



Pts in rows 3+ have distance at least δ from q, b/c diff in y-coordinate is at least δ.

So only 1st 2 rows are relevant

There are at most 7 pts other than q in 1st 2 rows, b/c only 8 box.